Chapter 5

A Power Allocation Technique for Underlay Communication

5.1 Introduction

The successful deployment of cognitive radio network (CRN) requires achieving the spectral efficiency or capacity rate maximization using the detected channels/spectrum opportunities in the licensed spectrum, provided the primary users (PUs) are protected from harmful interference. Depending on the accuracy of the spectrum sensing module of CR, the unused spectrum opportunities can be utilized for secondary transmission. Utilizing the detected spectrum opportunities by means of maximization of capacity rate requires optimal power allocation on the channels by secondary user (SU). The power allocation for secondary transmission is needed to be performed such that the allowed aggregated interference on PUs is maintained as per the given constraint. It is revealed in [2, 87, 88] that, the overlay and underlay modes of secondary transmission access the spectrum with knowledge of PU activity and coexisting with PU. In such a CR transmission scenario, the maximization of SUs capacity rate requires optimum power allocation, which is regulated by the interference power constraint of the PUs. In underlay mode of communication, implementing the requirement of strict PU protection together with optimization of power allocation is challenging. Again, the computation overhead incurred due to iterative process involved in exhaustive search for allocating power on the channels is very high. In this direction, the traditional classic water-filling (WF) [25] based power allocation technique used for orthogonal frequency division multiplexing (OFDM) sub-channels, can offer realizable solution for CR transmission. The WF based technique improves the bandwidth utilization through utilizing the orthogonal sub-channels adaptively to allocate power, which in turn enhances the capacity rate for the transmission. For CR transmission, the PU protection depends on the interference tolerance behavior of the PU receiver. In underlay CRNs using OFDM framework, interference tolerance of PU can be achieved in two ways - (i) the peak interference power (PIP) limit of each of the sub-channels as a constraint or (ii) the average interference power (AIP) limit over all the sub-channels as another constraint [29]. Compared to PIP, AIP imposes lose constraint on SUs and offers larger instantaneous interference in a sub-channel providing larger throughput as long as the interference averaged over all the sub-channels is within the threshold limit.

In the literature, various power allocation techniques [4, 13, 26–28, 54–60] have been proposed. An optimization problem of capacity rate maximization is addressed in [26, 27] using water-filling (WF) [35] framework of power allocation, which uses a binary search method to iteratively invoke the classic WF algorithm. With assumption that the SU transmitter has the knowledge of the channel state information (CSI) [4, 13, 26] to the PU receiver, most of these approaches in the literature implement the power allocation scheme under the constraints of the total transmit power (the power budget of the transmitting SU) and then the interference to PU. It is also found that the classic WF based power allocation approaches face challenges [13, 26–28] in terms of ensuring strict PU protection and the computation overhead to find the water level for optimal solution, which indirectly affects the capacity rate of an SU. Therefore, in order to solve the optimal power allocation problem using the WF based technique, requires improving the search process for water level while ensuring the PU protection. To the best of our knowledge, in this direction no technique has been reported in the literature, which solves the optimization problem for capacity rate maximization for underlay communication using WF based power allocation framework (which consider the issue of computation overhead to find the water level and the PU protection in terms of AIP constraint).

In this chapter, the power allocation problem in OFDM-based underlay CRN for capacity rate maximization with AIP constraint for PU protection is investigated. The overall goal is to facilitate SU transmitter to allocate a certain power on a channel such that SU's capacity rate can be maximized while maintaining the average interference power to PU within the permissible limit. The power allocation problem is formulated as an optimization problem to maximize the SU's average throughput under the constraints of the PU receiver's AIP limit and total transmit power by SUs for transmission. Assuming the channel state information (CSI) [4,13,26] is known to the SU transmitter, the water-filling (WF) [25] based scheme is adopted to perform the power allocation on the channel. Under such assumptions about knowledge of CSI, the SU transmitter is able to adapt its transmission resources such as transmission rate, power, and spatial spectrum based upon the channel knowledge so as to optimally balance between maximizing its own transmit throughput and avoiding interferences at the PU receivers. In practice, the full knowledge of CSI from the SU transmitter to the PU receiver is difficult to obtain at the SU transmitter. Since primary and secondary networks are loosely coupled, the SUs usually need to rely on their own observations over the received signals from the PU terminals to estimate the the CSI. To this end, the techniques to estimate the CSI may be explicit or implicit [89]. A brute-force approach may be adopted using the explicit feedback between the two networks. The implicit approach allows SUs to periodically perform sensing the transmitted signal from the PU receiver provided that time-division-duplexing (TDD) is employed by the PU transmission. Use of the cognitive beamforming [90] can also be used to estimate the CSI. WF based scheme applies equalized power allocation strategies on the orthogonal sub-channels in a parallel fashion. The optimal power allocation is performed by deciding the water level that satisfies the PU's AIP constraint. The water level determines the maximum allowable power in the channel. The WF process allocates the optimal power to the sub-channels starting with the maximum noise level. The proposed optimization problem is realized by an underlay power allocation (UPA) algorithm, which maximizes the power allocation to the channel, leveraging the throughput optimization for transmission. The UPA algorithm improves the computational complexity of invoking the classic WF process. The optimization problem of the power allocation is solved through mathematical analysis. Simulation based numerical results demonstrate how the SUs achieve the maximum possible throughput with improved running time under the constraint of PU's AIP limit.

The rest of this chapter is organized as follows. Section 5.2 formally defines the problem. In that section, the model assumptions and the symbols and notations used are defined. The system model and formulation of objective function are presented in section 5.3. Section 5.4 presents the proposed technique for capacity rate maximization with the underlay power allocation (UPA) algorithm. In that section, the algorithm to perform negative power adjustment during power allocation is also presented. The observations and the analysis for finding the water-level is also discussed in that section. Section 5.5 evaluates the performance of the proposed technique and provides the simulation results. Finally, section 5.6 concludes this chapter.

