Chapter 4

Regional Annual Maximum Rainfall Frequency Analysis of North East India using LH-moment.

4.1. Introduction

In this chapter regional annual maximum rainfall analysis of North East India has been carried out using LH-moment of four orders. Three extreme probability distributions namely the generalized extreme value (GEV), generalized logistic (GLO) and generalized Pareto (GPA) are used.

4.2. LH-moments

LH-moments which is generalization of the L-moments defined by Wang [51] as follows:

$$\lambda_1^\eta = E\left[X_{(\eta+1):(\eta+1)}\right] \tag{4.2.1}$$

$$\lambda_2^\eta = \frac{1}{2}E\left[X_{(\eta+2):(\eta+2)} - X_{(\eta+1):(\eta+2)}\right] \tag{4.2.2}$$

$$\lambda_3^\eta = \frac{1}{3}E\left[X_{(\eta+3):(\eta+3)} - 2X_{(\eta+2):(\eta+3)} + X_{(\eta+1):(\eta+3)}\right] \tag{4.2.3}$$

$$\lambda_4^\eta = \frac{1}{4}E\left[X_{(\eta+4):(\eta+4)} - 3X_{(\eta+3):(\eta+4)} + 3X_{(\eta+2):(\eta+4)} - X_{(\eta+1):(\eta+4)}\right] \tag{4.2.4}$$

When $\eta = 0$, LH-moments reduces to L-moments of Hosking [23]. As η increases, LH-moments reflect more and more the characteristics of the upper part of distribution and larger events in data (Wang [51]). The LH-moments are denoted as L_1-moments, L_2-moments,….etc. for $\eta = 1,2, \ldots\ldots$, respectively. The LH-moments ratios (LHMRs) can be defined as

LH-coefficient of variation, $\tau^{\eta} = \lambda_2^{\eta}/\lambda_1^{\eta}$

LH-coefficient of skewness, $\tau_3^{\eta} = \lambda_3^{\eta}/\lambda_2^{\eta}$ (4.2.5)

LH-coefficient of skewness, $\tau_4^{\eta} = \lambda_4^{\eta}/\lambda_2^{\eta}$

For a given ranked sample, $x_{(1)} \le x_{(2)} \le \cdots \le x_{(n)}$, the sample estimates of LH-moments defined by Wang [51] as

$$\hat{\lambda}_1^{\eta} = \frac{1}{\binom{n}{\eta+1}} \sum_{i=1}^{n} \binom{i-1}{\eta} x_{(i)} \tag{4.2.6}$$

$$\hat{\lambda}_2^{\eta} = \frac{1}{2}\frac{1}{\binom{n}{\eta+2}} \sum_{i=1}^{n} \left\{ \binom{i-1}{\eta+1} - \binom{i-1}{\eta}\binom{n-i}{1} \right\} x_{(i)} \tag{4.2.7}$$

$$\hat{\lambda}_3^{\eta} = \frac{1}{3}\frac{1}{\binom{n}{\eta+3}} \sum_{i=1}^{n} \left\{ \binom{i-1}{\eta+2} - 2\binom{i-1}{\eta+1}\binom{n-i}{1} + \binom{i-1}{\eta}\binom{n-i}{2} \right\} x_{(i)} \tag{4.2.8}$$

$$\hat{\lambda}_4^{\eta} = \frac{1}{4}\frac{1}{\binom{n}{\eta+4}} \sum_{i=1}^{n} \left\{ \binom{i-1}{\eta+3} - 3\binom{i-1}{\eta+2}\binom{n-i}{1} + 3\binom{i-1}{\eta+1}\binom{n-i}{2} - \binom{i-1}{\eta}\binom{n-i}{3} \right\} x_{(i)} \tag{4.2.9}$$

Also, Wang [51] defined LH-moments as linear combination of normalized PWMs which can be written as:

$$\hat{\lambda}_1^{\eta} = B_{\eta} \tag{4.2.10}$$

$$\hat{\lambda}_2^{\eta} = \frac{1}{2}(\eta+2)\{B_{\eta+1} - B_{\eta}\} \tag{4.2.11}$$

$$\hat{\lambda}_3^{\eta} = \frac{1}{3!}(\eta+3)\{(\eta+4)B_{\eta+2} - 2(\eta+3)B_{\eta+1} + (\eta+2)B_{\eta}\} \tag{4.2.12}$$

$$\hat{\lambda}_4^{\eta} = \frac{1}{4!}(\eta+4)\{(\eta+6)(\eta+5)B_{\eta+3} - 3(\eta+5)(\eta+4)B_{\eta+2} + 3(\eta+4)(\eta+3)B_{\eta+1} - (\eta+3)(\eta+2)B_{\eta}\} \tag{4.2.13}$$

where,

$$B_r = \frac{\int_0^1 x(F)F^r dF}{\int_0^1 F^r dF} = (r+1)\int_0^1 x(F)F^r dF = (r+1)\beta_r \tag{4.2.14}$$

The sample LH-moment ratios can be defined as follows

$$\hat{\tau}^{\eta} = \hat{\lambda}_2^{\eta}/\hat{\lambda}_1^{\eta}, \quad \hat{\tau}_3^{\eta} = \hat{\lambda}_3^{\eta}/\hat{\lambda}_2^{\eta}, \quad \hat{\tau}_4^{\eta} = \hat{\lambda}_4^{\eta}/\hat{\lambda}_2^{\eta} \tag{4.2.15}$$

