Chapter 3

Qualitative Modeling of Movement Parameters for Rectangular Spatial Objects

In this chapter, qualitative modeling of two movement parameters, namely, direction of motion and spatial orientation will be discussed. For each of these parameters, new formalisms, in terms of sets of binary qualitative JEPD relations, have been proposed.

3.1 Qualitative Direction Relations

3.1.1 Need for Development

In chapter 2, under section 2.2, existing works on direction, orientation and distance have been reported. Formalisms for direction assume something about the dimensionality of the spatial objects. In oriented point algebra [43], spatial objects are abstracted as dimensionless points. In dipole relation algebra, objects are considered as dipoles. These formalisms do not scale if spatial objects are abstracted as two dimensional bodies. This is because of the fact that definition of qualitative direction relations rely on the dimensionality. For example, in the *lrrr* relation of dipole relation algebra (shown in Figure 2.5), relative position of the end points of the dipoles are considered for defining the relation. So, the definition inherently assumes that the object is a one dimensional spatial entity. Similarly for oriented point relation algebra, in the Figure 2.3, we can see that use of labels like *front*, *back* etc. is based on an assumption that the spatial object is a point. The 9+ intersection model [45], considers motion of an object with respect to a two dimensional region. This model does not handle the relationship between qualitative directions of two moving objects.

There is a need to develop a new formalism that can express the relationship between qualitative directions of two entities without assuming anything about their dimensions. Such a formalism will be useful for representation of qualitative direction even for two dimensional spatial objects. In this section, we propose a formalism for qualitative direction that uses an egocentric spatial reference frame. An egocentric reference frame is useful for modeling group intersections whether in video processing or in GIScience applications.

3.1.2 Definition of Qualitative Direction Relations

We begin by defining two concepts, namely, a direction line and a direction region. Intuitively, a direction line shows the direction of motion of an object at a time instant or during a time interval. Direction regions will be used later for presenting a spatial orientation model for rectangular objects.

Definition 3.1.1 *A Direction Line is a line segment in a two dimensional plane having a direction* dir*.*

Definition 3.1.2 *Direction Region : Let l_1 and l_2 be two direction lines having directions dir_1 and dir_2 respectively and having a point of intersection* o*. Let θ be the angle between dir_1 and dir_2 in an anticlockwise direction. Then, a direction region defines a set of direction lines that originate at* o *and the direction of any such line is bounded by the angle θ from dir_1 in an anticlockwise direction.*

In trigonometry, angles measured in anticlockwise direction are taken as positive and the clockwise ones as negative. Since all angle ranges used in the def-

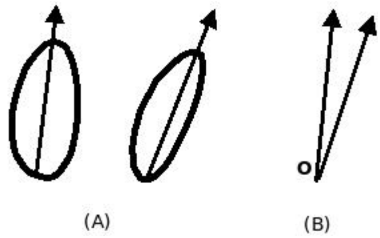

Figure 3.1: Objects and Their Direction

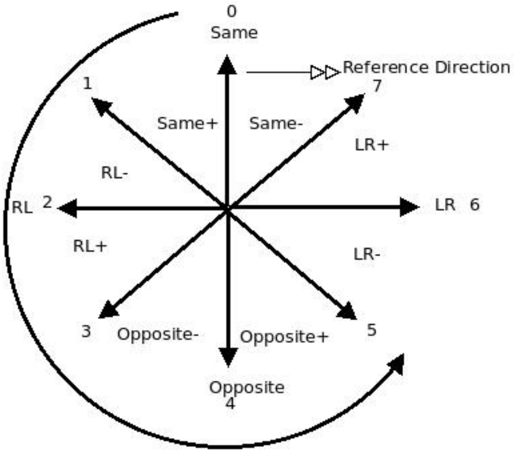

Figure 3.2: Direction Relations

inition of direction relations use positive angles, we choose to use anticlockwise direction for definition of relations.

In Figure 3.1, we have shown two objects whose egocentric heading is indicated by arrowheads (part A). In part (B), two direction lines are drawn parallel to the directions of the two objects. The point of intersection of these two direction lines is *o*.

For defining the qualitative direction relations, we start with four qualitative direction relations, namely, *Same* , *Opposite* , *LR* and *RL*. Intuitive meaning of *A Same B* is that the objects are directed in the same direction. In *LeftToRight* relation (abbreviated as *LR*) , one object is directed in a left-to-right orientation with respect to the other and in *RightToLeft* (abbreviated as *RL*), the situation is just the opposite.

We want to emphasize the fact that *LR* and *RL* are in no way related with spatial orientation of the objects. If an object moves along its egocentric direction, its course of motion divides the two dimensional plane into two parts, one

Table 3.1: Refined Direction Relations

Sl. No.	Base Relation	Angle Range	Sl. No.	Direction Relation	Angle Range
1	*Same*	$[0,0]$	7	*lr*	$[270,270]$
2	*Same+*	$]0,45]$	8	*lr+*	$]270,315]$
3	*Same−*	$]315,360[$	9	*lr−*	$]225,270[$
4	*Opposite*	$[180,180]$	10	*rl*	$[90,90]$
5	*Opposite+*	$]180,225]$	11	*rl+*	$]90,135]$
6	*Opposite−*	$]135,180[$	12	*rl−*	$]45,90[$

to the left and one to the right. Now, if the second object moves in *LR* direction, its course of motion is from the left to the right with respect to the first and intersects the first at 90 degrees. In Qualitative Spatial Reasoning, it is common to introduce abstractions and to discretise the domain under consideration. In order to refine each of the above relations, a span of 45 degrees counterclockwise is denoted by a + and the same in clockwise direction is denoted by a -. So, if the direction of the primary makes an angle less than or equal to 45 degrees anticlockwise with the direction of the reference object, the resulting relation is *Same+* and in the counterclockwise case it is *Same-*. Similar convention can be followed to arrive at relations like *Opposite+* , *Opposite-* , *LR+* , *Lr-* , *RL+* and *RL-*.