5.2 Problem Statement

To develop a power allocation technique in OFDM-based underlay CR network to maximize the average throughput of SU under the constraints of protecting the PU from harmful interference, maintaining the SNR of SU above the minimum threshold, and keeping the total transmit power of the SU within the allowable maximum.

5.2.1 Assumptions

- An ad-hoc Cognitive Radio Network (CRN) is considered
- SU uses OFDMA channel access and performs transmission using underlay mode communication
- A pair of SUs communicates using channel bandwidth subdivided into sub-channels according to OFDM
- A single pair of PUs is considered
- The PU transmitter uses a fixed constant power for transmission

- SUs are within the coverage area of the PU transmitter
- SU has the complete knowledge of Channel State Information [4, 13, 26]
- The channel gains from SU transmitter to PU receiver and SU transmitter to SU receiver are independent and identically distributed (i.i.d.) under uncorrelated fading [91]
- The Average Interference Power (AIP) constraint over all the sub-channels is considered for PU protection, as it supports larger SU throughput
- For each of the available sub-channels, the AIP threshold is known to SU transmitter
- The noise present in the wireless channel is Additive White Gaussian Noise (AWGN)

5.2.2 Notations and Symbols Used

For the remainder of this chapter, the notations and symbols used are summarized in Table 5.1.

Table 5.1: Notations and symbols used

Notations/Symbols	Comments
B	Represents the bandwidth of a channel
C	Set of sub-channels
N	Number of sub-channels with bandwidth of B/N each
n	Sub-channel number (n^{th} sub-channel)
PTx	PU transmitter
PRx	PU receiver
STx	SU transmitter
SRx	SU receiver
g_p	Channel State Information (CSI) in terms of channel gain between PTx and PRx
g_1	CSI vector between SU transmitter (STx) and PU receiver (PRx)
g_{1n}	Channel gain between STx and PRx in sub-channel n
g_2	CSI vector between SU transmitter (STx) and SU receiver (SRx)
g_{2n}	Channel gain between STx and SRx in sub-channel n
p_{max}	The total transmit power (maximum possible power) at STx
$\boldsymbol{P}$	Power allocation vector of the SU transmitter, where each component represents power allocation for the corresponding sub-channel

p_n	SU transmitter power in sub-channel n
N_0	AWGN noise power present in the channel
I_{max}	Average Interference Power (AIP) threshold
p_{int}	The allowable maximum transmission power at STx constrained by AIP threshold
μ	Lagrange multiplier. The inverse of μ represents the water level

5.3 System Model

A simple CR network consisting of single pairs of SUs and PUs is considered for the purpose of the systematic exposition of the power allocation strategy. As shown in Figure 5-1, an Orthogonal Frequency Division Multiplexing (OFDM) based single user underlay CRN is assumed, where $C = \{1, 2, \ldots, N\}$ denotes the set of OFDM sub-channels for a channel with bandwidth B. The vectors, $g_1 = [g_{11}, g_{12}, \ldots, g_{1N}]^T$ and $g_2 = [g_{21}, g_{22}, \ldots, g_{2N}]^T$ denote the CSI vectors between the SU transmitter and the PU receiver, and the SU transmitter and SU receiver respectively, where CSI is represented by channel gain. For a sub-channel $n \in C$, the channel gains g_{1n} and g_{2n}, are assumed to be independent and identically distributed (i.i.d.) chi-square random variables with uncorrelated fading [91] environment. The vector $\boldsymbol{P} = [p_1, p_2, \ldots, p_n, \ldots, p_N]^T$ denotes the power allocation vector of an SU, where p_n is the transmit power in sub-channel n. PU and SU share the same channel for communication, where the PU transmitter uses a fixed power for transmission. The SU link is assumed to be adaptive in the sense that it can adapt its transmission power based on the CSI of the channels/sub-channels. Water-filling (WF) [25, 28, 54] based technique is used by SUs for adaptive power allocation on the OFDM sub-channels due to its equalization strategies used for power allocation on sub-channels. Like water finds its level even when filled in one part of a vessel with multiple openings, as a consequence of Pascal's law [52, 53], the SU regulates transmission power up to an optimal level on each sub-channel compensating the channel impairments or noise floor. The power is regulated under the constraint of allowable power level due to the AIP limit of PU receiver. The water level is measured capturing the difference between the SU's allowable maximum power level to be allocated in a sub-channel and the channel impairment or noise floor present in the sub-channel.

5.3.1 Objective Function

The goal is to find the power allocation vector, $\boldsymbol{P}$ such that the capacity rate (or throughput) of an SU is maximized, subject to the interference constraint and the total transmit power constraint. The capacity rate, $\mathscr{C}$ can be computed using Shannons law and the objective function of the problem can be formulated as

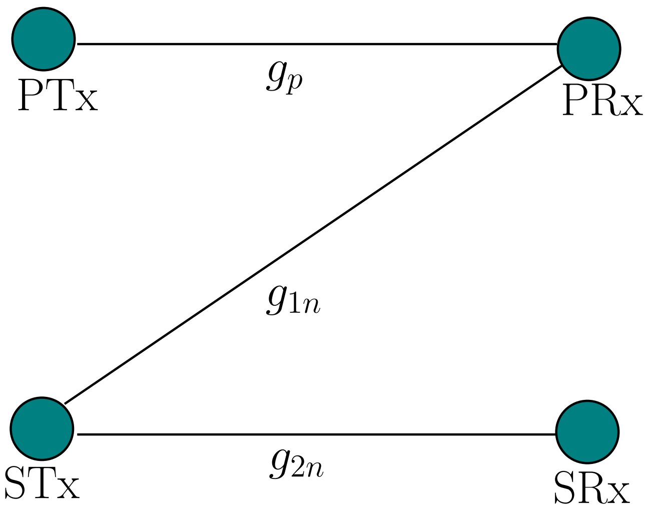

Figure 5-1: System Model for OFDM based CRN

follows:

$$\mathscr{C} = \max_{\boldsymbol{P}} \quad B\sum_{n=1}^{N} log_2\left(1+\frac{g_{2n}p_n}{N_0B}\right) \tag{5.1}$$

$$s.t. \quad \frac{1}{N}\sum_{n=1}^{N} g_{1n}p_n \leq I_{max}, \forall n \tag{5.2}$$

$$\sum_{n=1}^{N} p_n \leq p_{max} \tag{5.3}$$

$$p_n \geq 0, \forall n \tag{5.4}$$

where Eq.(5.2) represents the average interference power (AIP) constraint and Eq.(5.3) and (5.4) represent the total transmit power and positive power constraints respectively. The symbols used in these equations are defined in Table 5.1.

