Chapter 4

A Threshold Adaptive Cooperative Spectrum Sensing Technique

4.1 Introduction

Opportunistic access to license spectrum needs spectrum sensing by CR user in order to detect the presence of the primary user (PU) signal in licensed spectrum, so that the licensed users [2,13] are protected from harmful interference. Spectrum sensing is a crucial function for successful deployment of a CRN, which faces challenges [2] in terms of sensing efficiency. For instance, the sensing performance of secondary users (SUs) suffers due to the issues arising from spectral diversity [2,16,17,21,22]. Spectral diversity issues like multipath fading, shadowing, hidden terminal problem and receiver uncertainty problem [3,17–20,43] can be overcome by exploiting the spatial diversity [3] of the SUs. Cooperative spectrum sensing [3,17,32,33] has proven to emerge as an effective method to take advantage of spatial diversity of the SUs, where SUs share their individual sensing results [17] among themselves, and thereby eventually improving the sensing performance. In this direction, the design of a cooperation model for CSS is challenging. Cooperative behavior among SUs basically depends on the method of cooperation, which in turn has the impact on detection accuracy or performance. Cooperative gain and cooperation overhead are the two dominant factors that play important roles while choosing a technique to model the cooperation among the SUs [1]. The cooperative gain that can be achieved through the cooperation among SUs dictates the efficiency of the sensing/detection performance of the SUs, which is essential for protection of PUs in CRNs. On the other hand, the cooperation overhead is the measure of cost incurred due to cooperation among the SUs, which in turn is determined by sensing time, delay, energy and the operations involved to perform cooperation activity. Among the two widely used approaches namely *parallel fusion (PF) model* and *game theoretic model* [23,75,76], the game theoretic modeling offers capability to incorporate dynamic change in behavior and can offer better result in terms of detection performance and throughput [3].

The cooperative spectrum sensing schemes studied in the literature [3, 17, 32–34, 45, 46, 61–68, 77–79] mostly address to overcome the issues of multipath fading, shadowing and hidden node problem by modeling the framework for cooperation and using the techniques to perform data reporting and data fusion operations. The CSS technique by Saad *et al.* [34] proposes a distributed game theoretic collaboration strategies for the SUs using non-transferable coalitional game. The aim of their technique is to study the impact on the network topology of the inherent trade off that exists between the collaborative spectrum sensing gains in terms of detection probability and the cooperation costs in terms of false alarm probability. They also study the stability of the resulting coalition structure and show that a maximum coalition size exists for the proposed utility model. However, the CSS technique by Saad *et al.* [34] and most of the CSS techniques in the literature assume a fixed Energy Detection (ED)[15] threshold and the same probability of false alarm for all the secondary users (SUs). These assumptions affect detection performance. The accuracy of the detection of CSS can be improved by deciding the ED sensing threshold of the SUs adaptively. Depending on their location/position, SUs can adopt their ED sensing threshold values independently before they perform CSS. Spectral diversity plays an important role in sensing performance, since it is dependent on the locations/positions of the SUs. In practice, the choice of sensing threshold is affected by the spectral diversity. Moreover, with the increase in number of SUs in coalition, the sensing threshold requires to be as much accurate as possible. To the best of our knowledge, no CSS technique with consideration of adaptive ED sensing threshold for local sensing operations of SUs has been reported in the literature.

In this chapter, we formulate the problem of CSS as a non-transferable coalition game [23, 24, 76], where SUs organize themselves into disjoint partitions (also called coalitions) based on optimization of a utility function. The utility function of each coalition takes into account both detection accuracy and cooperation overhead. The utility/payoff function of the game collects the total revenue to be optimized, while incorporating the distance adaptive individual ED sensing thresholds, the individual probability of false alarms of the SUs. The utility/payoff function also takes into account the costs due to reporting error and reporting energy indirectly through selecting the head of a coalition. SUs in the coalitions resolve the spectral diversity problems due to their location diversity by utilizing the ED sensing threshold adaptively and thus accurately estimate the PU signal power. The revenue is collected in terms of probability of detection by means of minimizing the probabilities of false alarm and miss detection of a coalition. The game eventually establishes that with a given value of maximum tolerable probability of false alarm the optimal size of a coalition is decided. With the optimal size of the coalition the game establishes that the utility (that is, the probability of detection) of a coalition improves even for higher values of ED sensing thresholds. The cost of reporting energy is minimized by adopting a policy for selecting the head of a coalition while playing the game. A scheme for dynamic selection of head of a coalition is proposed, which is based on selecting an SU as head having its position at the minimum average distance from all other SUs. The distributed threshold adaptive CSS (TACSS) algorithm finds the optimal partition that maximizes the overall utility of all the coalitions in the network. The condition to achieve the coalition stability is established through mathematical analysis. Further, simulation based study is carried out to demonstrate how SUs can organized themselves into stable partitions with optimal utility and convergence property of the proposed scheme.

The rest of this chapter is organized as follows. Section 4.2 formally defines the problem. The assumptions taken and symbols and notations used throughout this chapter is also presented. The system model is presented in section 4.3. Section 4.4 presents the proposed game theoretic model for CSS and the distributed threshold adaptive CSS algorithm for its realization. In that section, the optimization of cost parameters and the head selection scheme are described. The stability of coalition is also studied and evaluated in that section. Section 4.5 evaluates the performance of the proposed model through simulation based studies. Finally, section 4.6 concludes this chapter.

4.2 Problem Statement

The main objective is to model the cooperative spectrum sensing for CR network incorporating the interaction behavior of SUs for higher detection accuracy, while mitigating the problem due to spatial diversity of the SUs and minimizing the cooperation overhead and hence to improve probability of detection. The cooperation overheads considered are the reporting error and reporting energy.

4.2.1 Assumptions

- A time slotted system is considered [2], where SUs and the PU synchronize themselves with a common clock. The SUs synchronize their spectrum access to the time-slot clock by, e.g., listening to the timing pilot on a broadcast control channel of the PU network [2, 69]
- Distances of SUs (SU transmitters) from the PU transmitter is known
- Energy Detector(ED) [3, 15, 70] based approach is used for local/individual spectrum sensing by the SUs because of its low computational and implementation complexity
- The SNR value of each of the SU in the network depends on its individual ED threshold value
- Reporting channel incurs error due to multipath fading and shadowing with infinite precision energy combining method used for sensing
- The noise present in the wireless channel is Additive White Gaussian Noise (AWGN) [17]

- SUs perform the spectrum sensing operation in Rayleigh fading [17] environment
- Voting based rule is used by coalition head for combining individual sensing results of SUs

4.2.2 Notations and Symbols Used

For the remainder of this chapter, the notations and symbols used are summarized in Table 4.1.

Table 4.1: Notations and symbols used

Notations/Symbols	**Comments**
S	Represents a coalition
N	Number of SUs in the network
P_{TX}	Primary transmitter
$P_{d,i}$	Probability of detection of the i^{th} SU
$P_{f,i}$	Probability of false alarm of the i^{th} SU
$P_{m,i}$	Probability of miss detection of the i^{th} SU
u	Time bandwidth product
$\Gamma(.,.)$	Incomplete Gamma function
$\Gamma()$	Gamma function
λ	Fixed sensing threshold (Energy Detection threshold) for an SU
$\gamma_{i,PU}$	Average SNR of the received signal from the PU at i^{th} SU
$h_{PU,i}$	Path loss between PU and the i^{th} SU
P_{PU}	PU signal power
σ^2	Gaussian noise variance
κ	Path-loss constant
μ	Path loss exponent
$d_{PU,i}$	Distance of i^{th} SU receiver from the PU transmitter
$P_{e,i,l}$	Probability of error of the channel from the i^{th} SU to coalition head l
$\gamma_{i,l}$	Average SNR at the coalition head l from the i^{th} SU
P_i	Transmission power of the i^{th} SU, used for reporting sensing information to coalition head
$h_{i,l}$	Path loss between the i^{th} SU and coalition head l
$d_{i,l}$	Distance between i^{th} SU receiver and the coalition head l
E_T	Energy consumed for reporting the local sensing result to coalition head by an SU
e_d	Energy dissipated per bit per $metre^2$
e_t	Energy spent by transmission circuitry per bit
b	Number of bits to be transfered/received/sensed

d	Distance between transmitter and receiver
$Q_{m,S}$	Probability of miss detection of a coalition S
$Q_{f,S}$	Probability of false alarm of a coalition S
λ_i	Adaptive sensing threshold (i.e. Energy Detection threshold) for the i^{th} SU
d_i	Distance between the i^{th} SU and the PU transmitter
D_{PU}	Distance vector containing the distances between SU transmitters and the PU transmitter
$V_{i,D}$	Distance vector maintained by i^{th} SU consisting of relative distances between this SU and all other SUs in a coalition
$d_{i,j}$	Distance between the i^{th} SU and the j^{th} SU
$d_{avg,i}$	Average distance between the i^{th} SU and all other SUs in a coalition
$V_{D,avg}$	Global vector consisting of average distance between every SU and all other SUs in a coalition
$d_{avg,min}$	Minimum of the average distance between each SU and all other SUs in a coalition
$E_{T,i}$	Energy consumed for reporting the local sensing information by the i^{th} SU to the coalition head
α	Maximum tolerable probability of false alarm
$\hat{P}_{d,i}$	Probability of detection of the i^{th} SU with adaptive sensing threshold
$\hat{P}_{f,i}$	Probability of false alarm of the i^{th} SU with adaptive sensing threshold
$\hat{P}_{m,i}$	Probability of miss detection of the i^{th} SU with adaptive sensing threshold
$\hat{P}_{e,i,l}$	Probability of error in the channel from the i^{th} SU to the coalition head l while adaptive sensing threshold is considered
$\hat{Q}_{m,S}$	Probability of miss detection of a coalition S when adaptive sensing thresholds of SUs are considered
$\hat{Q}_{f,S}$	Probability of false alarm of a coalition S when adaptive sensing threshold of SUs are considered
$P_{f,min}$	Minimum probability of false alarm for the network
$P_{e,min}$	Minimum probability of reporting error for the network
S_{max}	Maximum number of SUs in a coalition S
$\mathbb{D}$	Defection function