For defining the angle ranges for the direction relations, we assume that the direction line of the primary is rotating in a counterclockwise direction. When this line aligns with the direction line of the reference, both are moving in the *Same* direction. When the angle between these two lines is more than zero but less or equal to 45 degrees, the relation is *Same+*. Continuing in the counterclockwise direction, the next relation *rl-* will have the more than 45 degrees but less than 90 degrees. It is less than 90 degrees because when the angle is exactly 90 degrees, the relation is *rl*. The last relation in this counterclockwise rotation (*Same-*) will hold when the angle is more than 315 degrees but less than 360 degrees. It has to be less than 360 degrees because when it is equal to 360, both the direction lines align again and the relation becomes *Same*. In all these definitions, a span of 45 degrees is chosen because in that case each relation gets an equal direction range. Thus, we have obtained twelve base relations. At this level of granularity, changes in direction are noticed after a threshold of 45 degrees. In Table 3.1, these base relations are enumerated along with direction regions in terms of angles of the bounding direction lines. Out of these base relations, there are some relations that are exact (like Same, Opposite etc.) and there are others that are spread-out i.e. a range (like Same+, rl- etc.). For assigning angle ranges, We start with an angle of zero between primary and reference direction lines (Same relation), move in an anticlockwise direction and go on assigning ranges. For example, from an angle of more that zero degree and up to and including 45 degrees, the relation is *Same+*. Then, from an angle of more than 45 degrees and to less than 90 degrees, it is *rl-* and so on. The exact angle of 90 degrees is defined as the *rl* relation.When we follow this principle for assigning other angle ranges to direction relations, the relation *Same-* becomes] 315,360[. It cannot be] 315, 360] because then the direction lines are parallel and the angle between them is zero. This range is already assigned to the relation *Same*. This is the reason for which *Same+* gets an interval]...] where as *Same-* gets]...[. Had we chosen a clockwise direction, the situation would have been reverse i.e. *Same+* would have got]...[whereas *Same-* would have got]...]. In Figure 3.2, the relations are illustrated. Direction regions are expressed in terms of positive angles measured in counterclockwise direction for all relations.

It is important to note that out of the relations listed in Table 3.1, some relations are exact whereas others are spread-out i.e. a region. The relations *Same*, *Opposite*, *lr* and *rl* are exact relations. In such a relation, definition of the relation is based on exact equality of values in the context of definition. For example, for the exact relations in Table 3.1, the angle between the direction line of the primary and that of the reference is exactly equal to some value. The other relations in the table are spread-out in the sense that the angle between the direction line of the primary and that of the reference falls in a range. Similarly for the Allen's interval algebra relations listed in Figure 2.9, relations like *meets*, *equal* are exact relations.

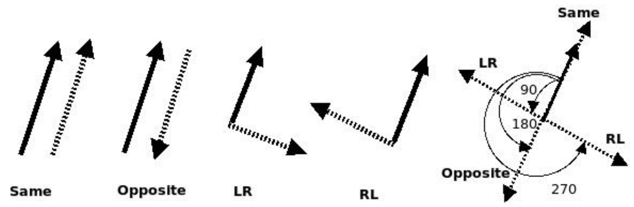

Figure 3.3: The Direction Relations

3.1.3 Refinement of Granularity

In defining the above relations, the wheel shown in Figure 3.2, divides 360 degrees into eight regions, each having a span of 45 degrees.

For certain applications, we may have to observe smaller changes in direction and as such, refinement of qualitative direction relations may be necessary. We will explain below how the direction relations defined above can be easily refined. The same process can be repeated further for additional refinements. We equally divide the + and - direction regions. For example, if we divide the + region for which the angle range is $]0, 45]$, we obtain two direction regions of span 22.5 degrees each. The region $[0, 22.5]$ is denoted by the symbol $+_1$ and the region $]22.5, 45]$ is denoted by $+_2$. As a result, we get twenty four base relations that are listed in Table 3.2. This time, change in direction is noticed after a threshold of 22.5 degrees. These refined relations are shown in Figure 3.4.

In order to apply constraint based reasoning to a set of spatial relations, we develop a partition scheme for the objects in the domain under consideration [132] and arrive at a set of Jointly Exhaustive Pairwise Disjoint (JEPD) base relations. General relations are obtained by taking the power set of base relations, with top, bottom, union, intersection and complement of relations defined in the set theoretic way. Moreover, an identity relation and a converse operation on base relations must be provided.

For the set of base relations introduced earlier, *Same* is the identity relation. Each relation is closed under converse operation.