5.4 The Proposed Power Allocation Technique

Analyzing the objective function presented in section 5.3.1, it can be stated that the Eq.(5.1), (5.3) and (5.4) constitute the classic water-filling (WF) framework of power allocation for OFDM based communication. For CRN, the prime requirement is to consider the interference to PU alongside optimal power allocation to SUs. The power profile of an SU therefore depends on the total interference power tolerance limit constrained by AIP expressed by the Eq.(5.2). Again, the amount of interference produced by an SU to the PU receiver always remains within the limit determined by maximum total transmit power capacity of the SU given by p_{max}. Therefore, for a sub-channel n, the maximum interference that can be tolerable by PU receiver, I_{max} can be given by equation, $I_{max} = g_{1n}p_n$. Then, the allowable maximum transmit power for the SU in sub-channel n can be calculated as follows:

$$p_n = \frac{I_{max}}{g_{1n}} \tag{5.5}$$

From Eq.(5.5), the allowable maximum transmission power p_{int} of an SU, can be derived as follows:

$$p_{int} = \sum_{n=1}^{N} p_n = \sum_{n=1}^{N} \frac{I_{max}}{g_{1n}} \tag{5.6}$$

Theorem 1: The power profile for which the total capacity rate in Eq.(5.1) is maximized subject to the constraints in Eqs.(5.2) and (5.4) can be derived as

$$p_n = \left[\frac{1}{\mu g_{1n}} - \frac{N_0 B}{g_{2n}}\right]^+, \forall n \tag{5.7}$$

where μ is a deterministic Lagrange multiplier and $[x]^+ = max(x, 0)$

Proof: The objective function in Eq.(5.1) is concave in $\boldsymbol{P}$, and constraints in Eq.(5.2) and (5.4) are convex. The Karush-Kuhn-Tucker (KKT) conditions for convex optimization are sufficient and necessary for the optimality [82]. Let μ and β be the Lagrange multipliers for constraints in Eq.(5.2) and (5.4) respectively. For the KKT conditions, the Lagrangian can be written as:

$$L(p_n, \mu, \beta) = -\sum_{n=1}^{N} B log_2\left(1 + \frac{g_{2n} p_n}{N_0 B}\right) + \mu\left(\sum_{n=1}^{N} g_{1n} p_n - N I_{max}\right) - \beta p_n \tag{5.8}$$

where the KKT conditions can be written as follows:

$$\begin{aligned} p_n &\geq 0, \forall n, \\ \sum_{n=1}^{N} g_{1n} p_n - N I_{max} &\leq 0, \\ \beta &\geq 0, \\ \beta p_n &= 0, \forall n, \\ \mu &\geq 0, \\ \mu\left(\sum_{n=1}^{N} g_{1n} p_n - N I_{max}\right) &= 0 \end{aligned}$$

$$\frac{\partial L}{\partial p_n} = -\frac{1}{\frac{g_{2n}}{N_0 B} + p_n} + \mu g_{1n} - \beta = 0, \forall n \tag{5.9}$$

Now, β can be eliminated, and can be written from the KKT conditions in Eq.(5.9) as follows:

$$-\frac{1}{\frac{g_{2n}}{N_0 B} + p_n} \leq \mu g_{1n}, \forall n \tag{5.10}$$

Multiplying Eq.(5.10) with p_n, the Eq.(5.11) can be expressed as:

$$p_n \mu g_{1n} - \frac{p_n}{\frac{g_{2n}}{N_0 B} + p_n} = 0, \forall n \tag{5.11}$$

If $\mu g_{1n} < \frac{1}{\frac{g_{2n}}{N_0 B}}$, the Eq.(5.10) can only hold for a power value, say $p_n^{''}$, if $p_n^{''} > 0$, which by solving Eq.(5.11) gives, $p_n^{''} = \frac{1}{\mu g_{1n}} - \frac{N_0 B}{g_{2n}}$. On the other hand, if $\mu g_{1n} > \frac{1}{\frac{g_{2n}}{N_0 B}}$ holds, $p_n^{''} > 0$ is impossible. Because, $p_n^{''} > 0$, implies $\mu g_{1n} \geq \frac{1}{\frac{g_{2n}}{N_0 B}} > \frac{1}{\frac{g_{2n}}{N_0 B} + p_n^{''}}$, which violates Eq.(5.11). Therefore, the only possible solution in this case is $p_n^{''} = 0$. Hence, the optimization solution can be rewritten as

$$p_n^{''} = \begin{cases} \frac{1}{\mu g_{1n}} - \frac{N_0 B}{g_{2n}} & \text{if } \mu g_{1n} < \frac{g_{2n}}{N_0 B} \\ 0 & \text{if } \mu g_{1n} \geq \frac{g_{2n}}{N_0 B} \end{cases}$$

which is equivalent to

$$p_n^{''} = \left[\frac{1}{\mu g_{1n}} - \frac{N_0 B}{g_{2n}} \right]^+, \forall n$$

This completes the proof of *Theorem 1.* □

The main principle of WF method [25, 28, 54] is to allocate power in sub-channels such that the water level is settled so as to satisfy the total power constraint of the node. The water level is denoted by the inverse of Lagrange multiplier for the total power constraint. According to [27], it is observed that the transmit powers allocated to individual sub-channels generally vary by the same amount if the total transmit power changes. The total transmit power of a node changes due to the change in channel gain (to maintain target capacity). Therefore, the power allocation formulation based on WF [25] needs to be simplified. Inspired by Son *et al.* [26], the proposed method separates the power allocation problem formulation in Eqs.(5.1-5.4) from total transmit power constraint given by Eq.(5.3) and derived the equation for optimal solution in Eq.(5.7). This is based on modifying the WF method with incorporation of two constraints namely the interference power (AIP) constraint given by Eq.(5.2) and minimum SNR constraint given by Eq.(5.4).