4.3 LH-moments of probability distributions

4.3.1 GEV Distribution

The probability weighted moments (PWMs) of GEV distribution developed by Hosking et al. [21] is given by

$$\beta_4 = \frac{1}{1+r}\left\{\xi + \frac{\alpha}{k}[1 - \Gamma(1+k)(r+1)^{-k}]\right\} \tag{4.3.1}$$

Wang [51] developed LH-moment for GEV distribution in terms of normalized PWMs which can be written as:

$$\lambda_1^\eta = \xi + \frac{\alpha}{k}[1 - \Gamma(1+k)(\eta+1)^{-k}] \tag{4.3.2}$$

$$\lambda_2^\eta = \frac{(\eta+2)\alpha\Gamma(1+k)}{2!k}[-(\eta+2)^{-k} + (\eta+1)^{-k}] \tag{4.3.3}$$

$$\lambda_3^\eta = \frac{(\eta+3)\alpha\Gamma(1+k)}{3!k}[-(\eta+4)(\eta+3)^{-k} + 2(\eta+3)(\eta+2)^{-k} - (\eta+2)(\eta+1)^{-k}] \tag{4.3.4}$$

$$\lambda_3^\eta = \frac{(\eta+3)\alpha\Gamma(1+k)}{3!k}[-(\eta+4)(\eta+3)^{-k} + 2(\eta+3)(\eta+2)^{-k} - (\eta+2)(\eta+1)^{-k}] \tag{4.3.5}$$

$$\lambda_4^\eta = \frac{(\eta+4)\alpha\Gamma(1+k)}{4!k}[-(\eta+6)(\eta+5)(\eta+4)^{-k} + 3(\eta+5)(\eta+4)(\eta+3)^{-k} - 3(\eta+4)(\eta+3)(\eta+2)^{-k} + (\eta+3)(\eta+2)(\eta+1)^{-k}] \tag{4.3.6}$$

Parameters

Wang [51] developed a relation between shape parameter k and LH-skewness for different level of LH-moments; the values of coefficients have been shown in Table 4.1.

Table 4.1 Coefficients of the relations for different levels of LH-moments

η	a_0	a_1	a_2	a_3
1	0.4823	-2.1494	0.7269	-0.2103
2	0.5914	-2.3351	0.6442	-0.1616
3	0.6618	-2.4548	0.5733	-0.1273
4	0.7113	-2.5383	0.5142	-0.1027

$$k = a_0 + a_1\tau_3^\eta + a_2[\tau_3^\eta]^2 + a_3[\tau_3^\eta]^3 \tag{4.3.7}$$

$$\alpha = \frac{k[(\eta+2)\beta_{\eta+1}-(\eta+1)\beta_\eta]}{\Gamma(1+k)[(\eta+1)^{-k}-(\eta+2)^{-k}]} \tag{4.3.8}$$

$$\xi = (\eta+1)\beta_\eta - \frac{\alpha}{k}[1-(\eta+1)^{-k}\Gamma(1+k)] \tag{4.3.9}$$

4.3.2 GPA Distribution

The PWMs of GPA distribution developed by Hosking [23] is

$$\beta_4 = \frac{1}{1+r}\left\{\xi + \frac{\alpha}{k}\left[1-\frac{\Gamma(1+k)\Gamma(1+r)(1+r)}{\Gamma(2+k+r)}\right]\right\} \tag{4.3.10}$$

The LH-moments for GPA distribution developed by Meshgi and Khalili [31] are given by

$$\lambda_1^\eta = \xi + \frac{\alpha}{k}\left[1-\frac{\Gamma(1+k)(\eta+1)!}{\Gamma(\eta+2+k)}\right] \tag{4.3.11}$$

$$\lambda_2^\eta = \frac{(\eta+2)\alpha\Gamma(1+k)}{2!k}\left[-\frac{(\eta+2)!}{\Gamma(\eta+3+k)}+\frac{(\eta+1)!}{\Gamma(\eta+2+k)}\right] \tag{4.3.12}$$

$$\lambda_3^\eta = \frac{(\eta+3)\alpha\Gamma(1+k)}{3!k}\left[-\frac{(\eta+4)(\eta+3)!}{\Gamma(\eta+4+k)}+2\frac{(\eta+3)(\eta+2)!}{\Gamma(\eta+3+k)}-\frac{(\eta+2)(\eta+1)!}{\Gamma(\eta+2+k)}\right] \tag{4.3.13}$$

$$\lambda_4^\eta = \frac{(\eta+4)\alpha\Gamma(1+k)}{4!k}\left[-\frac{(\eta+6)(\eta+5)(\eta+4)!}{\Gamma(\eta+5+k)}+3\frac{(\eta+5)(\eta+4)(\eta+3)!}{\Gamma(\eta+4+k)}-3\frac{(\eta+4)(\eta+3)(\eta+2)!}{\Gamma(\eta+3+k)}+\frac{(\eta+3)(\eta+2)(\eta+1)!}{\Gamma(\eta+2+k)}\right] \tag{4.3.14}$$