4.3 System Model

An ad-hoc CRN consisting of N numbers of SUs and a single PU is considered. A time slotted system is considered, where SUs and the PU synchronize themselves with a common clock. It is also assumed that when the PU is sensed to be absent, it will remain absent for the entire period of the time slot and vice-versa. To enable the sharing of sensing information among SUs, it is considered that every SU exists within the coverage of every other SUs in the network. SUs in the CRN are assumed to have the knowledge about their distance from the PU transmitter and their relative distances from all other SUs within a coalition as shown in Figure 4-1 using techniques like Global Positioning System (GPS).

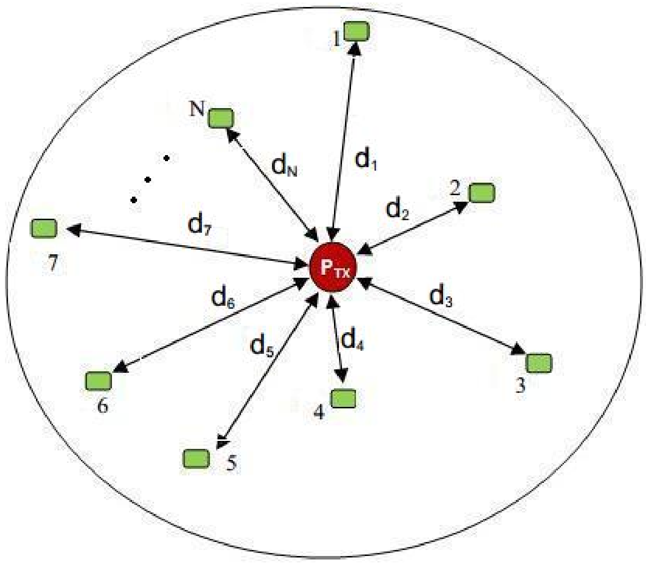

Figure 4-1: Network Architecture

In presence of Rayleigh fading, as stated in [17], for a cooperative spectrum sensing environment the probabilities of detection, miss detection, and false alarm of an individual SU i, can be given by Eq.(4.1), (4.2) and (4.3) respectively.

$$P_{d,i} = e^{-\frac{\lambda}{2}} \sum_{n=0}^{u-2} \frac{1}{n!} \left(\frac{\lambda}{2}\right)^n + \left(\frac{1+\gamma_{i,PU}}{\gamma_{i,PU}}\right)^{u-1}$$

$$\times \left[e^{-\frac{\lambda}{2\left(1+\gamma_{i,PU}\right)}} - e^{-\frac{\lambda}{2}} \sum_{n=0}^{u-2} \frac{1}{n!} \left(\frac{\lambda \gamma_{i,PU}}{2(1+\gamma_{i,PU})}\right)^n \right] \tag{4.1}$$

$$P_{m,i} = 1 - P_{d,i} \tag{4.2}$$

$$P_{f,i} = \frac{\Gamma\left(u, \frac{\lambda}{2\sigma^2}\right)}{\Gamma(u)} \tag{4.3}$$

where u is the time-bandwidth product, $\Gamma(.,.)$ is the incomplete gamma function, λ is the fixed sensing threshold and $\Gamma()$ is the gamma function. $\gamma_{i,PU}$ represents the average SNR of the received signal from the PU at i^{th} SU and is given by $\gamma_{i,PU} = \frac{P_{PU}h_{PU,i}}{\sigma^2}$ with P_{PU} be the PU transmitter signal power, σ^2 be the Gaussian noise variance and $h_{PU,i}$ be the path loss between the PU and the i^{th} SU. $h_{PU,i}$ is expressed by $\kappa/d^{\mu}_{PU,i}$ with κ, μ and $d_{PU,i}$ being the path loss constant, path loss exponent and the distance between the PU and the i^{th} SU respectively. The sensing parameters expressed by Eq.(4.1), (4.2) and (4.3) for individual sensing suffer from problem of spectral diversity. To overcome this problem during individual sensing, SUs can make use of collaboration among themselves to form coalitions and perform CSS. The cooperative decision is made by selecting a head as a fusion center of a coalition. The SUs in a coalition report their individual or local sensing information to the head using the reporting channel. The main task of the coalition head is to make a decision using the local sensing information of the SUs by applying a fusion rule. The majority/voting rule is assumed that the head uses for data fusion. Because the voting rule is more robust in situations of unpredictable noisy environment, not all the SUs need to be perfect in reporting to the head. While the CSS offers the enhancement of the sensing performance, it suffers from cooperation overhead during reporting phase of the SUs. This happens due to the presence of fading and shadowing over the reporting channel, leading to incur reporting error and extra energy to report. In a Rayleigh fading environment with BPSK modulation in use, the probability of reporting error (i.e. $P_{e,i,l}$) from the i^{th} SU to coalition head l of coalition S is given by Eq.(4.4), as stated in [53].

$$P_{e,i,l} = \frac{1}{2}\left(1 - \sqrt{\frac{\gamma_{i,l}}{1+\gamma_{i,l}}}\right) \tag{4.4}$$

where $\gamma_{i,l} = P_i h_{i,l}/\sigma^2$ is the average SNR at the coalition head l from i^{th} SU and P_i being the transmission power of i^{th} SU transmitter used for reporting the sensing result to l. $h_{i,l} = \kappa/d^{\mu}_{i,l}$ is the path loss between the i^{th} SU and the coalition head l, where $d_{i,l}$ is the distance between the head l and the i^{th} SU. As stated in [71], the energy spent during reporting can be given by Eq.(4.5)

$$E_T = e_d b d^{\mu} + e_t b \tag{4.5}$$

where e_d, e_t, b, d and μ represent the amount of energy dissipated per bit per $metre^2$, energy spent by transmission circuitry per bit, the number of bits to be transferred/received/sensed, distance between transmitter and receiver and the path loss exponent respectively. According to [80], using CSS the probability of miss detection and the probability of false alarm of a coalition S with coalition head l can be given by Eq.(4.6) and (4.7) respectively.

$$Q_{m,S} = \prod\nolimits_{i \in S} [P_{m,i}(1 - P_{e,i,l}) + (1 - P_{m,i})P_{e,i,l}] \tag{4.6}$$

$$Q_{f,S} = 1 - \prod\nolimits_{i \in S} [(1 - P_{f,i})(1 - P_{e,i,l}) + P_{f,i}P_{e,i,l}] \tag{4.7}$$

where $Q_{m,S}$ and $Q_{f,S}$ represent the probability of miss detection and probability of false alarm of coalition S with $P_{m,i}$, $P_{f,i}$ and $P_{e,i,l}$ being the probability of miss detection, probability of false alarm and probability of reporting error of the i^{th} SU respectively. From the Eq.(4.6), it is observed that for a coalition S, the

$Q_{m,S}$ depends on $P_{m,i}$ and $P_{e,i,l}$, $\forall i \in S$. The value of $P_{m,i}$ of i^{th} SU depends on ED sensing threshold (λ) and the average SNR ($\gamma_{i,PU}$) of the received signal from the PU at the i^{th} SU. $P_{e,i,l}$ of i^{th} SU is dependent on $\gamma_{i,l}$. Similarly, from Eq.(4.7), it can be stated that the false alarm probability of a coalition S has a dependency on $P_{f,i}$ and $P_{e,i,l}$, $\forall i \in S$, where $P_{f,i}$ depends on sensing threshold (λ), and $P_{e,i,l}$ depends on $\gamma_{i,PU}$ and $\gamma_{i,l}$ respectively. Further, the distances between the PU transmitter and i^{th} SU, and i^{th} SU and coalition head l have significant impact on $\gamma_{i,PU}$ and $\gamma_{i,l}$ respectively. Having the impact of sensing threshold (λ) on the accuracy of individual detection and false alarm probability, the choice of it's value is dependent on the distance between the i^{th} SU and the PU transmitter. Therefore, a formulation to adaptively estimate the value of sensing threshold (λ), based on the distance between i^{th} SU and the PU transmitter can be derived using Eq.(4.8) as stated in [81].

$$\lambda_i = P_{PU} - 10\mu log(d_i) \tag{4.8}$$

where P_{PU} and λ_i are the PU transmitter signal power and the adaptive sensing threshold of i^{th} SU respectively, with d_i being the corresponding distance between the i^{th} SU and the PU transmitter.