5.4.1 Observations

To estimate allowable transmit power on sub-channel n by the SU, p_n from Eq.(5.7), the WF algorithm needs to search for the value of μ, which satisfies the Eq.(5.6). This needs more computations as the classic WF method is an iterative process, offering allocation of optimal power in the sub-channels of the underlay CRN. The proposed technique aims to calculate p_n using the fact that the allocated transmit power to the individual sub-channels get changed by the same value given by $(p_{int} - p_{max})$, if the maximum transmit power p_{int} needs to be changed due to the adjustment required to keep the total transmit power limit p_{max} of the STx intake. Because of this adjustment, it would be computationally expensive to invoke WF with a new μ searching process for each newly changed p_{int} value. Initially, the WF is invoked with p_{int} to compute the power vector $\boldsymbol{P}$ and to determine the water level boundary at the maximum possible transmit power. From Eq.(5.7), the optimal transmission power value in sub-channel n, p_n is given by

$$p_n = max\left(\frac{1}{\mu g_{1n}} - \frac{N_0 B}{g_{2n}}, 0\right), \forall n$$

where μ is the positive Lagrange multiplier associated with the AIP constraint which is chosen such that Eq.(5.2) holds with equality. For $\forall n \in C$, the water level for the modified WF method can be derived as follows:

$$\frac{1}{\mu g_{1n}} = p_n + \frac{N_0 B}{g_{2n}} \tag{5.12}$$

$$\text{that is,} \quad \frac{1}{\mu} = g_{1n}\left(p_n + \frac{N_0 B}{g_{2n}}\right)$$

Since, μ is a constant satisfying Eq.(4.7) constrained by the Eq.(4.2), the water level can be derived based on Eq.(5.2) taking the average of the total power applied plus the total noise level present in each of the sub-channels. This is given as

$$\frac{1}{\mu g_{1n}} = \frac{1}{N}\left(\sum_{n=1}^{N}\left(p_n + \frac{N_0 B}{g_{2n}}\right)\right) = \frac{1}{N}\left(\sum_{n=1}^{N} p_n + \sum_{n=1}^{N}\frac{N_0 B}{g_{2n}}\right) \tag{5.13}$$

From Eq.(5.12) and (5.13), p_n can be expressed by

$$p_n = \frac{1}{N}\left(\sum_{n=1}^{N} p_n + \sum_{n=1}^{N}\frac{N_0 B}{g_{2n}}\right) - \frac{N_0 B}{g_{2n}} \tag{5.14}$$

Now, using Eq.(5.6), p_n becomes

$$p_n = \frac{1}{N}\left(p_{int} + \sum_{n=1}^{N}\frac{N_0 B}{g_{2n}}\right) - \frac{N_0 B}{g_{2n}} \tag{5.15}$$

Let Δ be the adjustment constant when p_{int} becomes larger than p_{max}. This needs adjusting the transmit power by an amount, $(p_{int} - \Delta)$ according to WF concept. Therefore, the adjustment constant Δ can be computed as, $\Delta = p_{int} - p_{max}$.

Lemma 1: If $p_n \geq \delta$ then $p_n^{'} = p_n - \delta$, $\forall n$, where $\delta = \frac{\Delta}{N}$

Proof: From Eq.(5.15) for sub-channel n, the p_n can be adjusted to get $p_n^{'}$ as below:

$$p_n^{'} = \frac{1}{N}\left(p_{int} - \Delta + \sum_{n=1}^{N}\frac{N_0 B}{g_{2n}}\right) - \frac{N_0 B}{g_{2n}} \tag{5.16}$$

From Eq.(5.15) and (5.16), $p_n^{'}$ can be derived as follows:

$$p_n^{'} = p_n - \frac{\Delta}{N} = p_n - \delta \tag{5.17}$$

This completes the proof of *Lemma 1.* □

5.4.2 Negative Power Adjustment

Lemma 1 shows that in situations when the total interference power p_{int} becomes larger than that of total transmit power limit p_{max}, the total interference power is decreased by an amount Δ. This leads to reduction of an equal amount δ from transmission powers of all the sub-channels to maintain the water level intact. In some sub-channels, this reduction operation may result in negative powers requiring to further reduce power from the sub-channels with positive transmit powers as a compensation to keep the optimal transmit power amount $(p_{int} - \Delta)$ unchange. With classic WF process, these negative transmit powers are set to zero according to Eq.(5.7). This process of required power adjustment, is defined as negative power adjustment.

