Chapter 5

Regional Annual Maximum Rainfall Frequency Analysis of North East India using TL-moment.

5.1. Introduction

In this chapter regional annual maximum rainfall analysis of North East India has been carried out using TL-moment. Three extreme probability distributions namely the generalized extreme value (GEV), generalized logistic (GLO) and generalized Pareto (GPA) have been used for the study.

5.2. TL-moment

Let $X_1, X_2, \ldots, X_n$ be a sample from a continuous distribution function $F(.)$ with quantile function $Q(F)$ and let $X_{1:n}, X_{2:n}, \ldots, X_{n:n}$ be the order statistics. Then the rth L-moment λ_r is given by

$$\lambda_r = \frac{1}{r}\sum_{k=0}^{r-1}(-1)^k \binom{r-1}{k} E(X_{r-k:r}), \qquad r = 1,2,\ldots \tag{5.2.1}$$

In TL-moment defined by Elamir et al. [18], the term $E(X_{r-k:r})$ in the above equation (5.2.1) is replaced by $(X_{r+t_1-k:r+t_1+t_2})$. That is for each r, the conceptual sample size will be increased from r to r+t$_1$+t$_2$ and work only with the expectation of r ordered statistics $Y_{t_1+1:r+t_1+t_2}, \ldots, Y_{t_1+r:r+t_1+t_2}$ by trimming the t$_1$ smallest and t$_2$ largest from the conceptual sample. Then the rth TL-moment is defined as:

$$\lambda_r^{(t_1,t_2)} = \frac{1}{r}\sum_{k=0}^{r-1}(-1)^k \binom{r-1}{k} E\left(X_{r+t_1-k:r+t_1+t_2}\right), \ r = 1,2 \ldots \tag{5.2.2}$$

For $t_1=t_2=0$, the TL-moment yields the original L-moment. When $t_1=t_2=t$, then the rth TL-moment is defined as:

$$\lambda_r^{(t)} = \frac{1}{r}\sum_{k=0}^{r-1}(-1)^k \binom{r-1}{k} E(X_{r+t-k:r+2t}), \ r = 1,2 \ldots \tag{5.2.3}$$

Taking t=1 the first four TL-moments can be expressed as

$$\lambda_1^{(1)} = E[X_{2:3}] = 6\beta_1 - 6\beta_2 \tag{5.2.4}$$

$$\lambda_2^{(1)} = \frac{1}{2}E[X_{3:4} - X_{2:4}] = 6(-2\beta_3 + 3\beta_2 - \beta_1) \tag{5.2.5}$$

$$\lambda_3^{(1)} = \frac{1}{3}E[X_{4:5} - 2X_{3:5} + X_{2:5}] = \frac{20}{3}(-5\beta_4 + 10\beta_3 - 6\beta_2 + \beta_1 \tag{5.2.6}$$

$$\lambda_4^{(1)} = \frac{1}{4}E[X_{5:6} - 3X_{4:6} + 3X_{3:6} - X_{2:6}] = \frac{15}{2}(-14\beta_5 + 35\beta_4 - 30\beta_3 +$$
$$10\beta_2 - \beta_1) \tag{5.2.7}$$

The TL-co-efficient of variation, TL-co-efficient of skewness and TL-co-efficient of kurtosis are defined as

$$\tau_2^{(1)} = \frac{\lambda_2^{(1)}}{\lambda_1^{(1)}},$$

$$\tau_3^{(1)} = \frac{\lambda_3^{(1)}}{\lambda_2^{(1)}} \text{ and} \tag{5.2.8}$$

$$\tau_4^{(1)} = \frac{\lambda_4^{(1)}}{\lambda_2^{(1)}}$$

The rth sample TL-moment is given by

$$l_r^{(t)} = \frac{1}{r}\sum_{k=0}^{r-1}(-1)^k \binom{r-1}{k} \hat{E}(X_{r+t-k:r+2t}), \ r = 1,2 \ldots \tag{5.2.9}$$

where unbiased estimator is given by

$$\hat{E}(X_{r+t-k:r+2t}) = \frac{1}{\binom{n}{r+2t}}\sum_{i=1}^{n} \binom{i-1}{r+t-k-1}\binom{n-i}{t+k}X_{i:n} \tag{5.2.10}$$

