Chapter 6: A Comparative Study of Nonlinear Growth Models on Forestry

6.1 Introduction

Teak (*Tectona grandis*) is an all-around premier species of many favorable properties and will remain as one of the most admired and precious trees. The teak plant has a very economical important as its wood are very durable, resistant to fungi. It is indigenous to only four countries namely India, Myanmar, Thailand and Lao People's Democratic Republic [27]. In this Chapter, an attempt is made to analyze the growth (height and DBH) of Teak in India with the help of a set of suitable growth functions.

Babul (*Acacia Nilotica*) is a multipurpose tree native to Africa, the Middle East and the Indian subcontinent. Its timber is valued by rural folks, its leaves and pod are used as food and gum have a number of uses. Though it is not as long-lasting as teak wood, furniture made from babul wood can still last for many years and hence serves as a cheaper alternative to teak wood. Babul wood furniture can also be used in the open air as the wood has good resistance to water and climatic changes. Other than furniture, the wood obtained from the tree is mainly used for making pulpwood and also for medicinal purposes.

A modelling methodology is a powerful tool for the study of growth. It provides smooth curves of age and growth, even from irregularly spaced measurements. The comparison between families of curves can be done using parameter estimates. The Chapmen Richards growth model along with its limiting cases has a wide application in forestry. Here the five limiting cases namely Von Bertalanffy, Monomolecular, Logistic and Gompertz growth model have been considered. Various researchers used these models and Weibull growth model in forestry. This study presents a comparative study of the most commonly used six growth models Monomolecular, Gompertz, Logistic, Weibull, Von Bertalanffy and Chapmen Richard growth models for describing the growth pattern of Teak (*Tectona grandis*) and Babul (*Acacia Nilotica*) in India. This study also presents a comparative study of six growth models for top height age, the mean diameter at breast height data and the cumulative basal area production originated from the Bowmont Norway spruce thinning experiment, sample plot 3661.

In this chapter, the GARCH family models are also used to fit for the data sets. GARCH is a useful generalization of ARCH model, introduced by T. Bollerslev in 1986 ([32], [33]). This model is a weighted average of part squared residuals and it has declining weights which never go completely to zero. The most commonly used GARCH specification, states that the best predictor of the variance in the next period is a weighted average of the long run average variance, the variance predicted for this period and the new information this period which is the most recent squared residual.

GARCH(1,1), GARCH(2,1), GARCH(2,2), EGARCH(1,1,1) and EGARCH(2,1,1) models have been considered for this study.

This chapter is organized as follows. Section 6.2 gives an overview of the data and the methodology of this study. The methodology used to estimate the parameters of these models are discussed in previous chapters. The three final sections (section 6.3 to 6.5) include a brief analysis of the results and some of the main conclusions.

6.2 Methods and materials

Height and DBH growth data from Teak trees in Warangal state and Hoshangabad division of India [27] have been used and presented in the **Table 6.1**. The maximum diameter data and top height growth of *babul (Acacia Nilotica) tree,* presented in **Table 2.1**, are based on the analysis of sample plot data of Uttar Pradesh, Maharashtra and Madhya Pradesh [37]. The top height age, the cumulative basal area production and the mean diameter at breast height data, originated from the Bowmont Norway spruce thinning experiment, sample plot 3661 [[21], [22]] are presented in **Table 2.2**.

Table 6.1: Height and DBH growth data from Teak trees in Warangal state and Hoshangabad division of India.

Age (Years)	Warangal state		Hoshangabad division	
	Height(m)	DBH(cm)	Height(m)	DBH(cm)
10	6.3	12	3.7	3.8
20	12.6	22	7.9	13.2
30	16.7	30	10.7	20.8
40	20	36	12.5	27.4
50	22.4	39	14	32.5
60	24.3	48	15.5	36.3
70	---	51	17.1	39.1
80	---	---	---	41.7
90	---	---	---	43.9

To check the stationarity of the data, the theoretical correlogram and unit root test have been used. The autocorrelation function (ACF) and partial autocorrelation function (PACF) of the data sets are plotted from Figure 6.1 to Figure 6.8.