Let $\Delta^{'}$ be the compensation amount and $k \subseteq C$ be a subset of sub-channels with positive transmit powers. If $p_n \geq \delta$ for sub-channel $n \in k$ and if $p_n < \delta$ for sub-channel $m \in \{C \backslash k\}$, the value of $\Delta^{'}$ can be computed as follows:

$$\Delta^{'} = \frac{1}{|k|} \sum_{n \in \{C \backslash k\}} \delta - p_n \tag{5.18}$$

Lemma 2: If $p_n \geq \delta + \Delta^{'}$ for sub-channel $n \in k$ and $p_n < \delta$ for sub-channel $n \in \{C \backslash k\}$, the modified value will be, $p_n^{'} = p_n - (\delta + \Delta^{'})$ for $n \in k$ and $p_n^{'} = 0$ for $n \in \{C \backslash k\}$

Proof: When the transmit power is decreased by $(p_{int} - \Delta)$, *Lemma 1* can be applied to state that $p_n^{'} = p_n - \delta$ holds though the transmit power poured on all the sub-channels $n \in \{C \backslash k\}$ are negative with assumption $p_n < \delta$. The $max(x, 0)$ function in Eq.(5.7) transforms the negative transmit power values of these sub-channels into zero. In order to maintain the optimum transmit power of $(p_{int} - \Delta)$ unchanged, the positive transmit powers in all the sub-channels $n \in k$ are decremented further by an amount expressed by Eq.(5.18) as a compensation. Using *Lemma 1*, consequently it will have, $p_n^{'} = p_n - (\delta + \Delta^{'})$ for $n \in k$ and $p_n^{'} = 0$ for $n \in \{C \backslash k\}$.
This completes the proof of *Lemma 2.* □

From the discussion in section 5.4.2 above, it can be understood that the negative power adjustment process can be formulated as an iterative power adjustment process until the allocated powers to all the sub-channels become positive. So, the power allocation can be carried out without requiring to search for the water level μ, which converges to the sub optimal solution much faster than that of classic WF algorithm. The detail of the negative power adjustment process is given in Algorithm 4.

```
Algorithm 4: Negative Power Adjustment
  Input: $p_n$ (STx power in sub-channel $n$), $\delta = \frac{\Delta}{N}$
  Output: The power vector $\boldsymbol{P} = [p_1, \ldots, p_n]^T$
  Step 1: Iterative negative power adjustment
          While $\exists p_n \leq 0, \forall n \in C$
              – Evaluate the compensation amount $\Delta^{'}$ as per Eq.(5.18) and calculate the total compensation amount from all the respective sub-channels with negative power.
              – Adjust the total compensation amount against all the sub-channels with positive transmission power.
          Repeat until all $p_n$ values become positive for all the sub-channels.
  Step 2: Estimate the power allocation vector $\boldsymbol{P} = [p_1, p_2, \ldots, p_n]^T$.
```

Time complexity analysis of Algorithm 4 (Negative Power Adjustment)

The main task of the algorithm is performed in Step 1 which involve the adjustment of negative power and adjustment of compensation in sub-channels iteratively. The time complexity of the algorithm can be derived as $O(|C \backslash k| + |k|)$, where $|C \backslash k|$ is the maximum number of sub-channels with negative power and $|k|$ is the number of sub-channels with positive power.

5.4.3 The Underlay Power Allocation (UPA) Algorithm

As evidenced from the above discussions the water-filling process to pour power into the sub-channels can be determined at the beginning using Eq.(5.15), and then the WF results can be directly used as a reference to compute the power reduction whenever necessary. The power reduction is necessary to adjust the allocated powers to the sub-channels in order to maintain the total transmit power limit of the SU within p_{max}. During the water filling process, the sub-channels are ordered in the decreasing values of power allocated to the sub-channels. With this observation, the power allocation technique for underlay communication is modeled as a modified WF process and named as underlay power allocation (UPA) in short. The iteration in Step 3 of the UPA algorithm ensures that the power for each of the sub-channel is non-negative during the power reduction process. The details of the process involved in the proposed power allocation technique is given in Algorithm 5.

As the AIP threshold I_{max} is known to a STx, it will compute the maximum tolerable interference limit using Eq.(5.2). Using this limit, Algorithm 5

Algorithm 5: Underlay Power Allocation Algorithm (UPA)

```
Input: $N$(Number of sub-channels), $I_{max}$ (AIP threshold), $p_{max}$ (maximum
       total transmit power of STx), $N_0$ (AWGN noise power), $B$ (bandwidth of
       channel), $g_{1n}$ (channel gain between STx and PRx in sub-channel $n$) and
       $g_{2n}$ (channel gain between STx and SRx in sub-channel $n$)
Output: The power vector $\boldsymbol{P} = [p_1, \dots, p_n]^T$

Step 1: Estimate $p_{int}$ as per Eq.(5.6) and calculate power allocation, $p_n$, $\forall n \in C$
        as per Eq.(5.15).
        If $p_n \leq 0$, $\forall n \in C$ then set $\delta = 0$ and call Algorithm 4.
Step 2: Check whether total transmit power limit is satisfied or not using
        Eq.(5.3). If yes go to Step 4. Otherwise go to Step 3.
Step 3: Iterative power reduction
          – Evaluate $\Delta = p_{int} - p_{max}$
          – Reduce transmit power $p_n$, $\forall n \in C$ by an amount $\delta = \frac{\Delta}{N}$ and set
            $p_n^{'} = p_n - \delta$ as per Eq.(5.17) $\forall n \in C$
          – If $p_n^{'} > 0$, $\forall n \in C$ then set $p_n = p_n^{'}$, $\forall n \in C$ and go to Step 4.
          – Otherwise call Algorithm 4 with $p_n = p_n^{'}$, $\forall n \in C$
Step 4: Estimate the power allocation vector $\boldsymbol{P} = [p_1, p_2, \dots, p_n]^T$.
```

generates a power allocation vector which satisfies Eq.(5.2). Then power allocation is constrained by the PU's AIP limit as it is within the SU's total transmit power limit and thus generated $\boldsymbol{P}$ is the final solution. When the PU's AIP limit grows larger than that the SUs total transmit power limit, the power allocation will be constrained by the SUs total transmit power limit after $\boldsymbol{P}$ is decreased by Δ amount to satisfy Eq.(5.3).