Parameters

The parameters of GPA distribution in terms of LH-moments developed by Meshgi and Khalili [32] are given as follows:

$$k = \frac{-5-2\eta+\frac{(\eta+3)[(\eta+3)\beta_{\eta+2}-(\eta+1)\beta_\eta]}{(\eta+2)\beta_{\eta+1}-(\eta+1)\beta_\eta}}{-1+\frac{(\eta+3)\beta_{\eta+2}-(\eta+1)\beta_\eta}{(\eta+2)\beta_{\eta+1}-(\eta+1)\beta_\eta}} \tag{4.3.15}$$

$$\alpha = -\frac{k\Gamma(\eta+3+k)\Gamma(\eta+2+k)[(\eta+2)\beta_{\eta+1}-(\eta+1)\beta_\eta]}{(\eta+1)!\Gamma(1+k)[(\eta+2)\Gamma(\eta+2+k)-\Gamma(\eta+3+k)]} \tag{4.3.16}$$

$$\xi = (\eta+1)\beta_\eta - \frac{\alpha}{k}\left[1-\frac{(\eta+1)\Gamma(\eta+1)\Gamma(1+k)}{\Gamma(\eta+2+k)}\right] \tag{4.3.17}$$

4.3.3 GLO Distribution

The PWMs of GLO distribution developed by Hosking [22] is

$$\beta_r = \frac{1}{1+r}\left\{\xi + \frac{\alpha}{k}\left[1-\frac{\Gamma(1+k)\Gamma(1+r-k)}{\Gamma(1+r)}\right]\right\} \tag{4.3.18}$$

The LH-moments for GLO distribution developed by Meshgi and Khalili [31] are given by

$$\lambda_1^\eta = \xi + \frac{\alpha}{k}\left[1 - \frac{\Gamma(1+k)\Gamma(\eta+1-k)}{\eta!}\right] \tag{4.3.19}$$

$$\lambda_2^\eta = \frac{(\eta+2)\alpha\Gamma(1+k)}{2!k}\left[-\frac{\Gamma(\eta+2-k)}{(\eta+1)!} + \frac{\Gamma(\eta+1-k)}{\eta!}\right] \tag{4.3.20}$$

$$\lambda_3^\eta = \frac{(\eta+3)\alpha\Gamma(1+k)}{3!k}\left[-\frac{(\eta+4)\Gamma(\eta+3-k)}{(\eta+2)!} + 2\frac{(\eta+3)\Gamma(\eta+2-k)}{(\eta+1)!} - \frac{(\eta+2)\Gamma(\eta+1-k)}{\eta!}\right] \tag{4.3.21}$$

$$\lambda_4^\eta = \frac{(\eta+4)\alpha\Gamma(1+k)}{4!k}\left[-\frac{(\eta+6)(\eta+5)\Gamma(\eta+4-k)}{(\eta+3)!} + 3\frac{(\eta+5)(\eta+4)\Gamma(\eta+3-k)}{(\eta+2)!} - 3\frac{(\eta+4)(\eta+3)\Gamma(\eta+2-k)}{(\eta+1)!} + \frac{(\eta+3)(\eta+2)\Gamma(\eta+1-k)}{\eta!}\right] \tag{4.3.22}$$

Parameters

The parameters of GLO distribution for LH-moments developed by Meshgi and Khalili [32] are given as follows:

$$k = -\frac{(\eta+3)(\eta+2)\beta_{\eta+2} - [(\eta+2)^2 + (\eta+2)(\eta+1)]\beta_{\eta+1} + (\eta+1)^2\beta_\eta}{(\eta+2)\beta_{\eta+1} - (\eta+1)\beta_\eta} \tag{4.3.23}$$

$$\alpha = \frac{\Gamma(\eta+2)[(\eta+2)\beta_{\eta+1} - (\eta+1)\beta_\eta]}{\Gamma(\eta+1-k)\Gamma(1+k)} \tag{4.3.24}$$

$$\xi = (\eta+1)\beta_\eta - \frac{\alpha}{k}\left[1 - \frac{\Gamma(\eta+1-k)\Gamma(1+k)}{\Gamma(\eta+1)}\right] \tag{4.3.25}$$

4.4 Regional Rainfall Frequency Analysis using LH-moment

The procedure discussed in section 2.5 can be employed for LH-moment also. For this purpose, L-cv, L-skewness and L-kurtosis are replaced by LH-cv, LH-skewness and LH-kurtosis respectively. For all calculations Fortran 77 programs are used.

4.4.1 Screening of Data

As discussed in the section 2.5.1 Discordancy test D_i for LH-moment can be written as

$$D_i = \frac{1}{3}N(u_i^{LH} - \bar{u}^{LH})^T S_{LQ}^{-1}(u_i^{LH} - \bar{u}^{LH}) \tag{4.4.1}$$

where $S_{LH} = \sum_{i=1}^{N}(u_i^{LH} - \bar{u}^{LH})(u_i^{LH} - \bar{u}^{LH})^T$ and $u_i^{LH} = \left[\hat{\tau}^{\eta,i}, \hat{\tau}_3^{\eta,i}, \hat{\tau}_4^{\eta,i}\right]^T, \eta = 1,2,3,4$ for ith station, N is the number of stations, S_{LH} is covariance matrix of $u_i^{LH}$and $\bar{u}^{LH}$ is the mean of vector, u_i^{LH}. Critical values of discordancy statistics tabulated by Hosking and Wallis [25] are also used here. For $N = 12$, the critical value is 2.757.