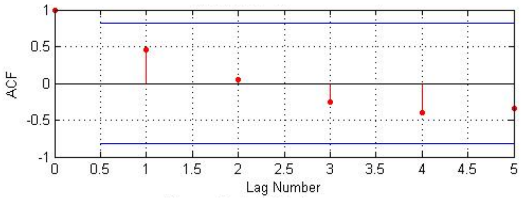


Figure 6.1: ACF of height growth data from Teak trees in Warangal state

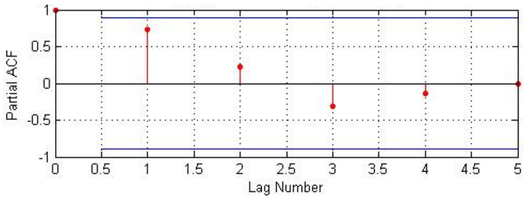


Figure 6.2: PACF of height growth data from Teak trees in Warangal state

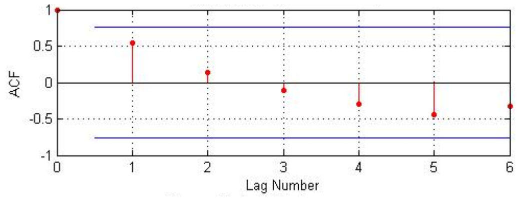


Figure 6.3: ACF of DBH growth data from Teak trees in Warangal state

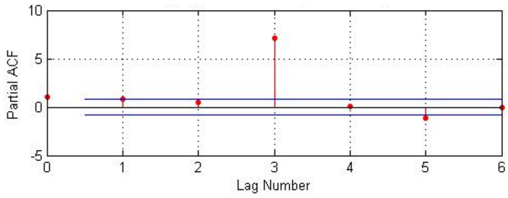


Figure 6.4: PACF of DBH growth data from Teak trees in Warangal state

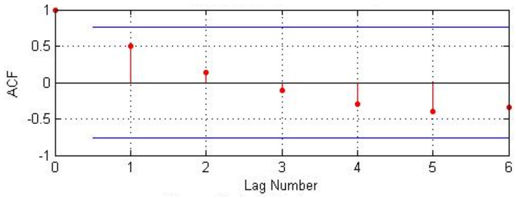


Figure 6.5: ACF of height growth data from Teak trees of Hoshangabad division

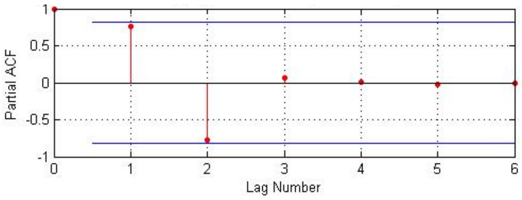


Figure 6.6: PACF of height growth data from Teak trees of Hoshangabad division

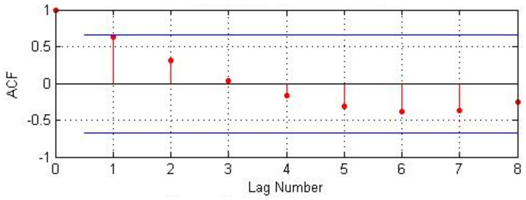


Figure 6.7: ACF of DBH growth data from Teak trees of Hoshangabad division

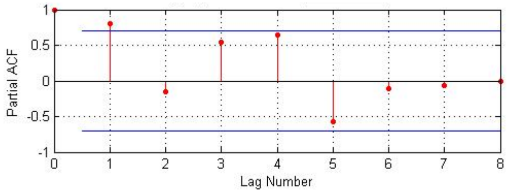


Figure 6.8: PACF of DBH growth data from Teak trees of Hoshangabad division

From the Figure 6.1 to Figure 6.8, it is clear that all the data sets have seasonal effect. Now the Augmented Dickey-Fuller (ADF) unit root test has been used to check the stationarity of the data sets, which are presented in **Table 6.2**. From the **Table 6.2**, it is observed that, for all data sets, the p −values are less then 5%. It means that, the null hypothesis H_0, considering the process is a unit root, can be rejected. And hence, all the data sets presented in this work are stationary.

Table 6.2: $p-$ values of ADF test for different data sets from Teak trees in Warangal state and Hoshangabad division of India.