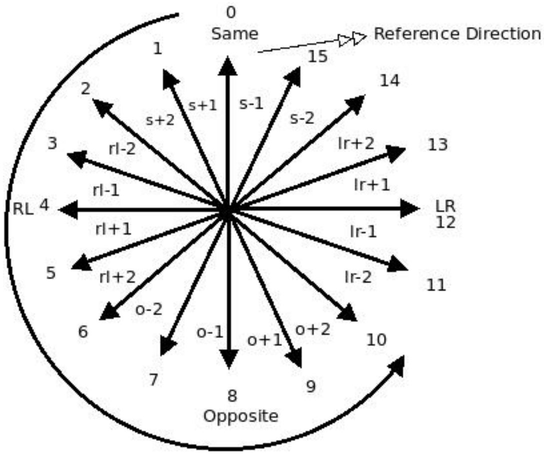

Figure 3.4: Direction Relations: One Level Refined

Table 3.2: Refined Direction Relations

Sl. No.	Base Relation	Angle Range	Sl. No.	Base Relation	Angle Range
1	*Same*	$[0, 0]$	13	*lr*	$[270, 270]$
2	$Same{+}_1$	$]0, 22.5]$	14	$lr{+}_1$	$]270, 292.5]$
3	$Same{+}_2$	$]22.5, 45]$	15	$lr{+}_2$	$]292.5, 315]$
4	$Same{-}_1$	$]337.5, 360[$	16	$lr{-}_1$	$]247.5, 270[$
5	$Same{-}_2$	$]315, 337.5]$	17	$rl{-}_2$	$]225, 247.5]$
6	*Opposite*	$[180, 180]$	18	*rl*	$[90, 90]$
7	$Opposite{+}_1$	$]180, 202.5]$	19	$rl{+}_1$	$]90, 112.5]$
8	$Opposite{+}_2$	$]202.5, 225]$	20	$rl{+}_2$	$]112.5, 135]$
9	$Opposite{-}_1$	$]157.5, 180[$	21	$rl{-}_1$	$]67.5, 90[$
10	$Opposite{-}_2$	$]135, 157.5]$	22	$rl{-}_2$	$]45, 67.5]$

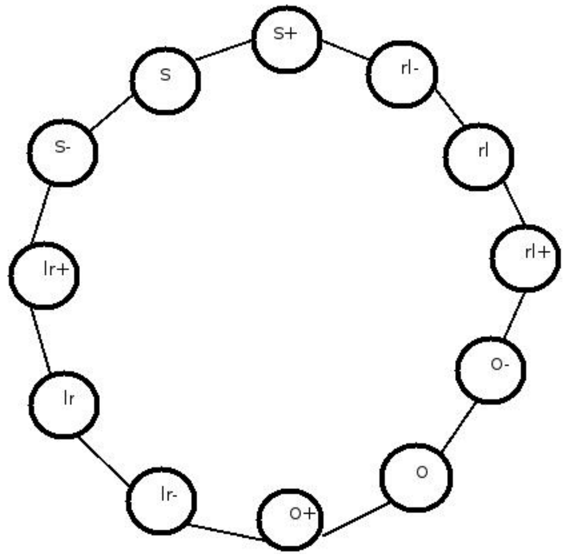

Figure 3.5: Conceptual Dependency of Base Relations

3.1.4 Conceptual Neighbourhood

When any direction relation R holds between two spatial objects A and B, an interesting question is what are the possible relations that may hold between the directions of A and B whenever any change is observed. For example, if at some point of time *Same* is the relation, then it is not possible that this relation will change directly to *Opposite* assuming that motion is continuous and spatio-temporal continuity is respected. This gives rise to a notion of spatio-temporal continuity which can be exploited in many application. The relations that may hold after the current relation are termed its conceptual neighbour and generally these conceptual neighbours are expressed by graph where nodes represent relations and from a node we draw edges to its conceptual neighbours. In Figure 3.5, conceptual dependency of 12 base relations enumerated in Table 3.1 is shown.

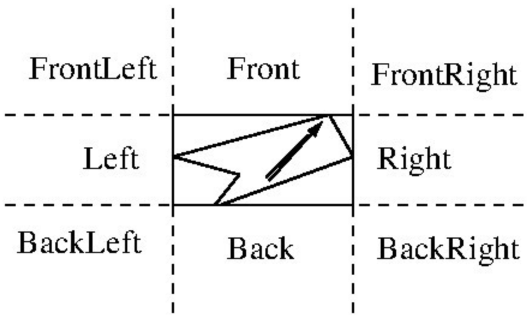

Figure 3.6: Effect of Egocentric Direction

3.2 Qualitative Spatial Orientation

3.2.1 Need For Development

Spatial orientation tells us where an object is located with respect to another. Well known formalisms on orientation of rectangular bodies are direction relation matrix and the formalism of rectangular cardinal direction that evolved from it (discussed in 2.2.2). If the spatial reference frame is external (allocentric), spatial orientation labels like *Front*, *Left* etc. can be used in the way it is done in rectangular cardinal directions (illustrated in Figure 2.8). When the frame of reference is egocentric, certain problems arise.

In Figure 3.6, rectangular cardinal directions set up by extending the *MBR* of an object are shown. The egocentric direction of the object is indicated by an arrowhead. When the direction of movement of the object is along the direction shown by the arrowhead, the region shown as *Front* actually lies to its left and similarly the region *FrontLeft* should be treated as back left. This is true because in such a *FoR*, it is inappropriate to preassign orientation labels. Concepts of *Front*, *Back* etc. depend on the direction in which the object is moving. Therefore, there is a need to develop a new spatial orientation model for rectangular objects for egocentric spatial reference frame.

3.2.2 Definition of Spatial Orientation Relations

In part (A) of Figure 3.8, a spatial orientation model is presented. The direction of motion is shown (represented by the direction line *OS*) and with the help of