5.3 TL-moments of Probability Distributions

5.3.1 GPA Distribution

Elamir and Sheult [18] developed TL-moment and parameters of GPA distribution as follows:

$$\lambda_1^{(1)} = \xi + (k-5)\alpha/(k+2)(k+3) \tag{5.3.1}$$

$$\lambda_2^{(1)} = 6\alpha/(k+2)(k+3)(k+4) \tag{5.3.2}$$

$$\tau_3^{(1)} = 10(1-k)/9(k+5) \tag{5.3.3}$$

$$\tau_4^{(1)} = 5\alpha(k-1)(k-2)/4(k+5)(k+6) \tag{5.3.4}$$

Parameters

$$k = \left(10 - 45\tau_3^{(1)}\right)/\left(10 + 9\tau_3^{(1)}\right) \tag{5.3.5}$$

$$\alpha = \lambda_2^{(1)}(k+2)(k+3)(k+4)/6 \tag{5.3.6}$$

$$\xi = \lambda_1^{(1)} - (k-5)\alpha/(k+2)(k+3) \tag{5.3.7}$$

5.3.2 GEV Distribution

Shabri A.B. et al. [43] developed TL-moment and parameters of GEV distribution as follows:

$$\lambda_1^{(1)} = \xi + \alpha\left[1 - \Gamma(1+k)\left(\frac{3}{2^k} - \frac{2}{3^k}\right)\right]/k \tag{5.3.8}$$

$$\lambda_2^{(1)} = 6\alpha\Gamma(k)\left(\frac{1}{2(4^k)} - \frac{1}{3^k} + \frac{1}{2(2^k)}\right) \tag{5.3.9}$$

$$\tau_3^{(1)} = 10\left(\frac{1}{5^k} - \frac{5}{2(4^k)} + \frac{2}{3^k} - \frac{1}{2(2^k)}\right)/9\left(\frac{1}{2(4^k)} - \frac{1}{3^k} + \frac{1}{2(2^k)}\right) \tag{5.3.10}$$

$$\tau_4^{(1)} = 5\left(\frac{7}{3(6^k)} - \frac{7}{5^k} + \frac{15}{2(4^k)} - \frac{10}{3(3^k)} + \frac{1}{2(2^k)}\right)/4\left(\frac{1}{2(4^k)} - \frac{1}{3^k} + \frac{1}{2(2^k)}\right) \tag{5.3.11}$$

Parameters

$$k = 0.2816 - 2.8825\lambda_3^{(1)} + 1.3744\left(\lambda_3^{(1)}\right)^2 - 0.8462\left(\lambda_3^{(1)}\right)^3 \tag{5.3.12}$$

$$\alpha = \lambda_2^{(1)}/6\Gamma(k)\left(\frac{1}{2(4^k)} - \frac{1}{3^k} + \frac{1}{2(2^k)}\right) \tag{5.3.13}$$

$$\xi = \lambda_2^{(1)} - \alpha\left[1 - \Gamma(1+k)\left(\frac{3}{2^k} - \frac{2}{3^k}\right)\right]/k \tag{5.3.14}$$

5.3.3 GLO Distribution

Shabri A.B. et al. [43] developed TL-moment and parameters of GEV distribution as follows:

$$\lambda_1^{(1)} = \xi + \frac{\alpha}{k} - \pi\alpha(1-k^2)/\sin(\pi k) \tag{5.3.15}$$

$$\lambda_2^{(1)} = -\pi\alpha k(k^2-1)/2\sin(\pi k) \tag{5.3.16}$$

$$\tau_3^{(1)} = -5k/9 \tag{5.3.17}$$

$$\tau_4^{(1)} = (7k^2 + 2)/24 \tag{5.3.18}$$

Parameters

$$k = -9\tau_3^{(1)}/5 \qquad\qquad (5.3.19)$$

$$\alpha = -\left\{2\lambda_2^{(1)} \sin(\pi k)\right\}/\{\pi k(k^2 - 1)\} \qquad\qquad (5.3.20)$$

$$\xi = \lambda_1^{(1)} + \frac{\pi\alpha(1-k^2)}{\sin(\pi k)} - \frac{\alpha}{k} \qquad\qquad (5.3.21)$$