Data	$\boldsymbol{p-}$ **value**
Height growth data from Teak trees in Warangal state	0.000062
DBH growth data from Teak trees in Warangal state	0.009505
Height growth data from Teak trees of Hoshangabad division	0.000000
DBH growth data from Teak trees of Hoshangabad division	0.017370

The integral forms of Monomolecular, Gompertz, Logistic, Weibull (two, three and four parameters), Von Bertalanffy (two, three and four parameters) and Chapmen Richard (three and four parameters) growth models are shown in **Table 1.1**. Here A, β, K, d, b_1, b and m are parameters to be estimated, y is the dependent growth variable, t is the independent variable and $\exp(e)$ is the base of the natural logarithms. The parameters of the growth models are defined as: A is the asymptote; K is the parameter governing the rate at which the regressand approaches its potential maximum; m is the allometric constant; d is the instant rate of growth in the inflection point, b is the value of the growth at the initial age and B, β and b_1 are biological constants.

To fit the models for Teak growth in India, the method of estimations have been used as discussed in previous chapters. For Babul growth in India and for the data originated from the Bowmont Norway spruce thinning experiment; the results use to compare are the best fit results obtained from previous chapters. To fit the GARCH class models MATLAB software has been used.

For Teak growth in India, after fitting the growth models using different methods of estimation, the best fit model has been selected based on the selection criteria described in section 1.6.2. The selection criteria consist of six distinct steps.

Step I: Logical and Biological consistency: In this step, the growth models with non-consistent and non-natural consistency and poor statistical properties are excluded.

Step II: Chi-Square Goodness-of-Fit Test (χ^2): In this study, only those results are considered which have 95% level of significance with their respective degree of freedom.

Step III: The Root Mean Square Error (RMSE): By comparing the RMSE, the ten best results are selected.

Step IV: Adjusted coefficient of determination(R_a^2): In this Chapter, only those results are considered which have R_a^2 value not less than 0.99.

Step V: Confidence interval: In this step, the confidence intervals of the estimated parameters are evaluated. The final estimate of the parameters with~95% confidence band excluding zero, indicate that there are only non-zero values of the parameters and then they are always significant. In this step, those results with negative confidence interval are eliminated.

Step VI: Coefficient of determination(R^2) and Approximate R^2 for prediction: Finally the value of Coefficient of determination(R^2) and approximate R^2 for prediction are calculated. If the value of $R^2_{prediction}$ and R^2 are r and m

> respectively, then one could expect from the model to explain about $r\%$ of the variability in predicting new observations, as compared to the approximately $m\%$ of the variability in the original data explained by the fitting [55]. Based on this statistics an attempt has been made to select the best fit model for different growth.

For Babul growth in India and for the data originated from the Bowmont Norway spruce thinning experiment; the results are already analyzed in their respective chapters. In this Chapter, the best fit growth model has been chosen by comparing the values of RMSE, $R^2_{prediction}$ and R^2.

6.3 Results

The eleven different forms of six growth models are fitted to height and DBH growth data from Teak trees in Warangal state and Hoshangabad division of India. The parameters of these models are estimated using a total of thirty one methods of estimation.

6.3.1 Height growth data of Teak from Hoshangabad division

The estimation of parameters for the growth models along with the summary of statistical analysis to height growth data from Hoshangabad division are presented in **Table 6.3**. For the height growth data from Hoshangabad division, Weibull model with four parameters unable to provide a fit due to having a singular matrix in the denominator during computation.

> **Step I**: The Gompertz growth model fitted by method B and D, the Logistic model estimated by method B, D, E and F are rejected due to the non-logical estimation of the parameters. All the methods have estimated the asymptotes smaller than the dominant height of Teak tree (17.10m). The estimated parameters of the rest of the models are logically consistent and biologically significant.

Step II: Based on step II, Gompertz growth model (method E), Logistic model (method A and C), Von Bertalanffy four parameters model (methods A and B), Chapmen Richards four parameters model (methods A and B) and Chapmen Richards three parameters model (method A) are rejected due to having less than 95% level of significance.

Step III: Considering the relative value of RMSE, the ten best results are selected in this step. Comparing the value of RMSE, Monomolecular growth models with all its methods of estimation, Gompertz growth model with method F, Weibull two and three parameters growth models along with Chapmen Richards three parameters model for method B are promoted to the next level.

Step IV: In the fourth step, Monomolecular growth model with method B, Gompertz growth model with method E, Weibull two parameters growth model are eliminated as they have R_a^2 value less than 0.99.

Table 6.3: Estimation of parameters along with the summary of statistical analysis to height growth data from Hoshangabad division.