4.4 Game Theoretic Formulation of the Proposed CSS

4.4.1 CSS using Game Theory

The proposed framework of CSS requires to deal with dynamic behavior of SUs during cooperation through the interaction among the SUs and to adapt the parameters like variations of distances of the SUs from the PU and other SUs in the network on run. It is revealed in the literature [44] that the game theory [23, 24] as a mathematical tool can be used to model and analyze such a collaborative framework efficiently. Between the two categories of game theoretic approaches (i.e. non-cooperative and cooperative game theory) [23, 24], the cooperative game theory considers the behavior of rational players with improvement of their mutual benefit via cooperation. Therefore, the proposed framework of CSS problem can be modeled efficiently using the cooperative game theoretic approach.

4.4.2 The Proposed Coalitional Game Model

Using game theory [23,24,76], the CSS framework is modeled as a non-transferable coalition game and is named as threshold adaptive CSS or TACSS in short. The game is represented by $(\mathcal{N}, \nu)$, where $\mathcal{N}$ and ν represent the finite set of players (SUs) and the payoff or utility associated with each of the players in the coalition respectively. As shown in Fig.4-2, in the framework of coalitional game, formation of coalitions are performed by partitioning the set of players into disjoint sets. The proposed model forms coalitions such that each player is a constituent member of exactly one coalition. The utility function captures the trade-off between the revenue generated in terms of probability of detection and the cost incurred due to cooperation. The utility function is derived considering the distance adaptive individual false alarm and reporting error of the SUs.

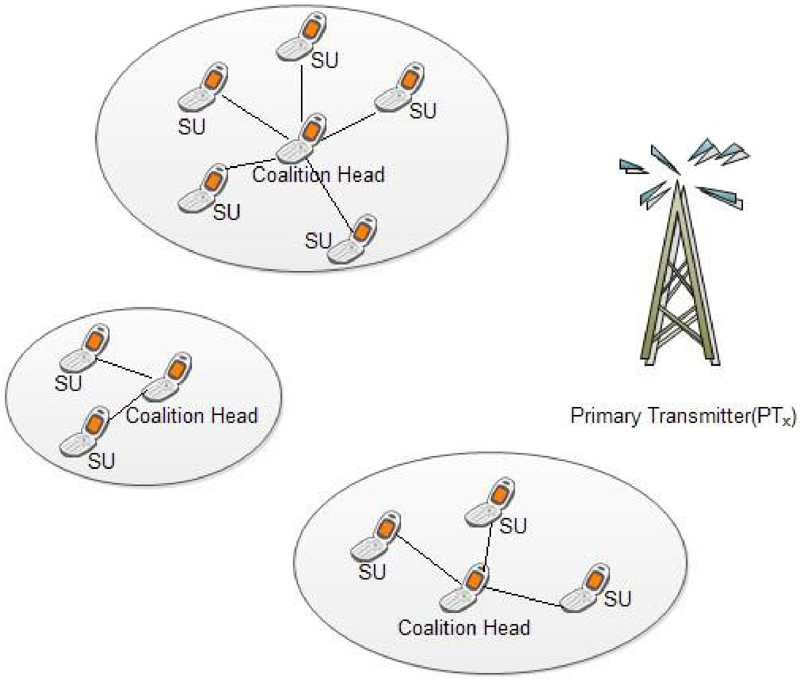

Figure 4-2: Distributed coalition formation by SUs

4.4.2.1 Design of Utility Function

Inspired by the work in [34], the utility function for a coalition S is designed to capture the result of detection (that is, probability of detection) against the trade-off in terms of probability of false alarm and is represented by $\nu(S)$, which can be given by Eq.(4.9)

$$\nu(S) = 1 - (Q_{m,S} + C(Q_{f,S})) \tag{4.9}$$

where $Q_{m,S}$ and $C(Q_{f,S})$ represent the probability of miss detection and the cost function of coalition S. The cost function is defined in terms of the probability of false alarm of the coalition S. During the game the optimal value of $\nu(S)$ can be obtained depending on minimization of $Q_{m,S}$ and $C(Q_{f,S})$. For a coalition S, the values of $Q_{m,S}$ and $C(Q_{f,S})$ depend on the probability of error during reporting to the coalition head. Therefore, the minimization of reporting error indirectly maximizes the utility of the coalition. Further, the cooperation overhead can be optimized by minimizing the reporting energy of the SUs. The constraints due to reporting energy and reporting error are indirectly considered in $\nu(S)$ through selecting the head of the coalition S in such a way that the average distance between every SU and the coalition head is minimized.

4.4.2.2 Selection of Coalition Head

Let S be a coalition consisting of K number of SUs denoted by $\{1, 2, \dots, K\}$. Using the formulation derived in 3.4.3.2 the selection of coalition head for a coalition S can be performed by calling the Algorithm 1 in Chapter 3.

4.4.3 Optimization of Cost Parameters

The maximization of utility eventually maximizes the probability of detection of coalition S, which depends on optimization of cooperation overhead by means of cost parameters reporting error over the channels and the reporting energy spent during reporting.

4.4.3.1 Optimization of Probability of Error during Reporting

Substituting $\gamma_{i,l}$ the Eq.(4.4) for probability of error can be expressed as

$$
\begin{aligned}
P_{e,i,l} &= \frac{1}{2}\left(1 - \sqrt{\frac{\frac{P_i h_{i,l}}{\sigma^2}}{1 + \frac{P_i h_{i,l}}{\sigma^2}}}\right) \\
&= \frac{1}{2}\left(1 - \sqrt{\frac{\frac{P_i \kappa}{\sigma^2 d_{i,l}^{\mu}}}{1 + \frac{P_i \kappa}{\sigma^2 d_{i,l}^{\mu}}}}\right) \\
&= \frac{1}{2}\left(1 - \sqrt{\frac{\frac{P_i \kappa}{\sigma^2 d_{i,l}^{\mu}}}{\frac{\sigma^2 d_{i,l}^{\mu} + P_i \kappa}{\sigma^2 d_{i,l}^{\mu}}}}\right) \\
&= \frac{1}{2}\left(1 - \sqrt{\frac{P_i \kappa}{\sigma^2 d_{i,l}^{\mu} + P_i \kappa}}\right) \qquad (4.10)
\end{aligned}
$$

Now, assuming that the values of κ, σ^2 and P_i are fixed for a particular instance of time, we can rewrite Eq.(4.10) as

$$
P_{e,i,l} = \frac{1}{2}\left(1 - \sqrt{\frac{C_1}{C_2 d_{i,l}^{\mu} + C_1}}\right) \qquad (4.11)
$$

where $C_1 = P_i \kappa$, $C_2 = \sigma^2$ are constant terms. So, Eq.(4.11) can be rewritten as

$$
P_{e,i,l} = \frac{1}{2}(1 - \sqrt{X}) \qquad (4.12)
$$

where $X = C_1/(C_2 d_{i,l}^{\mu} + C_1)$. Now, from Eq.(4.12) it can be observed that the term $X \propto 1/d_{i,l}^{\mu}$, that is, X is inversely proportional to the distance between i^{th} SU and the coalition head l, which implies that if the value of $d_{i,l}$ increases the value of X decreases polynomially. Considering this fact, the Eq.(4.12) can be expressed such that $P_{e,i,l} \propto d_{i,l}^{\mu}$, that is, the value of the error probability increases as the value of $d_{i,l}$ increases in polynomial order. Thus, it can be concluded that the minimization of the average distance between all the SUs and the respective coalition head minimizes the overall error probability of the coalition.

4.4.3.2 Optimization of Energy Consumption during Reporting

Considering the terms e_d, e_t, μ and b as constant, for a particular instance of time, the Eq.(4.5) can be rewritten as Eq.(4.13)

$$E_T = C_3 d^{\mu} + C_4 \tag{4.13}$$

where $C_3 = e_d b$, $C_4 = e_t b$ are constant terms. Using the Eq.(4.13), the energy spent during reporting by the i^{th} SU to the coalition head l at a distance $d_{i,l}$ can be rewritten as

$$E_{T,i} = C_3 d_{i,l}^{\mu} + C_4 \tag{4.14}$$

Therefore, $E_{T,i} \propto d_{i,l}^{\mu}$ for a large value of $d_{i,l}$. So, the value of $E_{T,i}$ can be optimized by minimizing the value of $d_{i,l}$, which can be estimated using Eq.(**??**).