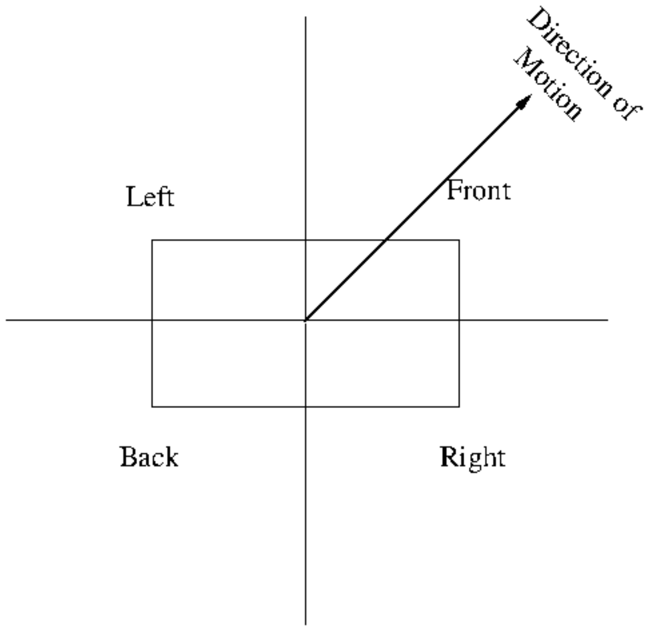

Figure 3.7: Spatial Orientation Regions For Rectangles

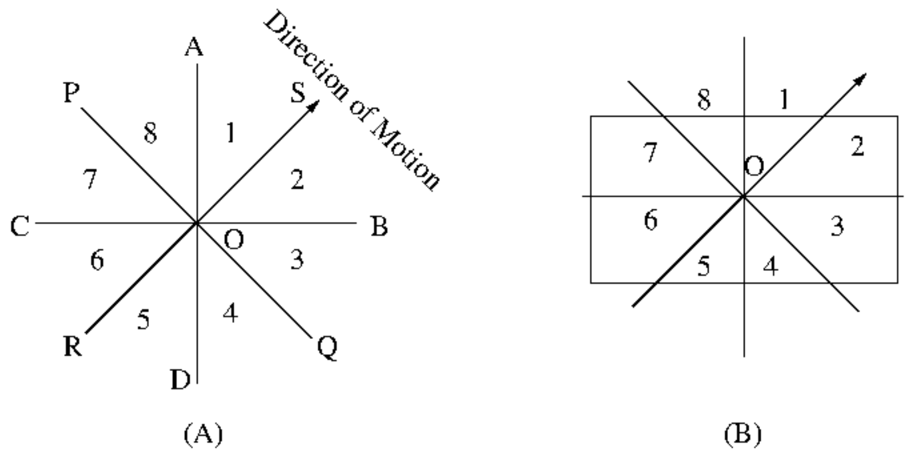

Figure 3.8: Spatial Orientation Model

this, spatial orientation labels like *Front*, *Back* etc. in egocentric reference frame can be defined.

The rays *OP* and *OQ* are at right angles to the direction line *OS*. The ray *OR* is obtained by extending the direction line backwards. These rays create four orientation regions. These orientation regions are *SOP*, *SOQ*, *ROP* and *ROQ*. These regions need to be refined further because we have not yet arrived at regions which can be labeled as *Front*, *Back* etc. depending on direction of movement. The orientation label *Front* (similarly other labels like *Back*, *Left* etc.) actually indicates a region that lies within an angular span measured from the