5.4 Regional Rainfall Frequency Analysis using TL-moment

The procedure discussed in section 2.5 can be employed for TL-moment also. For this purpose, L-cv, L-skewness and L-kurtosis are replaced by TL-cv, TL-skewness and TL-kurtosis respectively. For all calculations Fortran 77 programs are used.

5.4.1 Screening of data

As discussed in the section 2.5.1 Discordancy test D_i for TL-moment can be written as

$$D_i = \frac{1}{3}N(u_i^{TL} - \bar{u}^{TL})^T S_{TL}^{-1}(u_i^{TL} - \bar{u}^{TL}) \qquad\qquad (5.4.1)$$

where $S_{TL} = \sum_{i=1}^{N}(u_i^{TL} - \bar{u}^{TL})(u_i^{TL} - \bar{u}^{TL})^T$ and $u_i^{TL} = \left[\tau_2^{i(1)}, \tau_3^{i(1)}, \tau_4^{i(1)}\right]^T$ for i-th station, N is the number of stations, S_{TL} is covariance matrix of u_i^{TL} and $\bar{u}^{TL}$ is the mean of vector, u_i^{TL}. Critical values of discordancy statistics are tabulated by Hosking and Wallis [25]. For $N = 12$, the critical value is 2.757.

The calculated D_i values are given in the Table 5.1. It is observed from Table 5.1 that the D_i values of all the 12 stations of our study region are less than the critical value 2.757. Therefore, all the data from 12 stations can be considered for our study.

Table 5.1 Discordancy measures of each sites of the NE region using TL-moment

Site name	No. of observation	$\tau_2^{(1)}$	$\tau_3^{(1)}$	$\tau_4^{(1)}$	D_i
1. Imphal	30	0.0914	0.1555	0.0630	0.04
2. Agartala	30	0.1027	0.1190	-0.0356	1.75

3. Silchar	28	0.0768	0.0844	0.0491	0.52
4. Lengpui	13	0.0622	0.2307	0.0974	1.14
5. Dhubri	22	0.1306	0.1365	0.0649	2.23
6. Itanagar	26	0.0826	0.3286	0.2127	1.25
7. Jorhat	25	0.0773	-0.0738	-0.1000	1.50
8. Passighat	30	0.0954	0.1695	0.1244	0.52
9. Guwahati	30	0.0793	0.2080	0.0972	0.26
10 Mohanbari	30	0.0804	-0.0329	-0.0762	0.95
11. Lakhimpur	30	0.0778	0.1572	0.0158	0.71
12. Shilling	30	0.0947	0.1416	0.1298	1.13

(The D_i values of all the 12 stations of our study region are less than the critical value 2.757)

5.4.2 Identification of Homogeneous Region

As discussed in the section 2.5.2 The heterogeneity test H for TL-moment can be written as follows:

$$V_1 = \sqrt{\textstyle\sum_{i=1}^{N} n_i (\tau_2^{i(1)} - \tau_2^R)^2 / \sum_1^N n_i} \tag{5.4.2}$$

$$V_2 = \textstyle\sum_{i=1}^{N} \{n_i [(\tau_2^{i(1)} - \tau_2^R)^2 + \left(\tau_3^{i(1)} - \tau_3^R\right)^2]^{\frac{1}{2}}\} / \sum_{i=1}^{N} n_i \tag{5.4.3}$$

$$V_3 = \textstyle\sum_{i=1}^{N} \{n_i [(\tau_3^{i(1)} - \tau_3^R)^2 + \left(\tau_4^{i(1)} - \tau_4^R\right)^2]^{\frac{1}{2}}\} / \sum_{i=1}^{N} n_i \tag{5.4.4}$$