The calculated D_i values using L_1, L_2, L_3 and L_4-moment are given in the Table 4.2, Table 4.3, Table 4.4 and Table 4.5 respectively. It is observed that the $D_i$values of all the 12 stations of our study region are less than the critical value 2.757. Therefore, all the data from 12 stations can be considered for our study.

Table 4.2 Discordancy measures of each sites of the NE region using L_1-moments.

Name of sites	No. of observation	$\hat{\tau}^1$	$\hat{\tau}_3^1$	$\hat{\tau}_4^1$	D_i
1. Agartala	30	0.1284	0.1518	0.0386	1.78
2. Dhubri	22	0.1328	0.1931	0.1510	0.33
3. Guwahati	30	0.1209	0.2782	0.1108	0.27
4. Imphal	30	0.1356	0.2757	0.1458	0.10
5. Itanagar	26	0.1493	0.3966	0.1807	0.58
6. Jorhat	25	0.0760	0.1261	0.0298	2.56
7. Lakhimpur	30	0.1111	0.2264	0.0893	0.25
8. Lengpui	13	0.0903	0.2151	0.0180	1.58
9. Mohanbari	30	0.1100	0.1953	0.2101	1.69
10. Passighat	30	0.1880	0.4665	0.3527	2.25
11. Shillong	30	0.1363	0.2572	0.1355	0.19
12. Silchar	28	0.1084	0.2152	0.1346	0.43

(Table shows that $D_i$values of all the 12 stations are less than the critical value 2.757. Therefore, all the data from 12 stations can be considered for our study)

Table 4.3 Discordancy measures of each sites of the NE region using L_2-moments.

Name of sites	No. of observation	$\hat{\tau}^2$	$\hat{\tau}_3^2$	$\hat{\tau}_4^2$	D_i
1. Agartala	30	0.1054	0.1464	0.0732	2.52
2. Dhubri	22	0.1117	0.2448	0.2274	0.90
3. Guwahati	30	0.1082	0.2710	0.0933	0.28
4. Imphal	30	0.1201	0.2903	0.1525	0.07
5. Itanagar	26	0.1409	0.3724	0.1363	1.05
6. Jorhat	25	0.0456	0.1429	0.1035	1.68
7. Lakhimpur	30	0.0968	0.2273	0.1064	0.07
8. Lengpui	13	0.0791	0.1765	-0.0276	1.17
9. Mohanbari	30	0.0940	0.2830	0.3164	1.75
10. Passighat	30	0.1804	0.4952	0.3569	2.14
11. Shillong	30	0.1192	0.2736	0.1085	0.22
12. Silchar	28	0.0939	0.2483	0.1483	0.15

(Table shows that $D_i$values of all the 12 stations are less than the critical value 2.757. Therefore, all the data from 12 stations can be considered for our study)

Table 4.4 Discordancy measures of each sites of the NE region using L_3-moments.

Name of sites	No. of observation	$\hat{\tau}^3$	$\hat{\tau}_3^3$	$\hat{\tau}_4^3$	D_i
1. Agartala	30	0.0907	0.1615	0.1235	2.44
2. Dhubri	22	0.1011	0.3111	0.2817	0.49
3. Guwahati	30	0.0995	0.2579	0.0905	2.27
4. Imphal	30	0.1108	0.3004	0.1724	0.04
5. Itanagar	26	0.1342	0.3408	0.1108	0.81
6. Jorhat	25	0.0387	0.1120	0.1349	1.42
7. Lakhimpur	30	0.0875	0.2362	0.1291	0.06
8. Lengpui	13	0.0701	0.1287	-0.0665	1.39
9. Mohanbari	30	0.0875	0.3800	0.3892	2.23

10. Passighat	30	0.1788	0.5087	0.3512	2.17
11. Shillong	30	0.1092	0.2674	0.0881	0.28
12. Silchar	28	0.0860	0.2719	0.1620	0.41

(Table shows that $D_i$values of all the 12 stations are less than the critical value 2.757. Therefore, all the data from 12 stations can be considered for our study)

Table 4.5 Discordancy measures of each sites of the NE region using L_4-moments.

Name of sites	No. of observation	$\hat{\tau}^4$	$\hat{\tau}_3^4$	$\hat{\tau}_4^4$	D_i
1. Agartala	30	0.0811	0.1949	0.1694	2.73
2. Dhubri	22	0.0963	0.3707	0.3205	0.52
3. Guwahati	30	0.0926	0.2489	0.0875	0.05
4. Imphal	30	0.1046	0.3147	0.1975	0.07
5. Itanagar	26	0.1278	0.3133	0.0941	0.78
6. Jorhat	25	0.0402	0.0336	-0.2104	1.82
7. Lakhimpur	30	0.0810	0.2516	0.1521	0.16
8. Lengpui	13	0.0622	0.0773	-0.1209	0.81
9. Mohanbari	30	0.0862	0.4594	0.4253	2.34
10. Passighat	30	0.1795	0.5132	0.3399	2.27
11. Shillong	30	0.1016	0.2544	0.0676	0.19
12. Silchar	28	0.0809	0.2911	0.1736	0.27