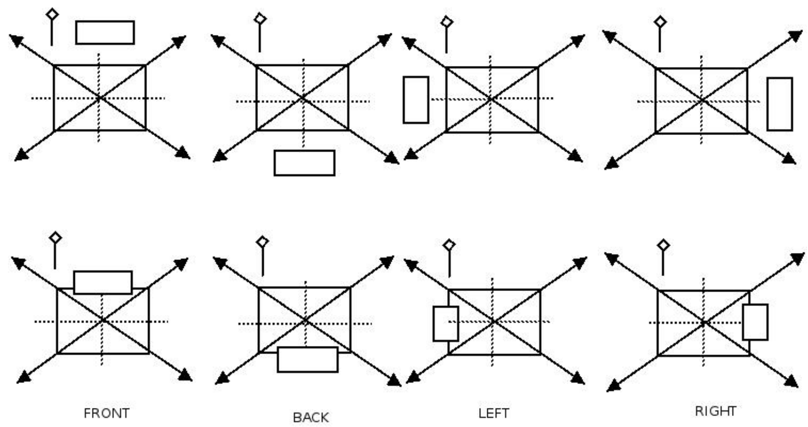

Figure 3.9: Spatial Orientation: One Region Span

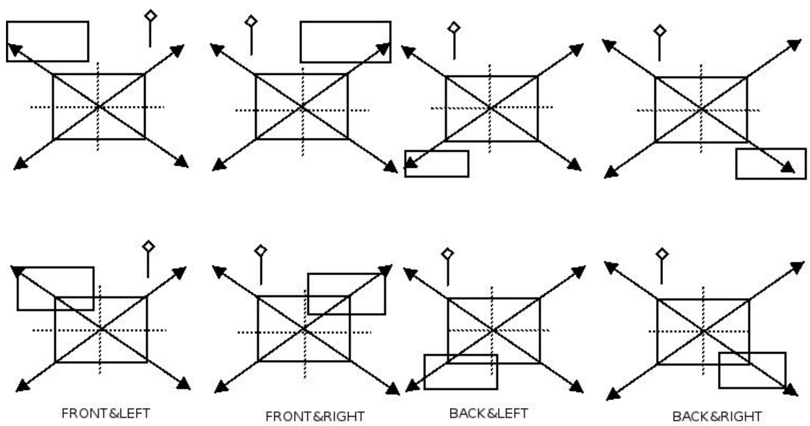

Figure 3.10: Spatial Orientation: Two Region Span

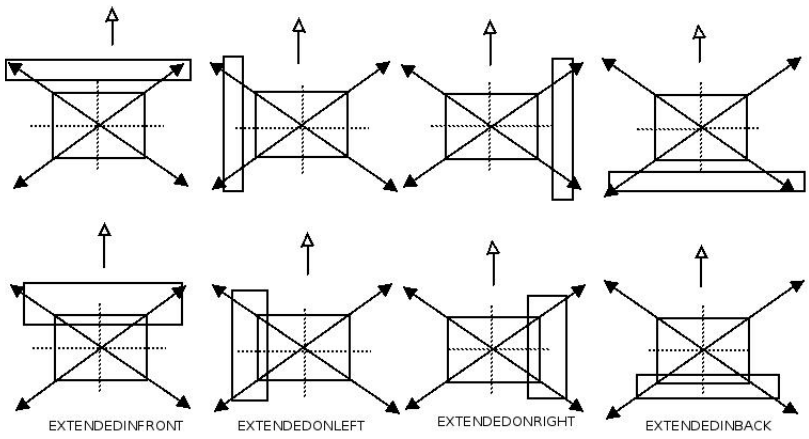

Figure 3.11: Spatial Orientation: Three Region Span

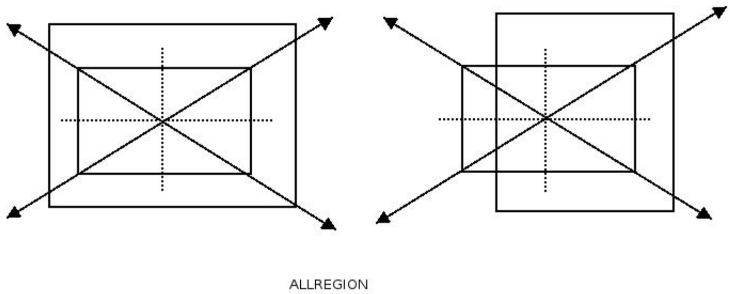

Figure 3.12: Spatial Orientation: Four Region Span

direction of motion. For example, a span of 45 degrees measured in clockwise and also in anticlockwise direction can indicate a *Front* region. The most appropriate span may depend on the size and shape of the object. In a general model, it will be reasonable to keep this span same for all the four orientation labels i.e. *Front*, *Back*, *Left* and *Right*. Therefore, the rays OA and OB divide the right angles equally and the region AOB is labeled as the *Front* region. Similarly, the rays OC and OD divide the other two right angles equally and the region COD is marked as the *Back* region. Once these two regions are labeled, the other two orientation regions i.e. *Left* and *Right* are automatically marked. In Figure 3.8, orientation regions are indicated by integers. For example, the regions *1* and *2* are indicate *Front*. Regions *3* and *4* indicate *Right*. Though we have used equal span for all the four spatial orientation regions, this does not mean that spans can not be different. The spatial orientation relations that we are going to define are unaffected by this issue of span. In part (B) of Figure 3.8, the spatial orientation model is used for a rectangle. The point O is aligned with the point of intersection of the two lines joining the mid points of the opposite sides of the rectangle.

In Figure 3.7, the spatial orientation model is redrawn for a reference rectangle showing the orientation regions. It can be seen that when direction regions are used as orientation regions, a cone based orientation model results. In addition to the four orientation relations, namely, *Front*, *Back*, *Left* and *Right*, more number of relations will result because rectangles are two dimensional and a rectangle may span more than one region. If the primary rectangle is completely included in one of the conical regions, then there will be four such cases. These cases are shown in Figure 3.9. The resulting orientation relations can be named as *Front*, *Back*, *Left* and *Right*. If the primary spans two such conical regions, then also four new relations result. These relations can be named considering the two cones the primary has spanned across. In Figure 3.10, these cases are shown. This figure has two lines. In the top line, the non-overlapping span is illustrated and in the bottom line, overlap of primary and reference rectangles is shown.The relations have been named as $Front\&Left$, $Front\&Right$, $Back\&Left$ and $Back\&Right$. Figure 3.11 shows how a primary rectangle may span three conical regions. Both overlapping and non-overlapping cases are drawn. The resulting relations are named as *ExtendedInFront*, *ExtendedInBack*, *ExtendedOnRight* and *ExtendedOnLeft*.

It is possible that the primary may span all the four cones and this situation is sown in Figure 3.12. The resulting spatial orientation relation is named as *AllRegion*. At this point *13* binary qualitative spatial orientation relations have resulted and these relations have been enumerated in Table 3.3.

3.2.3 Refinement of Spatial Orientation Relations

In defining the orientation relations listed from serial number *1* to *13* in Table 3.3, overlapping and non-overlapping orientations have not been distinguished.

If these cases are distinguished, then for each of the relations listed in Table 3.3, each of the relation will be broken into two, one indicating the overlap and the other for the non-overlap. The total number of spatial orientation re-