4.4.4 The Threshold Adaptive CSS Game

Using Eq.(4.8), the Eq.(4.1), (4.2) and (4.3) can be rewritten as

$$\hat{P}_{d,i} = e^{-\frac{\lambda_i}{2}} \sum_{n=0}^{u-2} \frac{1}{n!} \left(\frac{\lambda_i}{2}\right)^n + \left(\frac{1+\gamma_{i,PU}}{\gamma_{i,PU}}\right)^{u-1}$$

$$\times \left[e^{-\frac{\lambda_i}{2\left(1+\gamma_{i,PU}\right)}} - e^{-\frac{\lambda_i}{2}} \sum_{n=0}^{u-2} \frac{1}{n!} \left(\frac{\lambda_i \gamma_{i,PU}}{2(1+\gamma_{i,PU})}\right)^n \right] \tag{4.15}$$

$$\hat{P}_{m,i} = 1 - \hat{P}_{d,i} \tag{4.16}$$

$$\hat{P}_{f,i} = \frac{\Gamma\left(u, \frac{\lambda_i}{2\sigma^2}\right)}{\Gamma(u)} \tag{4.17}$$

Using Eq.(4.15), (4.16), (4.17) and the probability of reporting error to be $\hat{P}_{e,i,l}$ the Eq.(4.6) and (4.7) can be rewritten as follows.

$$\hat{Q}_{m,S} = \prod\nolimits_{i \in S} \left[\hat{P}_{m,i}(1 - \hat{P}_{e,i,l}) + (1 - \hat{P}_{m,i}) \hat{P}_{e,i,l} \right] \tag{4.18}$$

$$\hat{Q}_{f,S} = 1 - \prod\nolimits_{i \in S} \left[(1 - \hat{P}_{f,i})(1 - \hat{P}_{e,i,l}) + \hat{P}_{f,i} \hat{P}_{e,i,l} \right] \tag{4.19}$$

Now using Eq.(4.18) and (4.19), the utility function of the proposed model can be rewritten as

$$\hat{\nu}(S) = 1 - (\hat{Q}_{m,S} + C(\hat{Q}_{f,S})) \tag{4.20}$$

The cost function $C(\hat{Q}_{f,S})$ in Eq.(4.20) can be described by a logarithmic barrier penalty function given by [82] as follows.

$$C(\hat{Q}_{f,S}) = \begin{cases} -\sigma^2.\log\left(1 - \left(\frac{C(\hat{Q}_{f,S})}{\alpha}\right)^2\right), & \text{if } \hat{Q}_{f,S} < \alpha, \\ +\infty, & \text{if } \hat{Q}_{f,S} \geq \alpha, \end{cases} \tag{4.21}$$

where α represents the maximum tolerable probability of false alarm of a coalition.

The proposed game with the utility function given by Eq.(4.20), satisfies the following property and is proved for CSS.

Property 1: The proposed coalition game has a non-transferable utility.

Proof: Once the game arbitrates to form coalitions, the final decision of any SU within a coalition is based on the decision taken by the selected coalition head. Therefore, the miss detection and false alarm probabilities of a coalition S become the miss detection and false alarm probabilities for any SU $i, i \in S$, i.e. $\hat{P}_{m,i} = \hat{Q}_{m,s}$ and $\hat{P}_{f,i} = \hat{Q}_{f,s}$. Thus the utility of the coalition S becomes the utility of any SU $i, i \in S$ that is, $\nu_i(S) = \nu(S)$. So, the utility of the coalition S cannot be arbitrarily distributed among the participants of the coalition S, establishing that the game has non-transferable utility.
This completes the proof of *Property 1.* □

The proposed game partitions the CRN into multiple disjoint coalitions. The stability of the game in such a collaborative environment can be achieved while the coalition's formation follows the Pareto order conditions [83]. The concept of the Pareto order condition can be incepted from the description given in the book by Hossain *et al.* [83]

Definition 1: Pareto order - Consider two collections of coalitions $\mathcal{R} = \{R_1, \ldots, R_r\}$ and $\mathcal{S} = \{S_1, \ldots, S_k\}$, for $r, k \leq N$, which are partitions of the same subsets $\mathcal{A} \subseteq \mathcal{N}$ (same player in $\mathcal{R}$ and $\mathcal{S}$). For a collection $\mathcal{R} = \{R_1, \ldots, R_r\}$, let the utility of a player j in a coalition $R_j \in \mathcal{R}$ be denoted by $\phi_j(\mathcal{R}) = \phi_j(R_j) \in \nu(R_j)$. $\mathcal{R}$ is preferred over $\mathcal{S}$ by Pareto order, i.e. $\mathcal{R} \triangleright \mathcal{S}$, iff $\mathcal{R} \triangleright S \Leftrightarrow \phi_j(\mathcal{R}) \geq \phi_j(\mathcal{S}), \forall j \in \mathcal{R}, \mathcal{S}$ with at least one strict inequality ($>$) for a player k. Where $\triangleright$ is a preference operator or comparison operator.

Definition 2: Preference or comparison operator - A preference or comparison operator $\triangleright$ is defined to compare two collections of coalitions $\mathcal{R} = \{R_1, \ldots, R_r\}$ and $\mathcal{S} = \{S_1, \ldots, S_k\}$, which are partitions of the same subsets $\mathcal{A} \subseteq \mathcal{N}$. $\mathcal{R} \triangleright \mathcal{S}$ implies that the way $\mathcal{R}$ partition $\mathcal{A}$ is preferred to the way $\mathcal{S}$ partitions A.

Using Definition 1 for Pareto order, we propose a coalition formation mechanism involving merge and splits rules as follows [84].

Definition 3: Merge Rule - Merge any arbitrary set of coalitions $\{S_1, \ldots, S_k\}$, where $\{\bigcup_{j=1}^{k} S_j\} \triangleright \{S_1, \ldots, S_k\}$.
Definition 4: Split Rule - Split any coalition $\hat{S} = \{\bigcup_{j=1}^{k} S_j\}$, where $\{S_1, \ldots, S_k\} \triangleright \{\bigcup_{j=1}^{k} S_j\}$.

4.4.5 The Distributed Threshold Adaptive CSS Algorithm

The proposed game model is realized by a distributed algorithm, which is named TACSS in short. The algorithm consists of four main phases - (i) Individual local sensing by SUs, (ii) Adaptive coalition formation by an iterative merge and split operation for maximizing coalition utility, (iii) Selection of coalition heads in the resultant coalitions, and (iv) Performing coalition based sensing for making cooperative decision per coalition. The algorithm for distributed TACSS assumes that at any given time slot T_l, $l \neq 0$, the CRN is constituted by M number of coalitions given by $\{S_1, S_2, S_3, \ldots, S_m\}$ except at the first time slot T_0. At T_0, there is no coalition and each SU performs non-cooperative spectrum sensing individually. The steps of the proposed CSS algorithm are given as in Algorithm 3.

Time complexity analysis of Algorithm 3 (Distributed TACSS)

The time complexity of the algorithm can be determined by approximating the number of comparisons (attempts) for merge and split operation. Considering the worst case scenario of convergence, where there exist n singleton coalitions[1] in the network, denoted by $\{S_1, S_2, \ldots, S_n\}$, the complexity of the algorithm can be estimated as follows.

To approximate the number of merge attempts, suppose there are N number of singleton coalitions in $\mathcal{N}$. During the first iteration of the algorithm, any coalition $S_i, \forall S_i \in \mathcal{N}$ attempts to form coalition with any other coalition $S_j, \forall S_j \in \mathcal{N}$ and $i \neq j$. In worst case scenario, S_i attempts to merge with S_j, requiring at most $(N-1)$ comparisons. The merge attempts are repeated for all members of $\mathcal{N}$ since each singleton coalition may try to form a larger coalition requiring $(N(N-1))$ number attempts. At the end of the first iteration, in worst case, only one coalition will be formed consisting of only two members of $\mathcal{N}$. The rest of the members of $\mathcal{N}$ will fail to form any more coalitions and will remain singleton. In the second iteration, all the member of $\mathcal{N}$ except those members whoever already formed a larger coalition in the first iteration will try to merge with the previously formed larger coalition. This may take at most $(N-2)$ number of comparisons. At the end of second iteration, in worst case, only one coalition will be formed consisting of only three members of $\mathcal{N}$. Similarly, the iteration

[1] In game theory [23], a coalition having only one player is called a singleton coalition.

Algorithm 3: Distributed TACSS

Input: P_{PU} (PU signal power), D_{PU} (Distance vector containing the distances of SUs from the PU), α (tolerable probability of false alarm)
Output: Final/cooperative sensing decision

Step 1: Each SU i computes the individual sensing threshold (λ_i) using Eq.(4.8) and locally senses (that is, non-cooperative sensing using energy detection) the licensed spectrum using its λ_i.

Step 2: Start coalition formation considering individual SUs as coalitions with single SU and go to Step 3. Each coalition with single SU i, is called singleton coalition.

Step 3: Merge operation: Two coalitions S_i and S_j, $i \neq j$ can merge to form a large coalition if and only if $\{S_i \bigcup S_j\} \triangleright \{S_i, S_j\}$

Step 4: Split operation: A coalition $\hat{S} = \{S_i \bigcup S_j\}$ can be split to form two sub-coalitions if and only if $\{S_i, S_j\} \triangleright \{\hat{S}\}$ i.e. $\{S_i, S_j\} \triangleright \{S_i \bigcup S_j\}$

Step 5: Repeat the Step 3 and 4 until no more merge and split operations take place , that is, the coalitions become stable.

Step 6: Select coalition head for coalition/s using Algorithm **??**.

Step 7: Each SU i, $\forall i \in S$ reports its local sensing information to their Coalition head within the coalition S.

Step 8: Coalition head combines the local sensing information received from all the SUs of the coalition S and the final decision about the sensing is obtained.

process continues until coalitions are formed and become stable. Therefore, in the worst case scenario the total time required for all the merge attempts can be given as follows.

$$T(N) = N(N-1)+(N-2)+(N-3)+\ldots+1 = N(N-1)+(N(N-1))/2 = O(N^2) \tag{4.22}$$

In case of split attempts, any coalition S having N number of SUs, will try to find/form all possible disjoint subsets of coalition S. Since the power set of any set of size N contains all the possible subsets of that set, finding any two disjoint subsets from that power set will take at most 2^N number of comparisons. Therefore, in worst case scenario for any coalition S of size N, the time complexity for split attempts will be at most $O(2^N)$.