The regional average TL-moment ratios are calculated using the following formula

$$\tau_2^R = \textstyle\sum_{i=1}^{N} n_i \, \tau_2^{i(1)} / \sum_{i=1}^{N} n_i$$
$$\tau_3^R = \textstyle\sum_{i=1}^{N} n_i \, \tau_3^{i(1)} / \sum_1^N n_i \tag{5.4.5}$$
$$\tau_4^R = \textstyle\sum_{i=1}^{N} n_i \, \tau_4^{i(1)} / \sum_1^N n_i$$

where N is the number of stations and n_i is the record length at ith station. The heterogeneity test is then defined as

$$H_j = \frac{V_j - \mu_{V_j}}{\sigma_{V_j}}, \quad j = 1,2,3 \tag{5.4.6}$$

where μ_{V_j} and σ_{V_j} are the mean and standard deviation of simulated V_j values respectively. The region is acceptably homogeneous, possibly homogeneous and

definitely heterogeneous with a corresponding order of TL-moments according as H<1, 1≤H<2 and H≥2.

From the heterogeneity measures it is found that the values of $H_1 = 0.69, H_2 = 0.49$ and $H_3 = 1.04$. It has been observed from heterogeneity measures that, the study region can be considered as a possibly homogeneous one.

5.4.3 Choice of a Distribution

To select the best fit distribution Z-statistic criteria and L-moment ratio diagram proposed by Hosking and Wallis [25] can be used for TL-moment also.

(a) Z-statistic Criteria

As in the section 2.5.3 the Z-statistic for each distribution is calculated as follows

$$Z^{DIST} = (\tau_4^{(1)DIST} - \tau_4^R)/\sigma_4 \tag{5.4.7}$$

where DIST refers to a particular distribution, $\tau_4^{(1)DIST}$ is the TL-kurtosis of the fitted distribution while the standard deviation of τ_4^R is given by

$$\sigma_4 = \left[(N_{sim})^{-1}\sum_{m=1}^{N_{sim}}(\tau_4^{(1)(m)} - \tau_4^R)^2\right]^{1/2}$$

$\tau_4^{(1)(m)}$ is the average regional TL-kurtosis and has to be calculated for the mth simulated region. This is obtained by simulating a large number of kappa distribution using Monte Carlo simulations. The value of the Z-statistics is considered to be acceptable at the 90% confidence level if $|Z^{DIST}| \leq 1.64$. If more than one candidate distribution is acceptable, the one with the lowest $|Z^{DIST}|$ is regarded as the best fit distribution.

(b) TL-Moment Ratio Diagram

It is a graph of the TL-skewness and TL-kurtosis which compares the fit of several distributions on the same graph. As discussed in the section 2.5.3 the expression of $\tau_4^{(1)}$ in terms of $\tau_3^{(1)}$ for an assumed distribution can be written as

$$\tau_4^{(1)} = \sum_{k=0}^{8} A_k \tau_3^{(1)} \tag{5.4.8}$$

where the coefficients A_k are calculated by Shabri et al. [43]. Coefficients are given in Table A.6.

The Z-statistics values of three distributions used for the study region are given in Table 5.2. It has been observed that the absolute Z-statistic value of GPA distribution is less than 1.64. also from Figure 5.1 it has been observed that the point of regional TL-skewness and TL-kurtosis lies nearer to the GPA distribution curve. Therefore, the GPA distribution is identified as the best fit distribution for maximum rainfall frequency estimates of North East India.