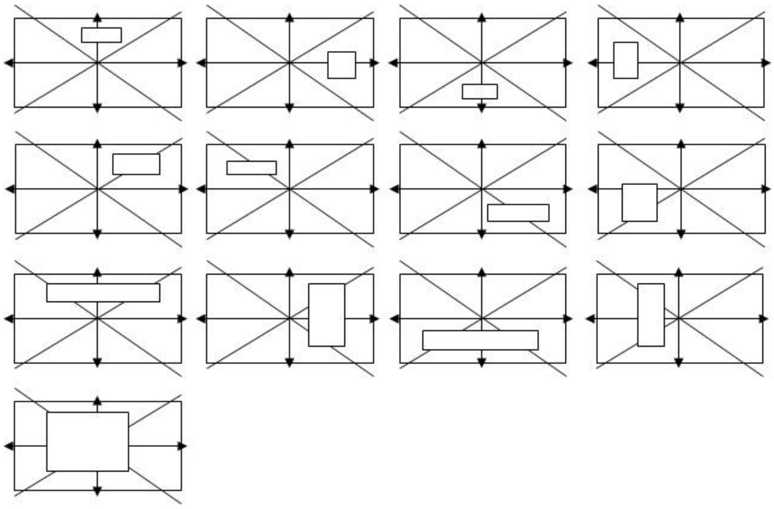

Figure 3.13: Inclusion of The Primary Inside the Reference

Table 3.3: Spatial Orientation Relations

Sl. No.	Relation	Abbrv	Sl. No.	Relation	Abbrv
1	Front	F	14	FrontOverlap	FO
2	Left	L	15	LeftOverlap	LO
3	Back	B	16	BackOverlap	BO
4	Right	R	17	RightOverlap	RO
5	Front&Left	F&L	18	Front&LeftOverlap	F&LO
6	Front&Right	F&R	19	Front&RightOverlap	F&RO
7	Back&Left	B&L	20	Back&LeftOverlap	B&LO
8	Back&Right	B&R	21	Back&RightOverlap	B&RO
9	ExtendedInFront	EIF	22	ExtendedOverlapInFront	EOIF
10	ExtendedInBack	EIB	23	ExtendedOverlapInBack	EOIB
11	ExtendedOnRight	EOR	24	ExtendedOverlapOnRight	EOOR
12	ExtendedOnLeft	EOL	25	ExtendedOverlapOnLeft	EOOL
13	AllRegion	AR	26	AllRegionOverlap	AO
			27	Inside	I
			28	Equal	E

lations will increase to *26*. Further refinement is possible if the cases where the primary is inside the reference are treated separately. Various ways in which the primary may be inside the reference are shown in Figure 3.13. A single relation with the name *Inside* is introduced to take care of the cases where the primary is included inside the reference. For constraint-based spatial reasoning [19], it is necessary to have an identity relation over the set of base relations. A relation named *Equal* is introduced with the semantics that this is the relation that each has rectangle has with itself. The *Equal* relation is the identity relation for the set of spatial orientation relations. All the base relations are listed in Table 3.3. From the definition of the spatial orientation relations in terms of the number of conical regions spanned, it is obvious that the set of spatial orientation relations is Jointly Exhaustive and Pairwise Disjoint (*JEPD*).

Lemma 3.2.1 *The set of spatial orientation relations listed in Table 3.3 is complete i.e. Jointly Exhaustive. Proof:The spatial orientation relations are defined according to the number of conical regions spanned by the primary rectangle. There are four conical regions and out of these four, the primary rectangle can span only one region, two regions, three regions or all the four regions. This problem is same as finding combinations out of four objects taking one, two, three and four object(s) at a time. Since the sides of the primary as well as the reference rectangles are parallel to the axes of projection, some combinations are included in others. For one region, there are* $\binom{4}{1}$ *i.e. four ways and corresponding relations are* Front, Back, Left *and* Right. *For two regions, there are* $\binom{4}{2}$ *i.e. six ways. Four out of these are relations* Front&Left, Front&Right, Back&Left *and* Back&Right. *The other two that combine regions* Left *with* Right *and regions* Front *with* Back *are ruled out because the rectangles have finite width and in order to span these regions, the primary has to pass through a third region too. When three regions are taken, there are* $\binom{4}{3}$ *i.e. four ways and corresponding relations are* ExtendedInFront, ExtendedInBack, ExtendedOnRight *and* ExtendedOnLeft. *This constitutes the list of relations from serial number* 1 *to* 12 *in Table 3.3. When all the four regions are spanned, there is only one combination i.e. the relation* AllRegion. *This proves that the set of spatial orientation* *relations is complete.*

The other relations (listed from serial number 14 *to* 26 *in Table 3.3), are special cases of the relations listed from serial number* 1 *to* 13 *when the overlap is given special attention. Moreover, the relation* AllRegion *is refined further into* Inside *and* Equal.

The conceptual dependency of orientation relations is shown using a conceptual dependency graph in Figure 3.14.