(Table shows that $D_i$values of all the 12 stations are less than the critical value 2.757. Therefore, all the data from 12 stations can be considered for our study)

4.4.2 Identification of Homogeneous Region

The heterogeneity test H for LH-moment is derived from the heterogeneity test proposed by Hosking and Wallis [25] given in section 2.5.2. The test can be written as follows:

$$V_1 = \sqrt{\sum_{i=1}^{N} n_i (\hat{\tau}^{\eta,i} - \tau^{\eta,R})^2 / \sum_1^N n_i} \tag{4.4.2}$$

$$V_2 = \sum_{i=1}^{N}\{n_i[(\hat{\tau}^{\eta,i} - \tau^{\eta,R})^2 + \left(\hat{\tau}_3^{\eta,i} - \tau_3^{\eta,R}\right)^2]^{\frac{1}{2}}\} / \sum_{i=1}^{N} n_i \quad (4.4.3)$$

$$V_3 = \sum_{i=1}^{N}\{n_i\left(\hat{\tau}_3^{\eta,i} - \tau_3^{\eta,R}\right)^2 + \left(\hat{\tau}_4^{\eta,i} - \tau_4^{\eta,R}\right)^2]^{\frac{1}{2}}\} / \sum_{i=1}^{N} n_i \quad (4.4.4)$$

The regional average LH-moment ratios are calculated using the following formula

$$\tau^{\eta,R} = \sum_{i=1}^{N} n_i\, \hat{\tau}^{\eta,i} / \sum_{i=1}^{N} n_i$$

$$\tau_3^{\eta,R} = \sum_{i=1}^{N} n_i\, \hat{\tau}_3^{\eta,i} / \sum_{1}^{N} n_i \quad (4.4.5)$$

$$\tau_4^{\eta,R} = \sum_{i=1}^{N} n_i\, \hat{\tau}_4^{\eta,i} / \sum_{1}^{N} n_i$$

where N is the number of stations and $n_i$is the record length at ith station. The heterogeneity test is then defined as

$$H_j = \frac{V_j - \mu_{V_j}}{\sigma_{V_j}} \ , \quad j = 1,2,3 \quad (4.4.6)$$

where $\mu_{V_j}$and $\sigma_{V_j}$are the mean and standard deviation of simulated V_j values, respectively. The region is acceptably homogeneous, possibly homogeneous and definitely heterogeneous with a corresponding order of LH-moments according as $H_j<1$, $1\leq H_j<2$ and $H_j\geq 2$.

The calculated values of H_j are given in the Table 4.6. From Table 4.6 it is observed that for L_1 moment the region can be taken as homogeneous one and for other order of LH-moment the region can be taken as possibly homogeneous region

Table 4.6: Heterogeneity measures using LH-moments (L_1, L_2, L_3& L_4)

Methods	H_1	H_2	H_3
L_1-moment	0.77	0.20	-0.13
L_2-moment	1.72	-0.92	-0.43
L_3-moment	1.57	-0.73	-0.23
L_4-moment	1.57	-0.12	0.68

(Table shows that values of H_1 for L_1-moment is less than 1 and for other order of LH-moment it lies between 1 and 2. Also values of H_2 and H_3 are less than 1for all orders of LH-moment)

4.4.3 Choice of a Distribution

Z-statistic criteria and L-moment ratio diagram proposed by Hosking and Wallis [25] to select the best fit distribution are applied in the similar manner for LH-moment also.

(a) Z-statistic Criteria

As in the section 2.5.3 the Z-statistic for each distribution is calculated as follows:

$$\mathrm{Z}^{\mathrm{DIST}} = (\tau_4^{\eta,\mathrm{DIST}} - \tau_4^{\eta,\mathrm{R}})/\sigma_4 \qquad (4.4.7)$$

where DIST refers to a particular distribution, $\tau_4^{\eta,\mathrm{DIST}}$ is the L_i-kurtosis of the fitted distribution while the standard deviation of $\tau_4^{\eta,\mathrm{R}}$ is given by

$$\sigma_4 = \left[(N_{sim})^{-1}\sum_{m=1}^{N_{sim}}(\tau_4^{\eta(m)} - \tau_4^{\eta,\mathrm{R}})^2\right]^{1/2}$$

$\tau_4^{\eta(m)}$ is the average regional L_i -kurtosis and has to be calculated for the m^{th} simulated region. This is obtained by simulating a large number of kappa distribution using Monte Carlo simulations. The value of the Z-statistics is considered to be acceptable at the 90% confidence level if $\left|\mathrm{Z}^{\mathrm{DIST}}\right| \leq 1.64$. If more than one candidate distribution is acceptable, the one with the lowest $|Z^{DIST}|$ is regarded as the best fit distribution.

(b) LH-Moment ratio diagram

It is a graph of the LH-skewness and LH-kurtosis which compares the fit of several distributions on the same graph. As discussed in the section 2.5.3 the expression of τ_4^{η} in terms of τ_3^{η} for an assumed distribution can be written as

$$\tau_4^{\eta} = \sum_{k=0}^{8} A_k\left(\tau_3^{\eta}\right)^k \qquad (4.5.8)$$

where the coefficients $A_k$are calculated by Meshgi and Khalili [32]. Coefficients are given in Table A.5.

