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List of Publications

(A) Publications in Peer Reviewed Journals

- [1] Saikia, P. and Borah, M. Construction of Complete Life Tables for 23 Districts of Assam for both Male and Female, *International Journal of Science and Research*, 5(12): 1-11, 2016.
- [2] Saikia, P. and Borah, M. A Comparison of Models for Projecting Survivors Past Beyond the Last Age for Assam, *International Journal of Mathematics Trends and Technology*, 39(2): 99-109, 2016.
- [3] Saikia, P. and Borah, M. A Comparison of Parametric Models for Graduation of Mortality of Districts of Assam, *Journal of Computer and Mathematical Sciences*, (Accepted).
- [4] Saikia P. and Borah, M. A study on mathematical modelling for oldest-old mortality rates, *International Journal of Mathematical Archive*, 4(11):104-112, 2014.
- [5] Saikia P. and Borah, M. A comparative study of parametric models of old-age mortality, *International Journal of Science and Research*, 3(1):406-410, 2014.

(B) Paper Communicated in Peer Reviewed Journals

- [1] Saikia, P. and Borah, M. A Study of Mortality Models for Oldest-Old Force of Mortality in Assam.
- [2] Saikia, P. and Borah, M. Fitting and Forecasting Mortality for Assam Population by Applying the Lee-Carter Model.

(C) Publications in Papers Presented in International/National Conferences

- [1] A Study on Mathematical Models for Population Growth of Assam, presented at National Seminar on Contemporary Issues of Mathematics and Statistics (NSCIMS), at Gargaon College, Sivasagar, Assam during 22nd and 23rd March, 2013.
- [2] Mortality Modelling for Assam Female Population, presented at the National Conference on Pure and Applied Mathematics, 2013, at Royal Group of Institutions, Guwahati, Assam during 9th 10th May, 2013.
- [3] Selection of Best Fit Model for Estimation of Oldest- Old Mortality Rates, presented at recent trends in mathematics and its applications (RTMA-2013), at Digboi College, Digboi, Assam during 7th and 8th October, 2013.