Abstract

A ring is called clean if every element can be written as a sum of an idempotent and a unit, further a ring is called strongly clean if every element can be written as a sum of an idempotent and a unit that commute. A ring is called nil clean if every element can be written as a sum of an idempotent and a nilpotent, further a ring is called strongly nil clean if every element can be written as a sum of an idempotent and a nilpotent that commute. The aim of this work is to study clean and nil clean properties of rings viz. nil clean index of rings, weak clean index of rings, weak nil clean rings and nil clean graphs of rings.

In Chapter 2, we have introduced nil clean index of rings, motivated by the notion of clean index of rings. We characterized arbitrary rings with nil clean index 1, 2 and gave a necessary condition for a ring to have nil clean index 3. We ended the chapter with a characterization of formal triangular matrix rings with nil clean index 4.

In Chapter 3, we have introduced weak clean index of rings and characterized arbitrary rings with weak clean index 1, 2, and 3.

Chapter 4 consists of further investigation of weak nil clean rings. Also we have studied S-weak nil clean rings and finally we have ended the chapter with the introduction of weak $J-$clean rings .

Chapter 5 is devoted to the study of the nil clean graph of rings. Graph theoretic invariants like girth, dominating set, diameter etc. of the nil clean graph have been studied for finite commutative rings.