The calculated Z-statistics are given in Table 4.7. From Table 4.7 it is observed that for L_1-moment $\left|\mathrm{Z}^{\mathrm{DIST}}\right|$ of GEV and GPA distributions are less than 1.64. But GPA distribution has occurred lowest $\left|\mathrm{Z}^{\mathrm{DIST}}\right|$. Hence for L_1-moment GPA distribution is selected as the best fit distribution. Similarly for L_2, L_3 and L_4-moments the lowest $\left|\mathrm{Z}^{\mathrm{DIST}}\right| < 1.64$ are occurred by GLO distribution. Hence for L_2, L_3 and L_4-moments GLO distribution is selected as the best fit distribution.

From L_1-moment ratio diagram (Figure 4.1) it is observed that the point of regional L_1-skewness and kurtosis lies nearer to the GPA distribution curve. Also from L_2, L_3 and L_4-moment ratio diagram (Figure 4.2, 4.3 and 4.4) it is observed that the points of respective regional skewness and kurtosis lie nearer to GLO distribution. Hence Z-statistic criteria and LH-moment ratio diagram show the same result.

Table 4.7 Z-statistics values of the distributions using LH-moments (L_1, L_2, L_3& L_4)

Methods	Name of the probability distribution	Z^{DIST}
L_1-moment	GLO	1.81
	GEV	0.83
	GPA	**-0.76**
L_2-moment	**GLO**	**-0.03**
	GEV	-0.64
	GPA	-1.51
L_3-moment	**GLO**	**-1.20**
	GEV	-1.65
	GPA	-2.20
L_4-moment	**GLO**	**-0.38**
	GEV	-0.72
	GPA	-1.19

(The lowest absolute Z-statistics is occurred by GPA distribution for L_1-moment and that by GLO distribution for other orders of LH-moments)

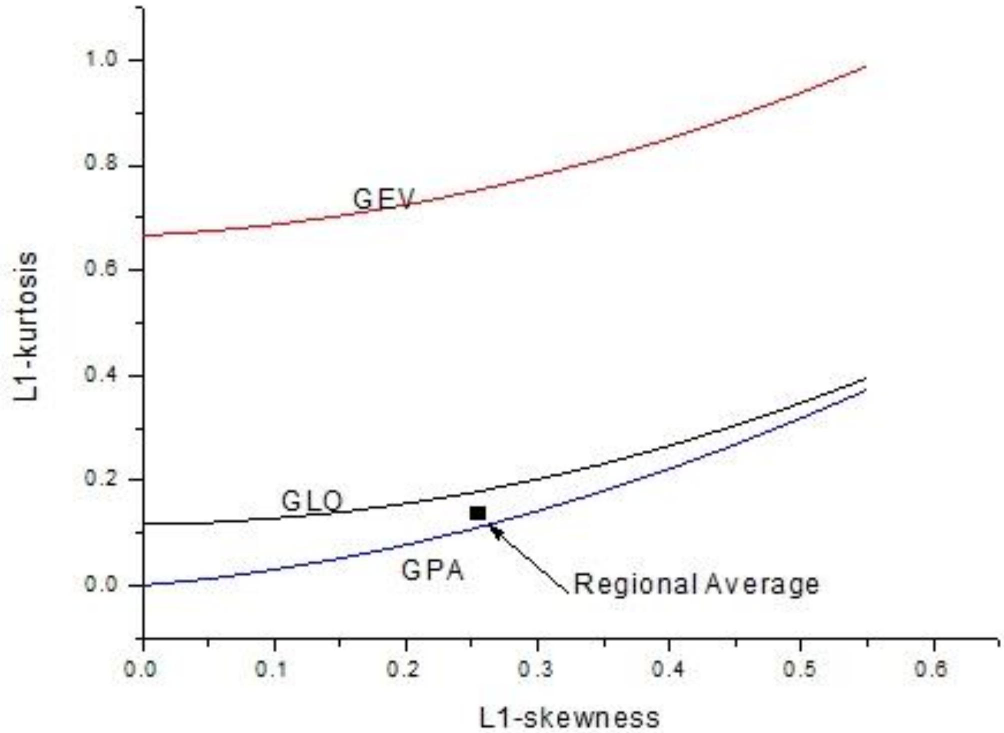

Figure 4.1 L_1-moment ratio diagram

(The regional average is nearer to the GPA distribution curve)