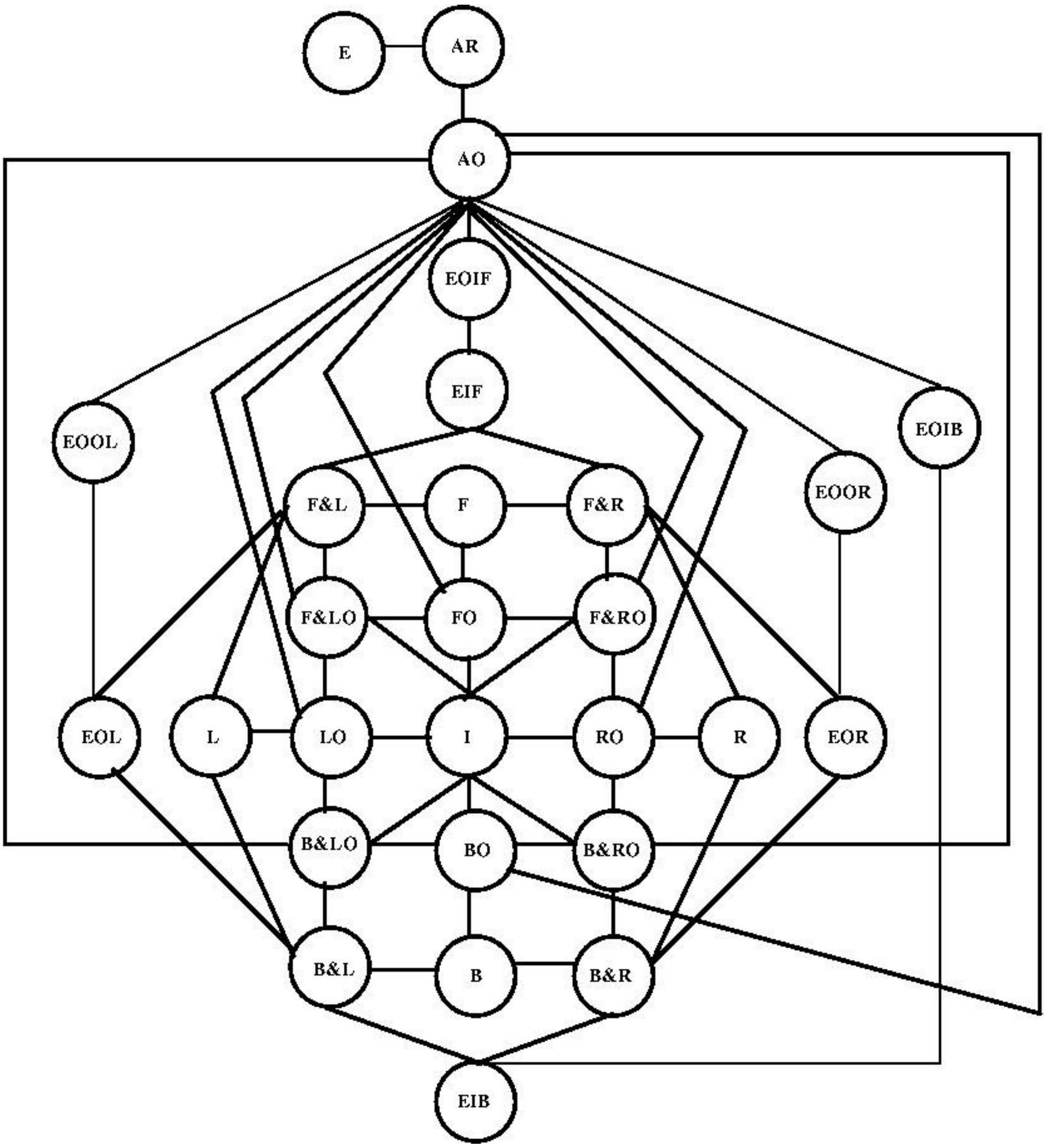

Figure 3.14: Conceptual Dependency of Spatial Orientation Relations

We would like discuss how conceptual dependencies have been found taking a few examples and illustrations. Transitions in this conceptual dependency graph can be caused by motion of a rectangle. Moreover, size variation can also result in transition. For example, let us consider the **B&L** relation. In this case, when the primary moves in positive Y-direction, it may move to the left of the reference. Another possibility is that it may overlap with the reference. If the primary moves in the positive X-axis direction, then the primary moves to the back of the reference. Some of the ways in which **B&L** can transform to its neighbours are illustrated in Figure 3.15.

In relations, **EOOR**,**EOOL**,**EOIF** and **EOIB**, the primary spans three regions and overlaps with the reference. When such a rectangle moves, it goes deeper into the reference and thereby spans one more region. Therefore, the resulting relation becomes **AO**. In Figure 3.16, in part (A), we have illustrated how **EOIF** can transform to **AO**.

Actually, this **AO** relation can result from any overlap relation. For example, when the relation is **LO**, the primary spans a single region and overlaps. When it moves in the positive X-axis direction, it finally it spans four regions.The primary still overlaps the reference. The resulting relation is **AO**. This is shown in part (B) of Figure 3.16. In part (C) of the same figure, it is shown how an **AO** relation can change to **E** relation.

In the next chapter, we establish some theoretical results regarding representation of movement parameters using *JEPD* sets of binary qualitative relations. Use of *JEPD* set of qualitative relations affects the type of grammar necessary for recognition. It also influences issues like learning and error handling.