Table 5.2 Z-statistics values of the distributions using TL-moment

Sl. No.	Probability distribution	Z-Statistic
1	GLO	2.94
2	GEV	2.16
3	GPA	-0.51

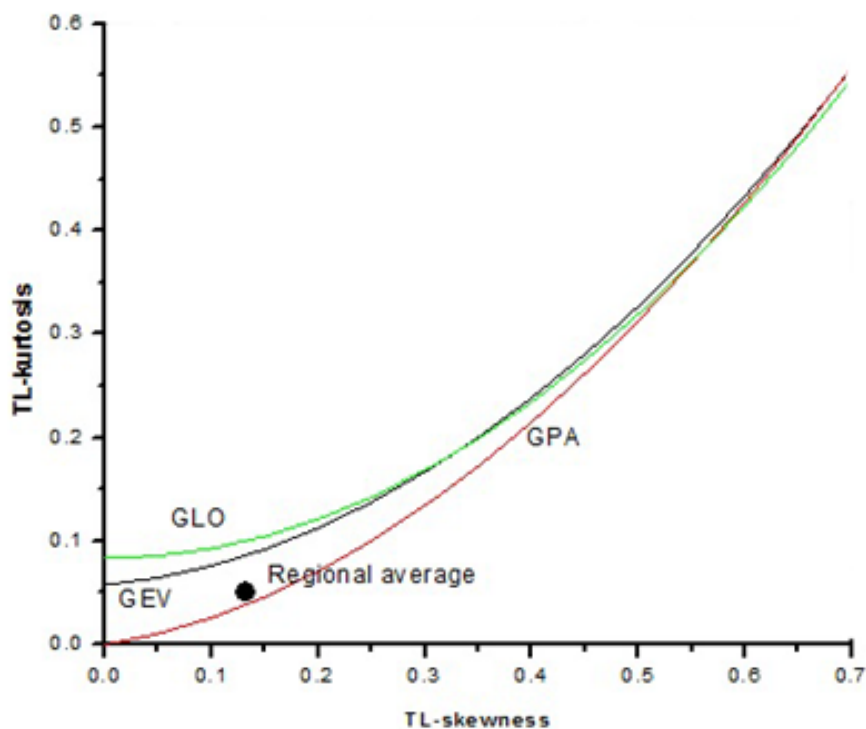(The absolute Z-statistic value of GPA distribution is less than critical value 1.64)

Figure 5.1 TL-moment ratio diagram for NE region
(Regional average is nearer to the GPA distribution curve)

5.4.4 Estimation of Frequency Distribution

The regional parameters of best fit distribution GPA are calculated using approximation relation given in section 5.3.1. Calculated parameters are given in the Table 5.3. Using the regional parameters in the quantile function of GPA given in section 2.3.1 the regional growth factors at different return periods are calculated. Calculated growth factors are given in Table 5.4.

Table 5.3 Parameters of the best fitting distribution using TL-moment

Best fit distribution	Parameters		
	Location(ξ)	Scale(α)	Shape(k)
GPA	0.656	0.510	0.365

Table 5.4 Quantile estimates by using GPA distribution

Best fit distribution	Return period (in years)				
	2	10	20	100	1000
GPA	0.968	1.451	1.586	1.794	1.942

5.5 Development of regional rainfall frequency relationship

The index flood procedure discussed in section 2.4 is used to develop regional rainfall frequency relationship. The form of regional rainfall frequency relationship or growth factor for the best fit distributions GPA can be expressed as

$$Q_T = [\xi + \frac{\alpha}{k}\{1 - (1 - F)^k\}] * \mu_i \tag{5.5.1}$$

where Q_T is the maximum rainfall for return period T, $F = 1 - 1/T$, μ_i is the mean annual maximum rainfall of the ith site, ξ, α and k are the parameters of the respective distributions. Substituting the regional values of the best fit distributions based on the data of 12 gauged sites the regional rainfall frequency relationship for gauged sites of study area is expressed as:

$$Q_T = [0.656 + 1.397\{1 - (1 - F)^{0.365}\}] * \mu_i \tag{5.5.2}$$

5.6 Conclusion

From discordancy test using TL-moment it is found that all the data of the 12 stations of the study region can be considered for the study. From heterogeneity test it is observed that the 12 stations of the study region form a possibly homogeneous region. Z-statistic criteria and TL-moment ratio diagram shows that GPA distribution is the best fit distribution for the study region. Parameters of GPA distribution are calculated using TL-moment. Substituting the regional parameters of GPA distribution in the quantile function GPA distribution of growth factors at different return periods are calculated. Finally using flood index procedure regional rainfall frequency relationships has been developed.

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