Time complexity analysis of Algorithm 5 (Underlay Power Allocation Algorithm)

The main tasks of the algorithm are performed in Step1 and Step 3 which involve iterative power reduction using Algorithm 4. The time complexity of the Algorithm 4 is $O(|C \backslash k| + |k|)$ as explained earlier. Therefore, the time complexity of the algorithm 5 can be derived as $O(|C \backslash k| + |k|) + O(|C \backslash k| + |k|)$, where the 1^{st} and 2^{nd} term of the complexity represent the tasks to perform negative power adjustment (in Step 1) and to reduce power for adjusting total transmit power limit (in Step 3) respectively.

5.5 Simulation Results and Observations

In this section, simulation results are presented to evaluate the performance of the proposed UPA with respect to the adjustable parameters. The performance of the proposed scheme is compared with classic water-filling scheme. We first present the results to show the relationship between the achieved spectral efficiency (that is, throughput) and total transmit power for a fixed number of sub-channels. Next, we show how the computational complexity in terms of running time changes with increase in number of sub-channels. Then we present the results to show how the spectral efficiency is regulated by the interference tolerance power (that is, AIP constraint of PU receiver), which is maintained within the total transmit power limit of the SU transmitter. Finally, the results are presented to study the effectiveness of the power adjustment scheme under the given constraints.

A MATLAB based simulation has been carried out to study the behavior and the efficacy of the proposed power allocation technique in an Intel i3 computer. The parameters taken for simulation are listed in the Table 5.2. For simulation purpose, a network is setup with a single PU pair in such a way that the PU transmitter is placed at the center of a circular area with 100m radius and a pair of SUs are placed randomly within the given range. The simulation is carried out considering that the total number of sub-channels N is assumed to be varied from 48 to 144. Based on 50 randomly generated CSI (the gain matrices), each of the performance point is approximated. The channel gains $\{g_{1n}|n \in C\}$ and $\{g_{2n}|n \in C\}$ are represented by i.i.d. chi-square random variables with a degree of freedom of one [27]. The phrase "spectral efficiency" is used to denote the capacity rate in Eq.(5.1) over the whole bandwidth B during simulation.

Parameter	**Value**
N (Number of sub-channels)	48-144
$(N_0B) \times N$	1 [27]
g_{1n} (Channel gain from STx to PRx)	i.i.d. chi-square random variable with a degree of freedom of one [27]
g_{2n} (Channel gain from STx to SRx)	i.i.d. chi-square random variable iwith a degree of freedom of one [27]
I_{max} (PU receiver's AIP threshold)	$1/N$

Table 5.2: Values of different parameters used in simulation

Figure 5-2 shows the result of experiment conducted to evaluate the performance of the proposed power allocation technique in terms of achieved spectral efficiency with varying total transmit power. During the experiment, the p_{int} is set to be equal to p_{max} to enable the proposed technique to behave analogous to the classic WF, which allows us to compare the results of the proposed UPA technique with the classic WF technique. It shows that the proposed technique achieves the spectral efficiency at par with the classic WF technique, even without resorting to searching for the water level μ. It confirms the correctness of the proposed

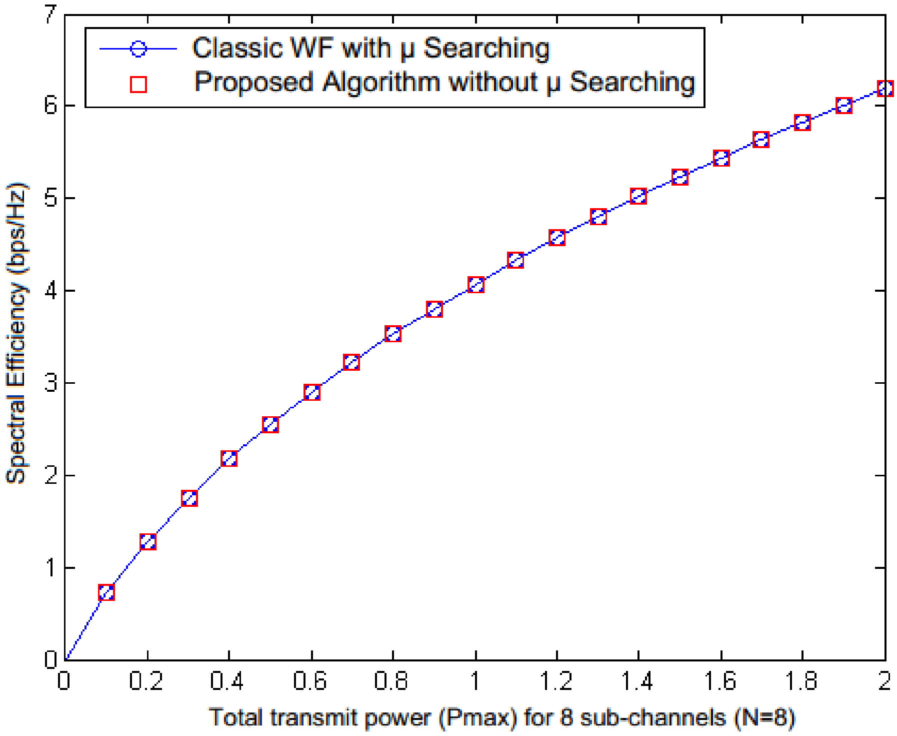

Figure 5-2: Spectral Efficiency compared to Classic WF for number of sub-channels, $N = 48$

technique. It also shows that while the transmission power of the SU increases from zero to its maximum value (i.e. total transmit power, p_{max}), the capacity rate in terms of spectral efficiency also increases. The value of p_{max} is assumed to be 2 watts for the SU with 48 sub-channels. The capacity rate in terms of spectral efficiency achieved at $p_{max} = 2$ watts is 6.1 bps/Hz.