However, in practice the worst case time complexity for the Algorithm 3 is not always withstanding. The merge process in practice requires significantly less number of attempts than in the worst case scenario; because in most of the instances the number of SUs in a coalition is reasonably small. In such a instance instead of going through all possible merge attempts with every coalition, whenever a coalition finds a partner satisfying the condition for merge, it merges. Therefore, in all cases it does not require to go through all the possible merge attempts.

Similarly once a coalition is heading towards split, the search for further splits is not necessary until the previous split attempt leads the coalition unstable.

4.4.5.1 Maximum Coalition Size

For the proposed algorithm, the maximum coalition size can be determined by the formulation given in *Theorem 1.*

Theorem 1: For the given minimum probability of false alarm ($P_{f,min}$) and minimum probability of error during reporting ($P_{e,min}$), any coalition S resulting from Algorithm 3 can have maximum S_{max} number of SUs, which can be approximated as follows.

$$S_{max} \leq \frac{\log(1-\alpha)}{\log[(1+2P_{f,min}P_{e,min})-(P_{f,min}+P_{e,min})]} \tag{4.23}$$

Proof: In order to decide the maximum number of nodes in any coalition S, the false alarm probability of every SU and the amount of error incurred during reporting the individual sensing information to the head should be minimal. Let, $P_{f,min}$ and $P_{e,min}$ represents the minimum probability of false alarm and minimum probability of error respectively. Suppose the values of each $\hat{P}_{f,i}$ and $\hat{P}_{e,i,l}$, $\forall i \in S$ is equal to the $P_{f,min}$ and $P_{e,min}$ respectively i.e. $\hat{P}_{f,1} = \hat{P}_{f,2} = \cdots = \hat{P}_{f,k} = \hat{P}_{f,min}$ and $\hat{P}_{e,1,l} = \hat{P}_{e,2,l} = \cdots = \hat{P}_{e,k,l} = \hat{P}_{e,min}$. Then, the Eq.(4.19) can be rewritten as

$$\begin{aligned}
\hat{Q}_{f,S} &= 1 - \prod\nolimits_{i\in S}[(1-P_{f,min})(1-P_{e,min}) + P_{f,min}P_{e,min}] \\
&= 1 - \prod\nolimits_{i\in S}[(1+2P_{f,min}P_{e,min})-(P_{f,min}+P_{e,min})] \\
&= [(1+2P_{f,min}P_{e,min})-(P_{f,min}+P_{e,min})]^{|S|}
\end{aligned}$$

Since α is a maximum tolerable value for $\hat{Q}_{f,S}$ for any coalition S.

$$\alpha \geq [(1+2P_{f,min}P_{e,min})-(P_{f,min}+P_{e,min})]^{|S|}$$

$$[(1+2P_{f,min}P_{e,min})-(P_{f,min}+P_{e,min})]^{|S|} \leq 1-\alpha$$

$$|S| \log[(1+2P_{f,min}P_{e,min})-(P_{f,min}+P_{e,min})] \leq \log(1-\alpha)$$

$$|S| \leq \frac{\log(1-\alpha)}{\log[(1+2P_{f,min}P_{e,min})-(P_{f,min}+P_{e,min})]}$$

$$S_{max} \leq \frac{\log(1-\alpha)}{\log[(1+2P_{f,min}P_{e,min})-(P_{f,min}+P_{e,min})]} \tag{4.24}$$

This completes the proof of *Theorem 1.* □

4.4.6 Stability of Coalition

In coalitional game theoretic formulation the stability of coalitions is characterized based on the defection measures in terms of defection function $\mathbb{D}$ [85].

Definition 5: Defection function - A defection function $\mathbb{D}$ is a function associated with each partition $\mathscr{P}$ of $\mathscr{N}$. $\mathscr{P}$ is a group of players in $\mathscr{N}$. A partition $\mathscr{P}$ is $\mathbb{D}$-stable if no group of players is interested to leave $\mathscr{P}$. Thus, the players can form the coalitions only allowed by the function $\mathbb{D}$.

With the proposed game model, the defection function $\mathbb{D}_p$ [85] can be used to describe the stability of the formed coalitions of the network. $\mathbb{D}_p$ function is a function which allows any group of players to leave the coalition S of $\mathscr{N}$ using merge and split operations to create another coalition in $\mathscr{N}$. Therefore, if a coalition S is $\mathbb{D}_p$ stable if no players in S are willing to leave S using merge and split operations to form other coalitions in $\mathscr{N}$.

Theorem 2: The coalitions resulted from the proposed Algorithm 3 are $\mathbb{D}_p$ stable.

Proof: As the proposed algorithm iterates to creates coalition using merge and split operations following the Pareto order optimality, the coalitions formed after $(i+1)^{th}$ iteration is always preferable over i^{th} iteration i.e. $\nu_{i+1}(S) \triangleright \nu_i(S)$ for any coalition S. As shown in [84, 85], any arbitrary merge and split operations performed during coalition formation terminates, leading to convergence to a stable coalition with respect to the merge and split rules. Therefore, the coalitions resulted from the proposed Algorithm 3 are $\mathbb{D}_p$ stable.
This completes the proof of *Theorem 2.* □

4.5 Simulation Results and Observations

In this section, simulation results are presented to evaluate the performance of the proposed scheme in terms of achieved probability of false alarm, probability of miss detection and the utility (the probability of detection) and compared with the model by Saad *et al.* [34]. Since the model proposed by Saad *et al.* [34] is similar to the proposed technique, we compare the performance of the proposed scheme with the scheme by Saad *et al.* The results to show how the increase in number of SUs in a coalition impacts the probability of false alarm and probability of miss detection using both the proposed technique and the model by Saad *et al.* are presented next. Then we present the results to show how the optimal size of a coalition varies in order to tolerate different target probability of false alarm values. The results to demonstrate how the distance between transmitter and receiver impacts in consumption of extra energy during reporting in a coalition is presented next. Further, we present the results to show the process of selection of coalition head and how the selection of the coalition head affects the reporting energy consumption. Finally, the results are presented to show how the selection of coalition head impacts the probability of error incurred during reporting by SUs.

A MATLAB based simulation has been carried out to study the behavior and the efficacy of the proposed TACSS model. The parameters taken for simulation environment are listed in the Table 4.2. For simulation purpose, a network

Parameter	Value
λ (fixed threshold)	20-80dB
R (radius of PU coverage)	2000m
P_{PU} (PU transmitter power)	1mW
e_d (energy dissipated per bit per $metre^2$)	0.0013 $pJ/bit/m^2$ as taken by Zheng *et al.* [74]
e_t (energy spent by transmission circuitry per bit)	5 $nJ/bit/signal$ as taken by Zheng *et al.* [74]
σ^2 (Gaussian noise variance)	-50dBm - -90dBm
u (time bandwidth product)	5 as taken by Ghasemi *et al.* [17]
κ (path loss constant)	1
μ (path loss exponent)	3
α (maximum tolerable probability of false alarm of a coalition)	0.1 as used in IEEE 802.22 [86]

Table 4.2: Values of different parameters used in simulation

with a single PU which is placed at the center of a circular area with a radius of 2000m . The SUs are placed randomly within the area of the network. The fixed spectrum sensing threshold value is set to be in between 20 dB to 80 dB for the model given by Saad *et al.* [34], whereas in the proposed TACSS model thresholds are determined adaptively using Eq.(4.8) based on the distance of SUs from the primary transmitter. In the experimental setup, the value of α, that is the maximum tolerable probability of false alarm of a coalition is maintained to be 0.1, and accordingly set the sensing threshold to be at least 20 dB.

Figure 4-3 to Figure 4-6 show the results of experiments conducted for evaluating the performance of the proposed model with respect to different sensing parameters and compared the performance with the model given by Saad *et al.* [34]. For the proposed scheme, the SUs independently adapt their sensing thresholds, whereas a fixed sensing threshold will be used by the SUs for the model by Saad *et al.* The results in these figures are based on a coalition of size 20 nodes. Figure 4-3 depicts the relationship between the adaptive sensing threshold and the achieved probability of false alarm using the proposed TACSS model and the fixed sensing threshold and the achieved probability of false alarm using the model by Saad *et al.* [34]. It shows that the proposed model achieves the probability of false alarm at per the results claimed by Saad *et al.* [34] while addressing the issues of spatial diversity. Compared to the model by Saad *et al.* [34], which uses the fixed sensing threshold for all the SUs, the proposed model chooses the sensing