Growth Models	Methods	A	B/b / b_1 / β	k	m / d	χ^2	RMSE	R^2(in %)	R_a^2	$R^2_{prediction}$ (in %)
Monomolecular	A	22.1381	1.0339	.2162	---	.098	.367	99.27	.99	99.08
	B	18.4943	1.0724	.3052	---	.076	.390	99.17	.98	98.17
	C	21.7349	1.0074	.2162	---	.086	.306	99.49	.99	99.33
	D	18.9265	1.0924	.3052	---	.050	.322	99.44	.99	98.93
	E	19.1200	1.0767	.2939	---	.047	.307	99.49	.99	99.01
	F	20.2819	1.0411	.2546	---	.052	.278	99.58	.99	99.34
Gompertz	A	19.0618	2.5772	.4524	---	.330	.635	97.81	.96	97.33
	B	16.4921	2.5281	.5725	---	.191	.591	98.10	.96	96.06
	C	18.8731	2.4331	.4524	---	.214	.529	98.48	.97	98.03
	D	17.0642	2.6513	.5725	---	.145	.508	98.59	.97	97.59
	E	21.8126	2.0489	.3145	---	.445	.655	97.67	.95	97.26
	F	17.9474	2.3339	.4802	---	.168	.452	98.89	.98	98.38
Logistic	A	17.9420	7.9593	.7265	---	.675	.888	95.71	.91	94.76
	B	15.5795	7.2402	.8921	---	.362	.796	96.56	.93	93.10

		C	18.8329	7.9532	.7265	---	.708	1.099	93.43	.87	89.93
		D	16.6592	8.2833	.8921	---	.345	.767	96.80	.94	95.30
		E	15.1116	9.3857	1.1181	---	.477	.983	94.75	.89	89.65
		F	15.1956	9.2801	1.1021	---	.454	.957	95.02	.90	90.28
Weibull	4	A	Not Fitted due to singular matrix occurs during computation								
	3	A	19.4958	---	.23053	1.0936	.070	.309	99.48	.99	99.19
	2	A	21.5015	---	.2186	---	.100	.311	99.47	.98	99.32
VB	4	A	20.8027	14.5914	.2284	.0909	.284	.481	98.74	.97	98.41
		B	21.7108	10.7162	.2192	.1866	.302	.486	98.72	.97	98.56
	3	A	21.1250	.8467	.2137	---	.256	.457	98.87	.98	98.56
		B	20.3753	.8530	.2154	---	.326	.639	97.85	.97	96.44
	2	A	25.9146	---	.1541	---	.312	.615	97.95	.98	97.68
		B	26.0046	---	.1628	---	.183	.535	98.45	.98	97.61
CR	4	A	21.5681	.9947	.1958	.9000	.220	.424	99.02	.98	98.73
		B	22.4362	.9530	.1885	.9898	.274	.459	98.85	.98	98.70
	3	A	33.3230	---	.1133	.9132	.745	1.294	90.92	.86	82.67
		B	20.7444	---	.2382	1.0370	.090	.311	99.48	.99	99.25

Step V: All surviving results along with the 95% confidence level are demonstrated in **Table 6.7**. It is observed that all parameters for all candidate growth models are significantly different from zero.

Step VI: The sixth and final selection criteria is based on R^2 and $R^2_{prediction}$, as this statistic gives some indication of the predictive capability of the growth models. From the final step, the best growth model has been selected. In case of height growth data from Hoshangabad division, the Monomolecular growth model (methods F) is found to be more suitable as the value of $R^2_{prediction}$ and R^2 (99.34 and 99.58 respectively) are better than the remaining surviving growth models. The observed and the estimated value are shown in **Figure 6.9**. The eliminated results in each step are highlighted accordingly in the **Table 6.3**.

6.3.2 DBH growth data of Teak from Hoshangabad division

The estimation of parameters for the growth models and the summary of statistical analysis to DBH growth data from Hoshangabad division are presented in **Table 6.4**. In this case, three parameters Von Bertalanffy growth model (method A and B) and two parameters Von Bertalanffy growth model (method A and B) are rejected due to the non-logical estimation of the parameters. In all the cases, some of their parameters estimate negative value, which violates the model assumption concerning the parameters. Logistic model (method B, E and F) is also eliminated due to having the estimates of asymptotic parameters smaller than the dominant DBH of Teak tree (43.90cm). The eliminated results in each step are also highlighted accordingly in **Table 6.4**. In the case of DBH growth data from Hoshangabad division, no results are eliminated in step IV and V, as all surviving results have 0.99 of R_a^2 value (**Table 6.4**) and all of their parameters are significantly different from zero (**Table 6.7**). Finally, it is find that the Monomolecular growth model (method F) and four parameters Weibull growth model give the best results with $R^2_{prediction}$ and R^2 values 99.98 and 99.98 respectively among all the surviving results. The two results are plotted in order to illustrate their differences (**Figure 6.10**). Both the results produce a very similar result for DBH growth data from Hoshangabad division.