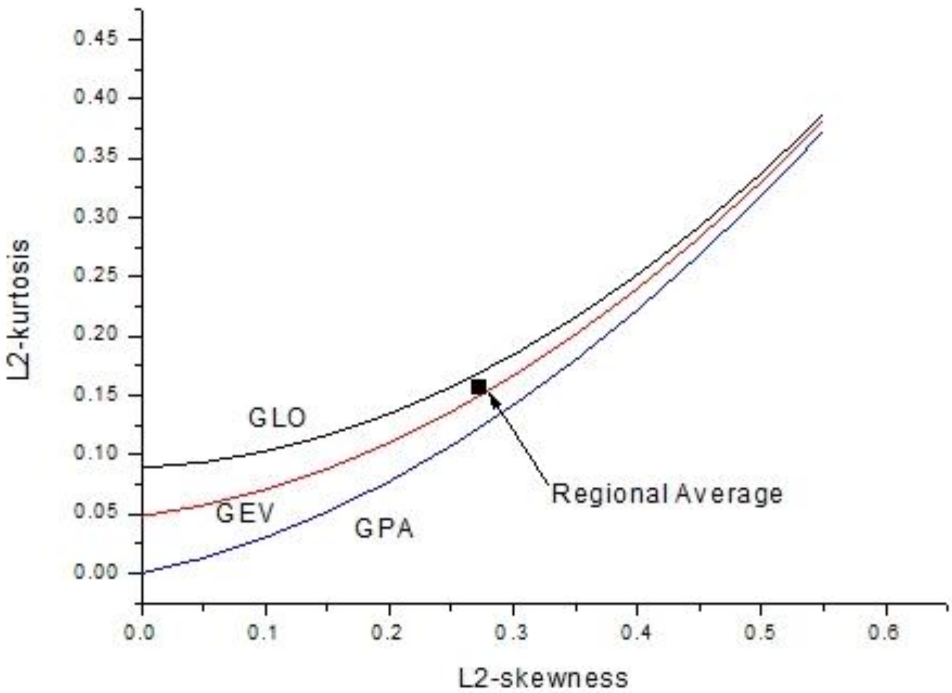

Figure 4.2 L_2-moment ratio diagram

(The regional average is nearer to the GLO distribution curve)

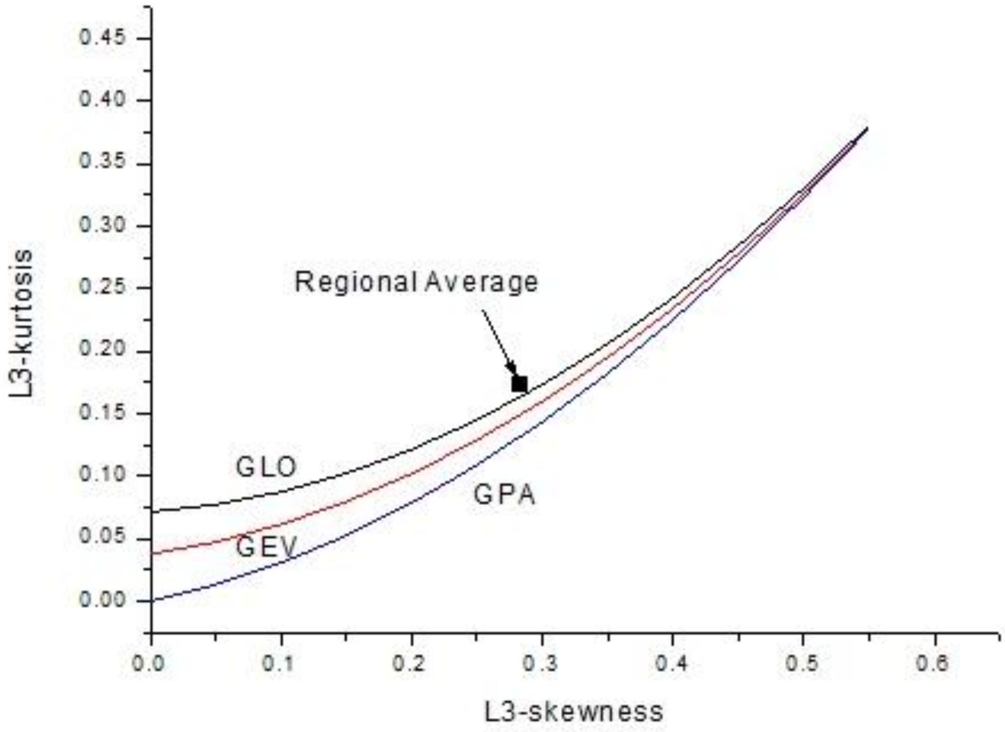

Figure 4.3 L_3-moment ratio diagram

(The regional average is nearer to the GLO distribution curve)