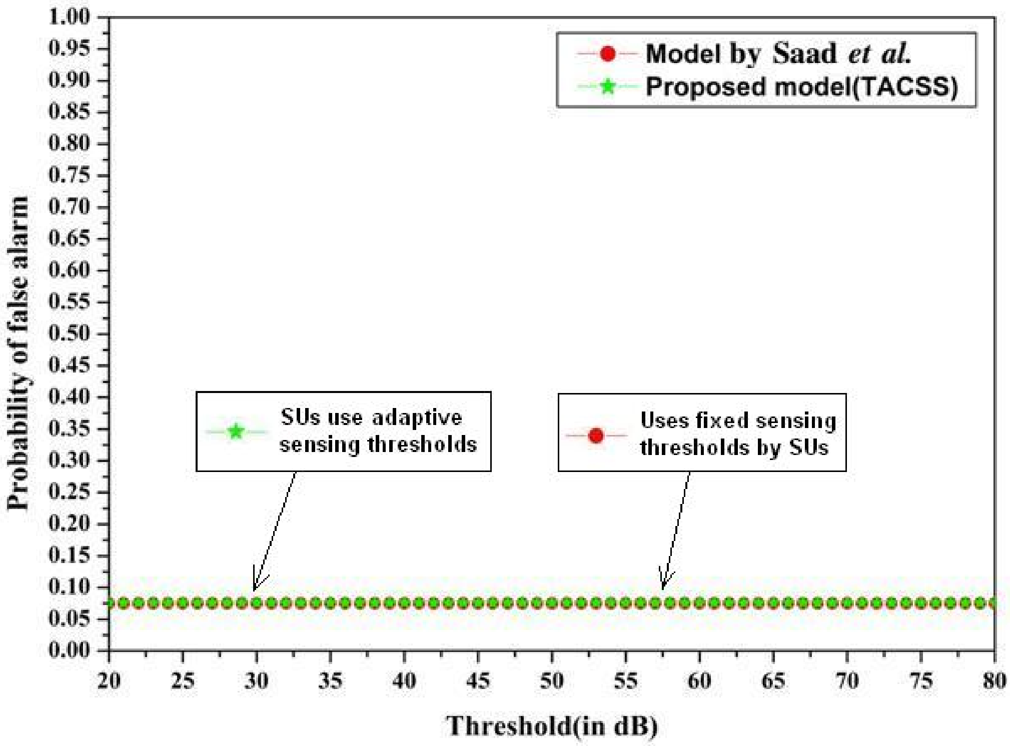

Figure 4-3: Threshold vs. Probability of False Alarm (with a coalition size of 20)

thresholds for SUs adaptively and diminishes the problem of spatial diversity.

Figure 4-4 shows the miss detection performance of both the proposed model and the model by Saad *et al.* [34]. It shows that for sensing threshold upto 35 dB, the proposed model performs similar to the model by Saad *et al.* [34] in terms of probability of miss detection. But, the miss detection performance of the model by Saad *et al.* [34] gradually suffers as the sensing threshold increases beyond 35 dB, whereas the proposed model performs consistently irrespective of the values of sensing threshold. It is due to TACSS's dynamic and adaptive nature in choosing the individual sensing thresholds of the SUs in the coalition.

Figure 4-5 demonstrates the performance of both the proposed TACSS model and the model by Saad *et al.* [34] in terms of utility. It shows that for feeble values of sensing threshold, the proposed model performs similar to the model by Saad *et al.* [34]. However, the utility achieved by the model by Saad *et al.* gradually suffers as the sensing threshold value increases beyond 35dB. On the other hand, the proposed model performs consistently. This consistently better performance of the TACSS can be attributed to its adaptive behavior of choosing distance adaptive sensing thresholds for the individual sensing of SUs in the coalitions.

Figure 4-6 and Figure 4-7, show the impact of coalition size (no. of SUs) on the probabilities of false alarm and miss detection in a coalition. Fig. 4-6 reveals

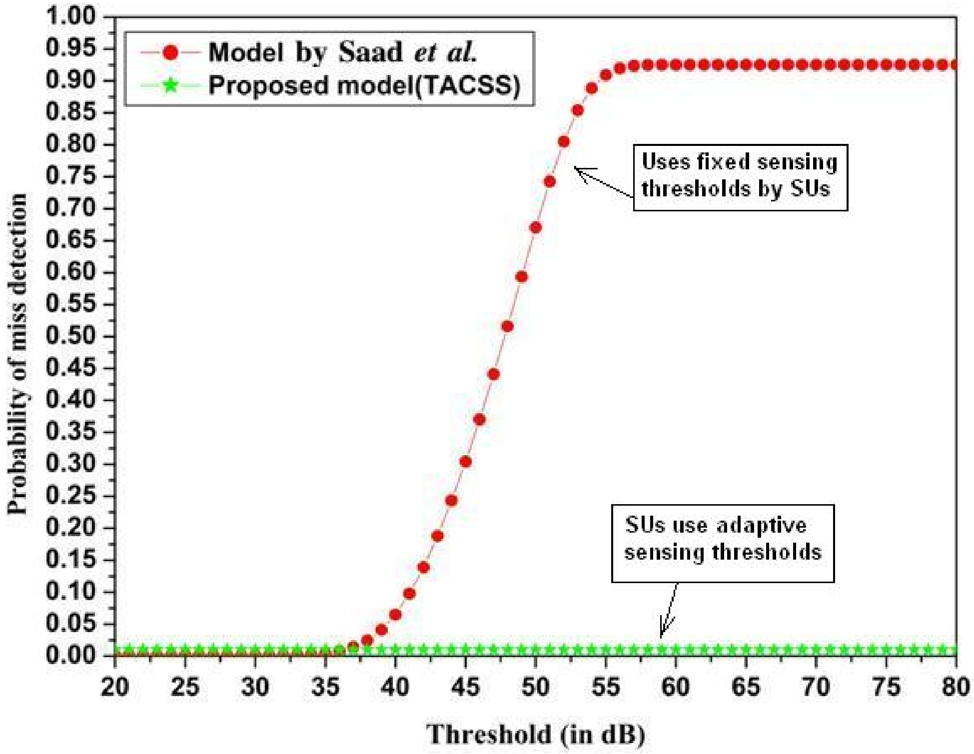

Figure 4-4: Threshold vs. Probability of Miss Detection (with a coalition size of 20)

that when the number of SUs in a coalition the probability of false alarm increases for both the models. However, the proposed model performs significantly better than the model by Saad *et al.* [34], and the difference in performance becomes higher in coalition with bigger size. Starting with coalition of 1 node as the number of nodes increases in the coalition up to 30, the probability of false alarm achieved is 0.062 for the proposed model, whereas it is 0.081 for the model by Saad *et al.* [34].

Figure 4-7 shows the behavior of both the methods in terms of probability of miss detection with increase in coalition size. With the increase in coalition size, the miss detection probability of both the models decreases. However, the proposed model outperforms the model by Saad *et al.* [34] when the coalition size is comparatively smaller (up to 11 in the figure). The result shows that the coalition with even 2 number of SUs can converge diminishing the probability of miss detection up to an optimal range than that of the model by Saad *et al.* [34].

Figure 4-8 demonstrates the determination of optimal number of SUs, which constitutes the optimal sized (i.e. S_{max}) coalition in order to tolerate a given maximum probability of false alarm value (i.e. α). The results show that with increasing values of probability of false alarm to be tolerated, more numbers of SUs are required in a coalition to maintain the given probability of false alarm. It can be seen that to maintain a target tolerable probability of false alarm of 0.1 (i.e. $\alpha =$

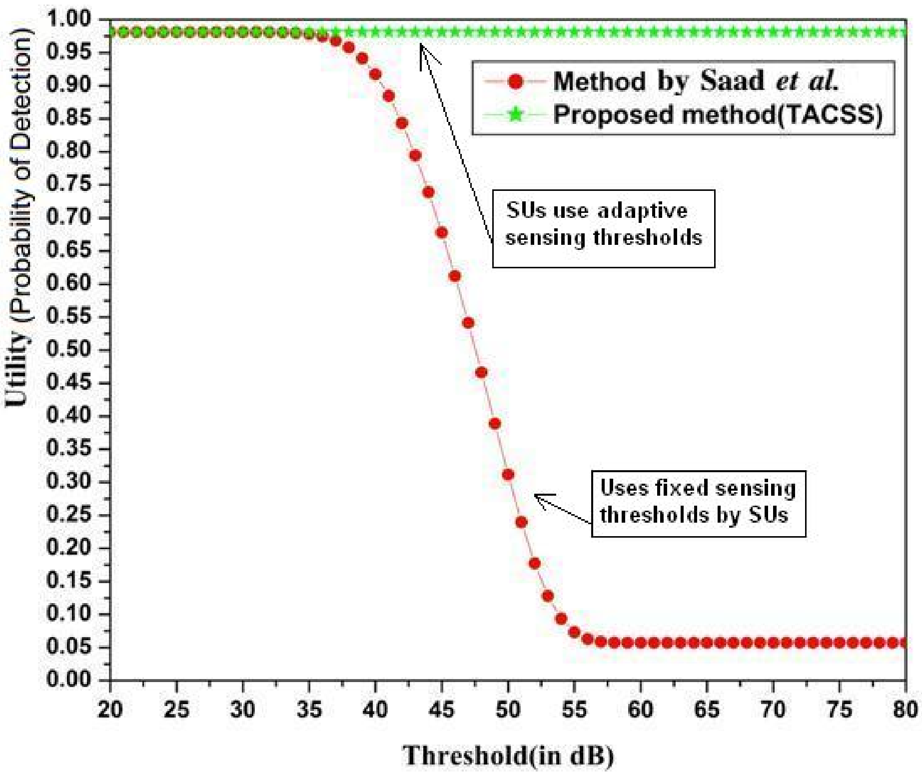

Figure 4-5: Utility achieved in terms of Probability of Detection against sensing threshold

0.1) with minimum probability of false alarm of 0.001 and minimum reporting error probability of 0.001 for the SUs, the optimal number of SUs required is 50. The optimal size of coalition increases to maintain smaller values of target probability of false alarm. This can be explained by the fact that, with optimal numbers of SUs, the cost of the coalition is minimized (that is, false alarm probability of the coalition).