Table 6.4: Estimation of parameters along with the summary of statistical analysis to DBH growth data from Hoshangabad division.

Growth Models		Methods	A	B/b / b_1 / β	k	m / d	χ^2	RMSE	R^2(in %)	R_a^2	$R^2_{predictio}$ (in %)
Monomolecular		A	51.4121	1.1665	.2308	---	.036	.297	99.95	.99	99.93
		B	50.6219	1.1890	.2391	---	.113	.275	99.95	.99	99.94
		C	51.2416	1.1728	.2308	---	.032	.202	99.98	.99	99.97
		D	50.5118	1.1856	.2391	---	.075	.267	99.96	.99	99.95
		E	52.5322	1.1543	.2179	---	.012	.207	99.97	.99	99.97
		F	51.8184	1.1635	.2246	---	.016	.185	99.98	.99	99.98
Gompertz		A	45.8708	2.9338	.4283	---	1.349	1.066	99.32	.99	99.26
		B	44.9708	3.6543	.4877	---	.683	1.001	99.40	.99	99.29
		C	49.8349	3.6416	.4283	---	1.703	2.006	97.59	.96	99.69
		D	46.4961	3.8454	.4877	---	.930	1.377	98.86	.98	98.59
		E	57.6666	2.5288	.2754	---	3.393	2.301	96.83	.95	96.03
		F	44.7922	3.3996	.4806	---	.681	.869	99.55	.99	99.44
Logistic		A	43.9088	8.2081	.6304	---	2.512	1.634	98.40	.97	98.26
		B	42.5763	19.7277	.8549	---	2.952	2.166	97.19	.95	96.75
		C	105.7693	43.7263	.6304	---	71.8	24.18	-249.86	-4.59	-503.11
		D	56.9020	30.4593	.8549	---	17.0	9.136	50.08	.20	23.18
		E	35.7249	37.4705	1.4975	---	5.047	4.326	88.80	.82	82.24
		F	36.0556	36.8190	1.4719	---	4.702	4.178	89.56	.83	83.55
Weibull	4	**A**	48.8618	54.3209	.1887	1.1452	.009	.176	99.98	.99	99.98
	3	A	44.7232	---	.1134	1.5251	.307	0.632	99.76	.99	99.68
	2	A	66.7442	---	.1248	---	2.433	1.670	98.33	.97	98.15

VB	4	A	50.8787	41.7167	.2482	.0909	.061	.247	99.96	.99	99.96
		B	50.2319	46.4833	0.2487	0.0585	.017	.211	99.97	.99	99.96
	3	A	51.2114	-10.8118	.2376	---	1.004	.558	99.81	.99	99.81
		B	51.4894	-8.9503	.2315	---	.044	.281	99.95	.99	99.94
	2	A	1.5551	---	-.3671	---	787.3	32.4	-528.2	-6.18	-815.0
		B	-1.0440	---	-.5844	---	-690.3	5.251	14566.5	166.33	27044.5
CR	4	A	51.6494	1.2477	0.2246	.9000	40.29	1.209	99.12	.99	99.11
		B	45.5156	1.4317	.2701	.558	-6.25	2.442	102.64	1.04	102.07
	3	A	2042.103	---	.0062	1.1607	20.17	11.58	19.67	-.07	-41.74
		B	84.3317	---	.1134	1.2323	2.28	2.568	96.06	.95	94.03

6.3.3 Height growth data of Teak from Warangal state

The estimation of parameters for the growth models along with the summary of statistical analysis to height growth data from Warangal state are presented in **Table 6.5**. The eliminated results in each step were also highlighted accordingly. Here, logistic growth model (method A, E and F) is eliminated due to non-logical estimates of one of its parameter. The methods have estimated for the asymptote (23.8857m, 22.9976m and 23.1506m respectively) smaller than the dominated height (24.30m). Three parameters Von Bertalanffy growth model (method A and B) are also eliminated in step I due to having negative parameter estimates. In the case of height growth data from Warangal state, it is also noticed that no results are eliminated in step IV and V, as all surviving results have 0.99 of R_a^2 value (**Table 6.5**) and all of their parameters are significantly different from zero (**Table 6.7**). And finally, based on R^2 and $R_{prediction}^2$, the better result is chosen and it is find as Monomolecular growth model (method F). The observed and the estimated value are shown in **Figure 6.11**.