Figure 5-3 demonstrates the performance enhancement of the proposed technique in terms of running time while varying the number of sub-channels. It shows that the proposed technique runs much faster in comparison with the classic WF technique. It shows that with increase in number of sub-channels, running time for classic WF technique increases reasonably, whereas only marginal or negligible changes can be observed using the proposed technique. It shows that, in the experiment as the value of N is allowed to vary from 48 to 144, the classic WF technique gradually suffers, whereas the proposed technique performs consistently efficiently. When N increases, the running time increases almost linearly in the case of classic WF technique, this increase is due to the extra μ searching operations. The proposed technique remains almost steady with high N and a slight variation that is seen in the result is due to the fluctuation of CSI.

Figure 5-4 shows the spectral efficiency of the proposed technique changes with total transmit power while the interference tolerance power (i.e. p_{int}) is within the total transmission power limit (i.e. p_{max}). It shows that the proposed technique gives optimal spectral efficiency, under the power allocation constraint by AIP as well as the total transmit power constraint of the SU. It can be observed

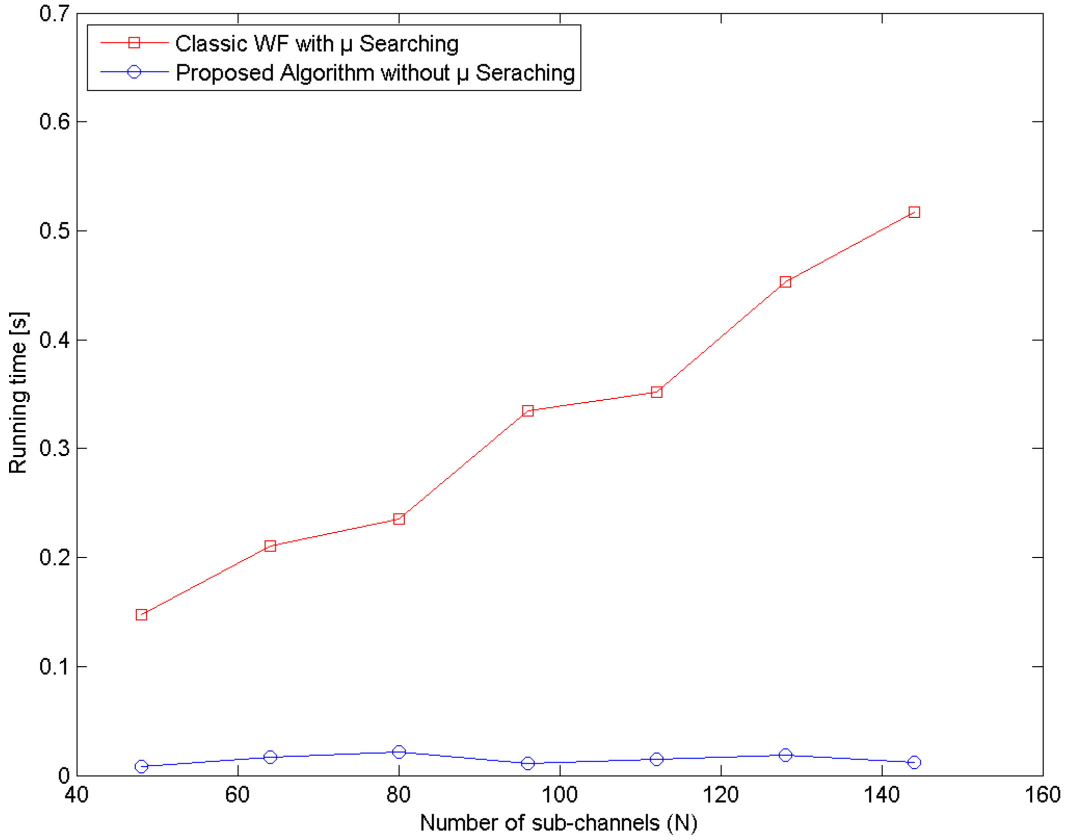

Figure 5-3: Running Time vs. Number of Sub-channels

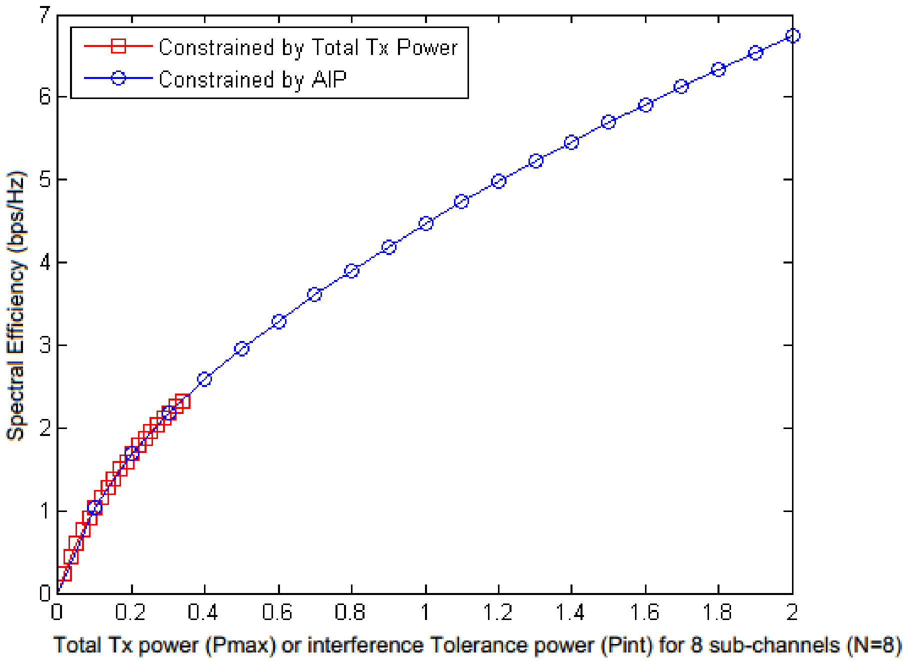

Figure 5-4: Spectral Efficiency while p_{int} is within p_{max}

that the spectral efficiency achieved for 8 sub-channels with the generated value of p_{int} to be 0.3 watts, is 2.2 bps/Hz. Since p_{int} is constrained by AIP, which is the prime requirement for PU protection in underlay CRN, the allowable maximum transmission power of the SU transmitter is p_{int} and the capacity rate achieved with this amount of allocated power is the optimal value, which is found to be 2.2 bps/Hz in the conducted experiment. On the other hand, for the same number of sub-channels with p_{max} set to be 2 watts, the spectral efficiency achieved is 6.9 bps/Hz. This is much higher as the allowable maximum transmission power of the SU transmitter regulated by p_{int} is its p_{max}. It also shows that, while p_{int} value increases close to p_{max} of the SU, the spectral efficiency improves further significantly. This results prove that the proposed technique can handle the issue of underlay power allocation dynamically depending on which of the constraints in Eq.(5.2) and (5.3) is applied during the power allocation.

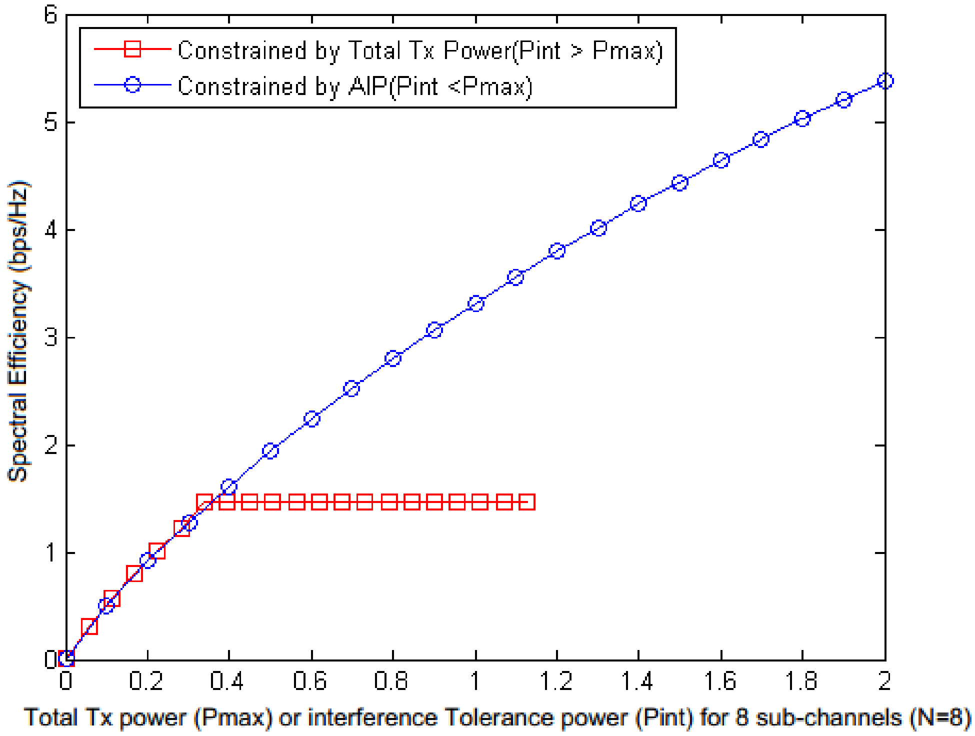

Figure 5-5: Spectral Efficiency while p_{int} is greater than p_{max}