Figure 4-9 shows the relationship between amount of energy consumption during reporting and the distance between transmitter and receiver. In this simulation setup, the coverage distance of maximum $5km$ is considered to evaluate the energy consumption by the SUs. The figure shows that the energy spent for reporting the local sensing results by an SU (transmitter) to the coalition head (receiver) increases with increase in distance between the transmitter and the receiver. It is observed that larger the distance between the SUs and the coalition head, higher is the amount of energy spent by the SUs to send their individual or local sensing information to the head. This in turn increases the energy consumption overhead due to reporting.

Figure 4-10 to Figure 4-12 illustrate about optimization of cost of reporting energy and reporting error as described in section 4.4.3. Figure 4-10 shows the scenario corresponding to the selection of coalition head. For simulation, a coalition of 15 SUs is assumed and SUs distances from the PU are generated randomly. The figure shows that the 12^{th} SU at a distance of $54m$ from the PU has

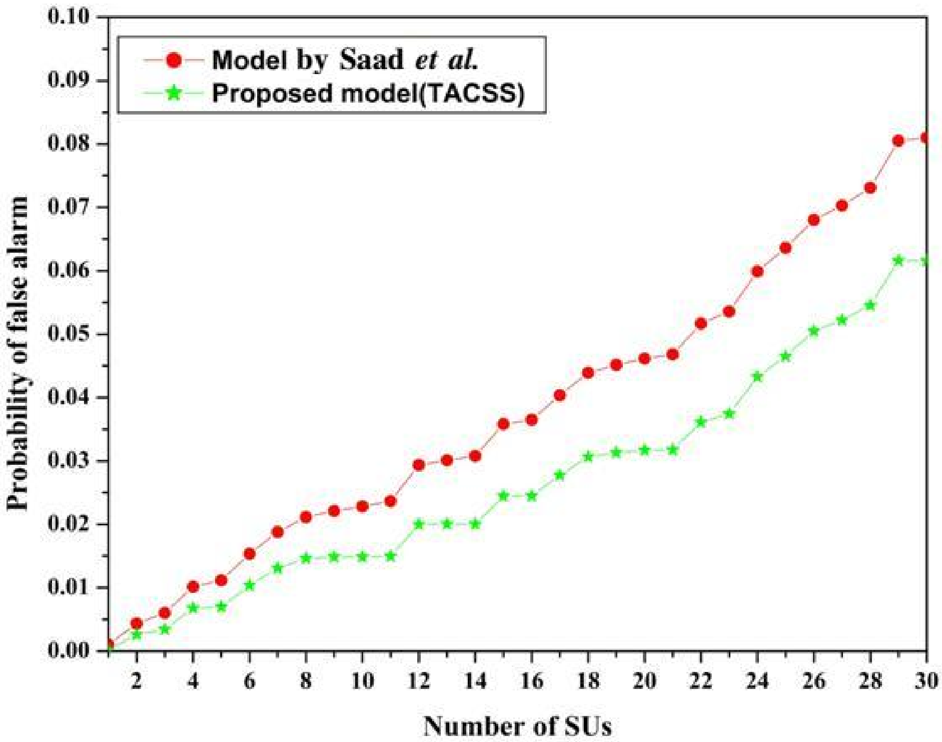

Figure 4-6: Number of SUs vs. Probability of False Alarm

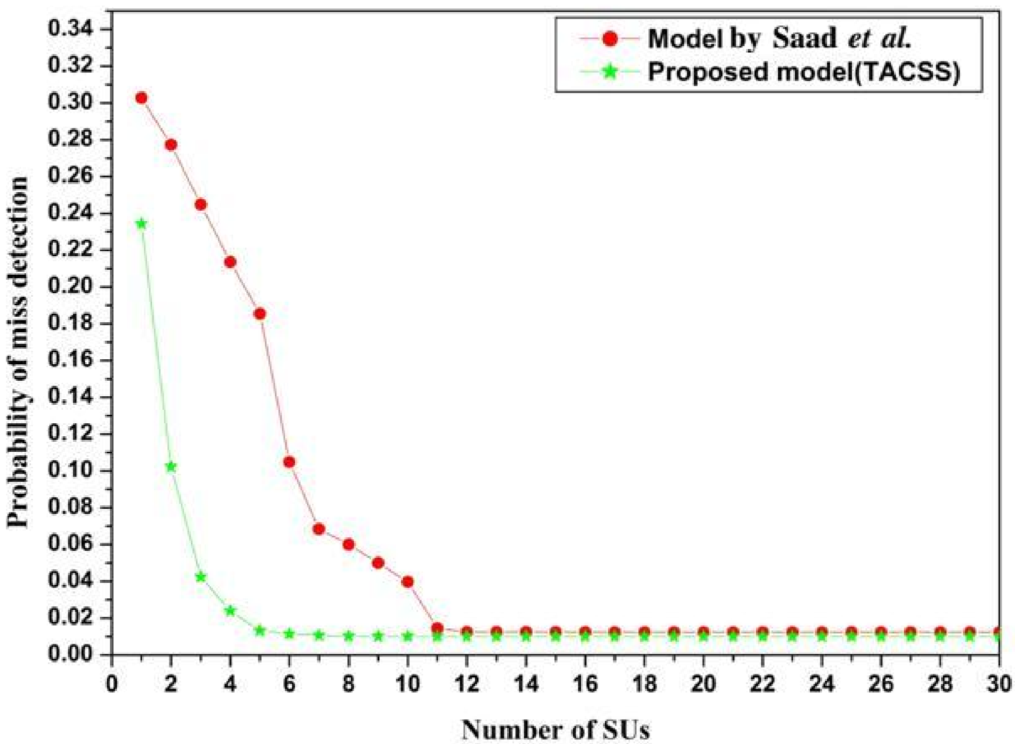

Figure 4-7: Number of SUs vs. Probability of Miss Detection

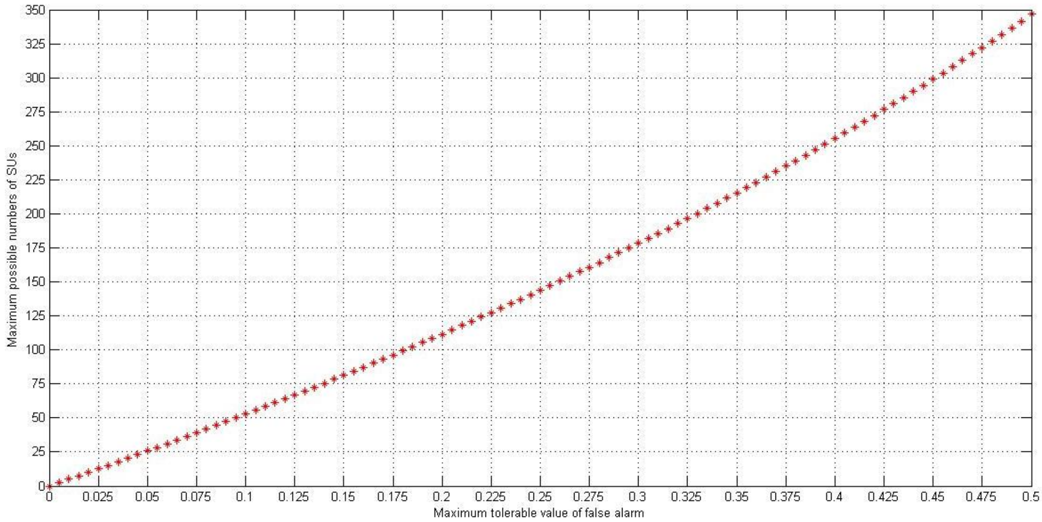

Figure 4-8: Maximum tolerable probability of false alarm (α) vs. Optimal Coalition Size

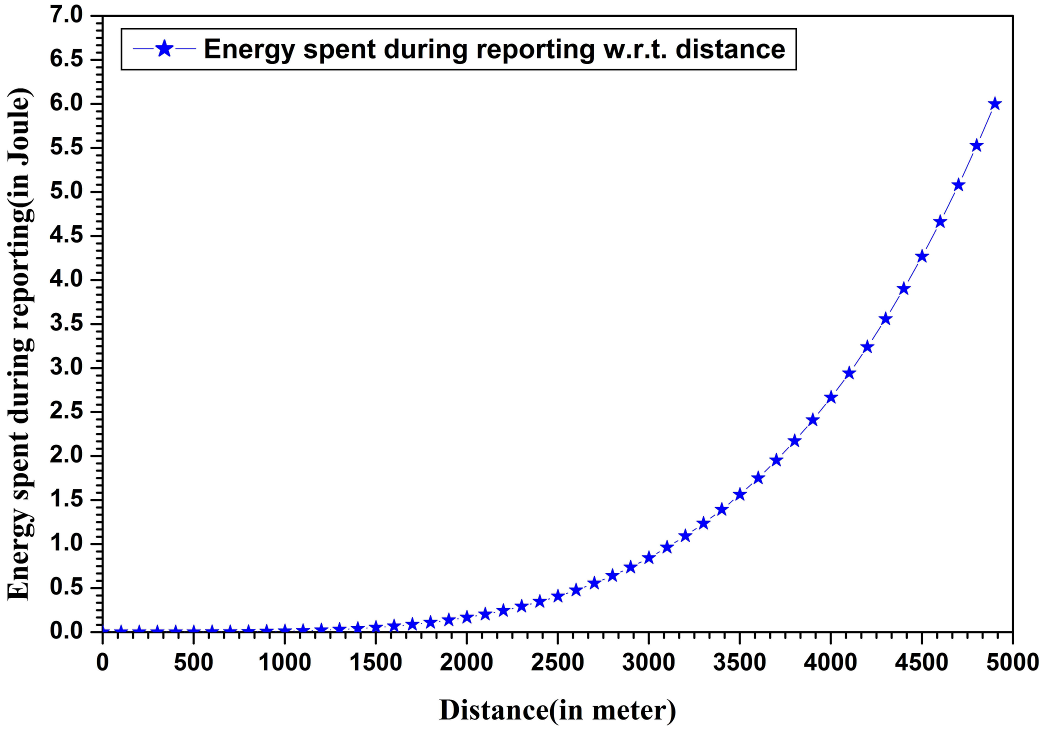

Figure 4-9: Distance vs. Reporting Energy

the minimum average distance and is selected as the coalition head.