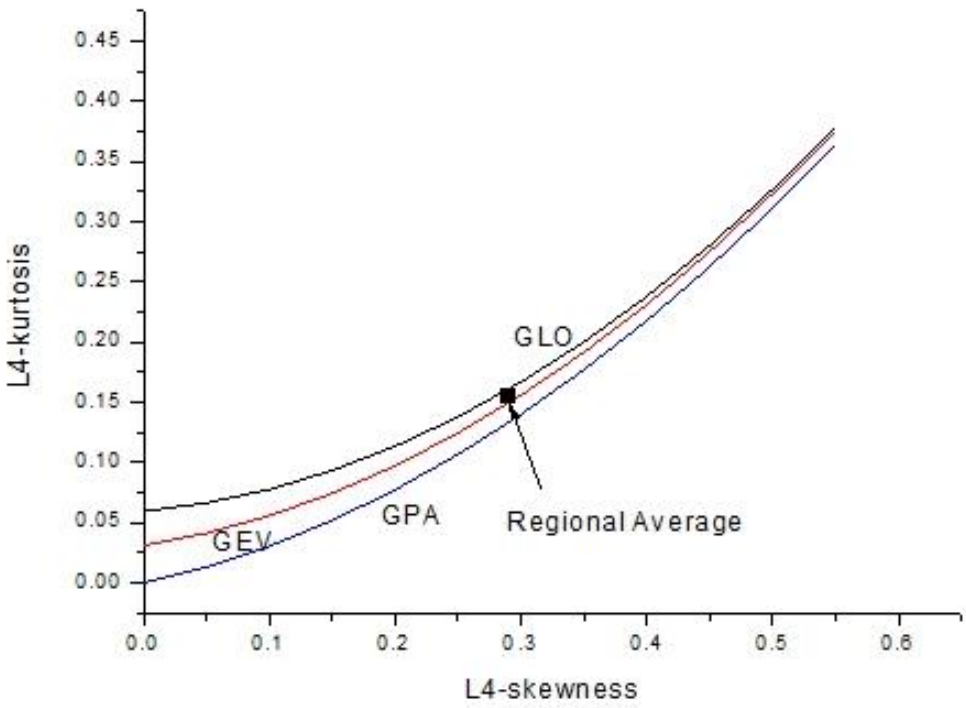

Figure 4.4 L_4-moment ratio diagram

(The regional average is nearer to the GLO distribution curve)

4.4.4 Estimation of Frequency Distribution

For L_1-moment the regional parameters of the best fit distribution GPA are calculated using the approximation expression given in section 4.3.2. Also for L_2, L_3 and L_4-moment the regional parameters of the best fit distribution GLO are calculated using approximation expression given in section 4.3.3. For L_1-moment using the parameters of GPA distribution in the quantile function of GPA distribution given in section 2.3.1, growth factors are calculated. Also using the respective regional parameters of GLO distribution for L_2, L_3 and L_4-moment, respective growth factors are calculated.

Calculated parameters of best fit distributions and growth factors are given in Table 4.8 and Table 4.9 respectively.

Table 4.8 Regional parameters of the best fit distributions

Methods	Best fit distributions	Parameters		
		Location	Scale	Shape
L_1-moment	GPA	0.555	0.386	0.254
L_2-moment	GLO	0.754	0.141	-0.100
L_3-moment	GLO	0.711	0.141	-0.074
L_4-moment	GLO	0.676	0.143	-0.056

Table 4.9 Quantile estimates by using best fitting distributions

Methods	Best fit dist.	Return period (in years)				
		2	10	20	100	1000
L_1-mom	GPA	0.801	1.229	1.366	1.604	1.813
L_2-mom	GLO	0.754	1.101	1.237	1.577	2.158
L_3-mom	GLO	0.711	1.048	1.175	1.483	1.983
L_4-mom	GLO	0.676	1.010	1.133	1.425	1.881

4.5 Development of Regional Rainfall Frequency Relationship

The index flood procedure discussed in section 2.4 is used to develop regional rainfall frequency relationship. The form of regional rainfall frequency relationship or growth factor for the best fit distributions GPA and GLO can be expressed as

$$Q_T = [\xi + \frac{\alpha}{k}\{1 - (1 - F)^k\}] * \mu_i \tag{4.5.1}$$

and

$$Q_T = \left[\xi + \frac{\alpha}{k}\left\{1 - \left(\frac{1-F}{F}\right)^k\right\}\right] * \mu_i \quad (4.5.2)$$

where Q_T is the maximum rainfall for return period T, $F = 1 - 1/T$, μ_i is the mean annual maximum rainfall of the ith site, ξ, α and k are the parameters of the respective distributions. Substituting the regional values of best fit distributions based on the data of 12 gauged sites the regional rainfall frequency relationship for gauged sites of study area is expressed as:

For L_1-moment

$$Q_T = [0.555 + 1.520\{1 - (1 - \mathrm{F})^{0.254}\}] * \mu_i \quad (4.5.3)$$

For L_2-moment

$$Q_T = \left[0.754 - 1.410\left\{1 - \left(\frac{1-F}{F}\right)^{-0.100}\right\}\right] * \mu_i \quad (4.5.4)$$

For L_3-moment

$$Q_T = \left[0.711 - 1.905\left\{1 - \left(\frac{1-F}{F}\right)^{-0.074}\right\}\right] * \mu_i \quad (4.5.5)$$

For L_4-moment

$$Q_T = \left[0.676 - 2.553\left\{1 - \left(\frac{1-F}{F}\right)^{-0.056}\right\}\right] * \mu_i \quad (4.5.6)$$

For estimation of rainfall of desired non-exceedance probability for a small to moderate size gauged catchments of the study area, above regional rainfall frequency relationships may be used.

4.6 Conclusion

From discordancy test using L_1, L_2, L_3, and L_4-moment it is found that all the data of the 12 stations of the study region can be considered for the study. From heterogeneity test it is observed that for L_1-moment the 12 stations of the study region form a homogeneous region whereas for L_2, L_3, and L_4-moment the region can be considered as a possibly homogeneous region. For L_1-moment Z-statistic criteria and LH-moment ratio diagram shows that GPA distribution is the best fit distribution for the study region. For L_2, L_3, and L_4-moment GLO distribution is selected as the best fit distribution. Parameters of GPA and GLO distributions are calculated using respective LH-moments. Substituting the regional parameters of GPA and GLO distributions in the respective quantile functions, growth factors at different return periods are calculated. Finally using flood index procedure regional rainfall frequency relationships has been developed.

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