Figure 5-5 shows the performance of the proposed scheme in terms of spectral efficiency when the interference tolerance power (i.e. p_{int}) is higher than the total transmission power limit (i.e. p_{max}). It can be seen that the proposed technique gives optimal spectral efficiency, considering that the power allocation is constrained by p_{max} of the SU. The power allocation is to be constrained according to p_{max}, when the interference tolerance power p_{int} is much higher than the total transmission power limit p_{max} of the SU. To verify this, the experiment is conducted by setting the value of p_{max} to be 0.3 watts while allowing the value of p_{int} to be varied maximum up to 2 watts. It shows that the spectral efficiency is constrained by the total transmission power limit and becomes steady for even higher values of generated interference power and thus protecting the PU from interference while keeping the SUs power allocation feasibility intact. At this point with $p_{max} = 0.3$ watts, the spectral efficiency achieved for 8 sub-channels is 1.6 bps/Hz. Further even with higher values of N the capacity rate in terms of spectral efficiency will be optimized as per the constraint in Eq.(5.3). On the other hand, for the same number of sub-channels with p_{max} allowed to be 2 watts, the spectral efficiency achieved is 5.3 bps/Hz, which is much higher as the allowable maximum transmission power of the SU transmitter regulated by p_{int} is its p_{max} only.

5.6 Conclusion

In this chapter, the power allocation problem in OFDM-based underlay CRN for capacity rate maximization is investigated to maximize the throughput (that is, capacity rate) under the constraints of average interference power (AIP) limit of PU receiver, minimum SNR threshold of SU transmitter and the total transmit power of SU. The problem of power allocation on a channel by an SU transmitter is solved as a convex optimization problem and modeled as a modified water-filling process. The proposed scheme is able to maximize the throughput of the SU through directly computing the power allocation vector while utilizing the initially allocated power onto the sub-channels. The water-filling process is used only for single initial iteration to pour the power onto the sub-channels. Using the water-filling process only once, the proposed scheme improves the computational complexity by avoiding the iterative search in water-filling. It eliminates the need to perform iterative search for adapting the possible change in allowable maximum transmit power. Therefore, the proposed scheme runs much faster, while implementing the PU protection dynamically using the AIP of PU over the sub-channels. The proposed scheme is shown to maximize the throughput analytically. Simulation results validate the effectiveness of the scheme compared to the classic water-filling technique.

With the enhanced capacity rate offered by the UPA technique for single user scenario, the next task is to develop an efficient power allocation scheme for the multiuser scenario, which will be addressed in the next chapter.