Figure 4-11 shows the overhead incurred in terms of reporting energy, while selecting different SUs as the coalition head. It shows that when an SU with minimum average distance that is the SU with 54m as the average distance is selected as the head of the coalition, the energy consumption by the coalition becomes minimal.

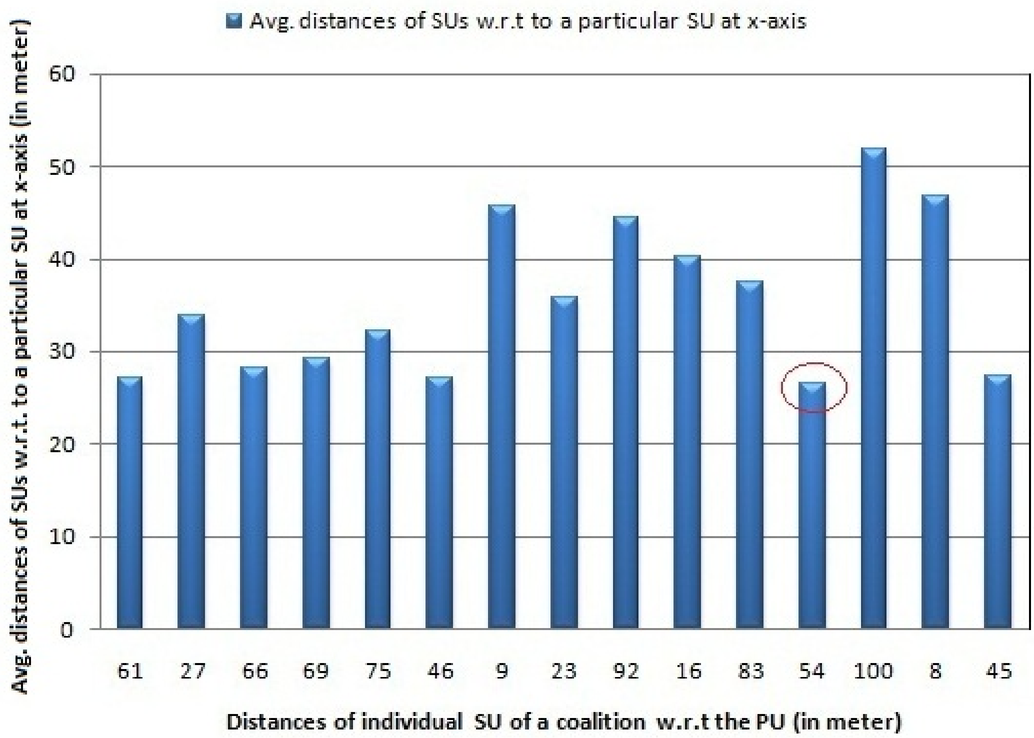

Figure 4-10: Distance of SUs w.r.t. the PU vs. Avg. distances of SUs

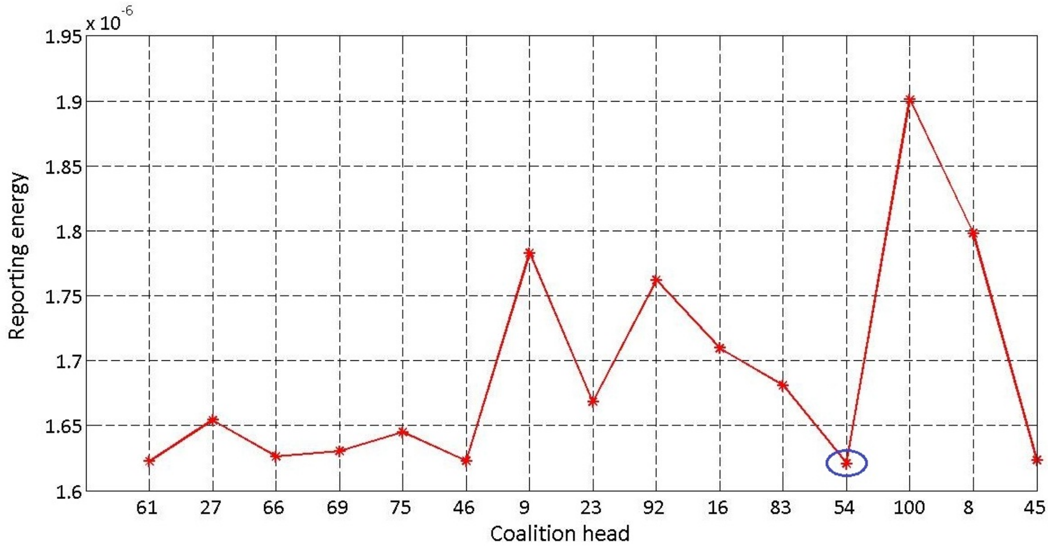

Figure 4-11: Reporting Energy vs. Coalition Head Selection

Figure 4-12 shows the probability of error incurred during reporting by the SUs against the selection of different SUs as the coalition head. It shows that the error probability can be minimized if an SU at a minimum average distance is selected as the coalition head. This in turn minimizes the false alarm probability of the coalition and hence enhances the performance of CSS.

Therefore, from the above experimental results it can be observed that the

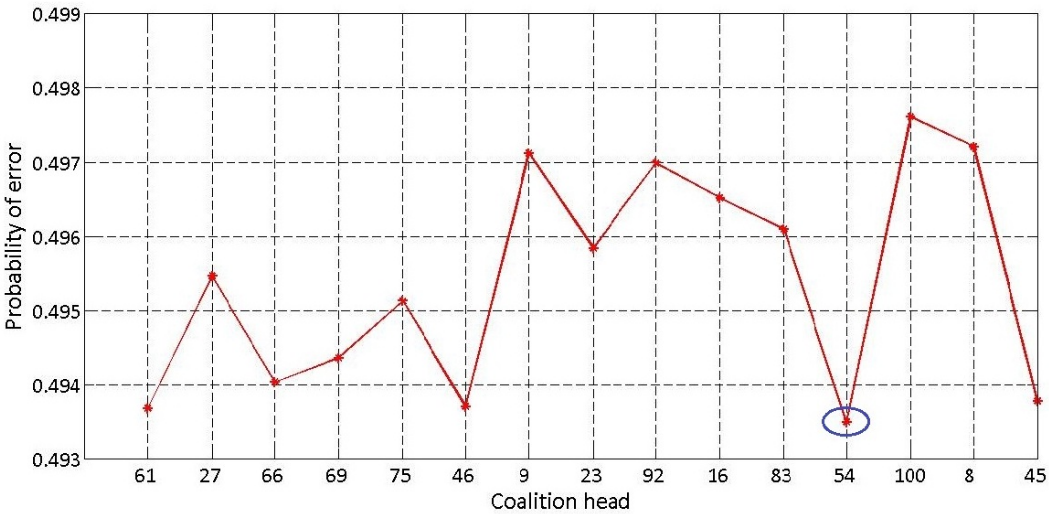

Figure 4-12: Probability of Error vs. Coalition head

proposed model can solve the spectral diversity problem while maintaining coalition overhead due to reporting energy and reporting error at minimum compared to the similar scheme in literature.

4.6 Conclusion

In this chapter, the cooperative spectrum sensing problem in CRN is investigated to improve the detection efficiency while overcoming the impact of spatial diversity problem during cooperation and reducing the cooperation overhead due to error in reporting channel and the reporting energy. The proposed CSS scheme is modeled as a cooperative game where SUs organize themselves into disjoint partitions while maintaining the overall utility function. The proposed CSS scheme overcomes the spatial diversity problem using a distance/location adaptive sensing threshold determination technique, which in turn improves the accuracy in detecting the PU signal. The proposed scheme is shown to reach stable partition analytically. Simulation results further validates the claim. The effectiveness of the proposed scheme is verified through its performance evaluation comparing against the model by Saad *et al.* [34].

With the enhanced detection performance of proposed CSS schemes, the next task is to use the detected opportunities efficiently for maximizing the capacity rate for SUs, which will be addressed in the next chapter.