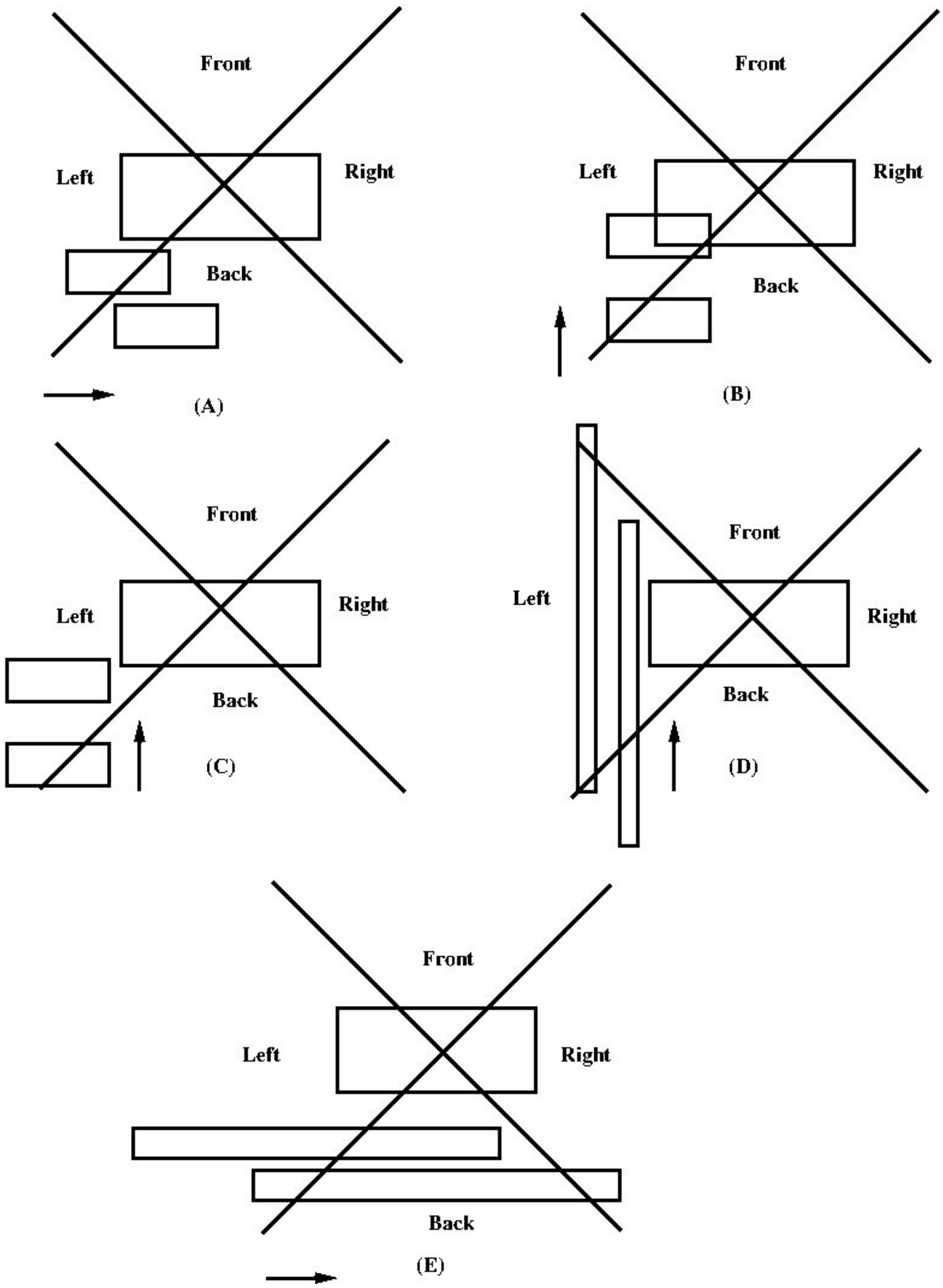

Figure 3.15: Illustration of Conceptual Dependency Transitions 1

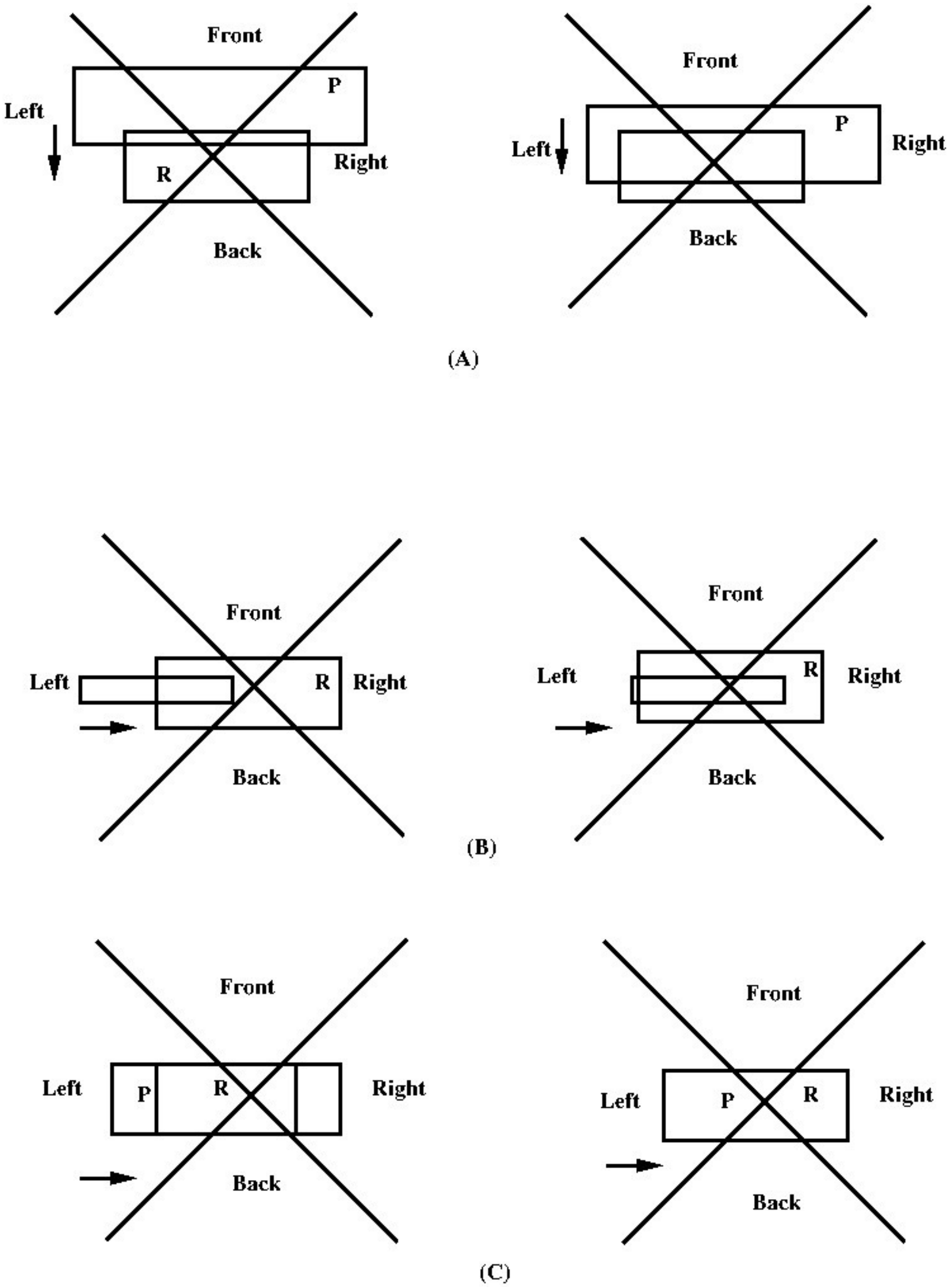

Figure 3.16: Illustration of Conceptual Dependency Transitions 2