6.3.4 DBH growth data of Teak from Warangal state

The estimation of parameters for the growth models and the summary of statistical analysis to DBH growth data from Warangal state are presented in **Table 6.6**. The best result is selected and found as Monomolecular growth model for method F. **Figure 6.12** represents the observed and the estimated values. The eliminated results in each step are highlighted accordingly in **Table 6.6**. For the DBH growth data, only eight results are promoted to step III as most of the results are failed to obtain 95% level of significance. In step IV, four results are eliminated due to having R_a^2 value less than 0.99.

Table 6.5: Estimation of parameters along with the summary of statistical analysis to height growth data from Warangal state.

Growth Models		Methods	A	B/b / b_1 / β	k	m / d	χ^2	RMSE	R^2(in %)	R_a^2	$R^2_{prediction}$ (in %)
Monomolecular		A	29.3128	1.0605	.3007	---	.009	.139	99.95	.99	99.94
		B	29.7602	1.0409	.2883	---	.013	.141	99.95	.99	99.94
		C	29.3508	1.0559	.3007	---	.006	.115	99.96	.99	99.96
		D	29.8473	1.0449	.2883	---	.010	.135	99.95	.99	99.95
		E	28.7071	1.0706	.3177	---	.005	.122	99.96	.99	99.93
		F	29.1420	1.0608	.3062	---	.006	.113	99.97	.99	99.96
Gompertz		A	25.4228	2.5419	.5999	---	.067	.404	99.56	.99	99.27
		B	26.4567	2.3586	.5396	---	.072	.361	99.65	.99	99.58
		C	25.6692	2.5242	.5999	---	.050	.356	99.66	.99	99.49
		D	26.8528	2.4250	.5396	---	.078	.408	99.55	.99	99.43
		E	31.9867	2.0577	.3597	---	.374	.836	98.14	.95	97.29
		F	25.9623	2.4093	.5702	---	.056	.337	99.69	.99	99.58
Logistic		A	23.8857	7.1097	.9349	---	.181	.668	98.81	.97	97.90
		B	24.9301	6.3350	.8361	---	.170	.585	99.08	.98	98.83
		C	24.6159	7.2631	.9349	---	.164	.665	98.82	.97	98.30
		D	26.1825	7.0499	.8361	---	.291	.892	97.87	.95	96.60
		E	22.9976	7.8323	1.0876	---	.217	.858	98.04	.95	95.87
		F	23.1506	7.7481	1.0704	---	.200	.818	98.22	.96	96.35
Weibull	4	A	31.6992	36.0614	.3519	.8383	.002	.067	99.96	.99	99.95
	3	A	27.0873	---	.2686	1.1733	.013	.186	99.90	.99	99.85
	2	A	31.5325	---	.2484	---	.066	.29	99.76	.99	99.74
VB	4	A	29.9822	21.2525	.2857	.0909	.119	.381	99.61	.99	99.60
		B	30.6153	17.2375	.2807	.1516	.080	.328	99.71	.99	99.69
	3	A	30.2645	-.1349	.2714	---	.089	.326	99.72	.99	99.71
		B	30.0835	-.7938	.2754	---	.029	.221	99.87	.99	99.84
	2	A	34.7985	---	.1997	---	.225	.735	98.56	.98	98.18
		B	34.8401	---	.2124	---	.081	.451	99.46	.99	99.03
CR	4	A	30.6413	1.0429	.2540	.9000	.054	.253	99.83	.99	99.82
		B	31.1110	1.0309	.2512	.9437	.036	.216	99.88	.99	99.87
	3	A	106.6136	---	.0328	.7828	.710	1.652	92.74	.88	82.43
		B	34.0534	---	.2112	.9737	.076	.386	99.60	.99	99.44

Table 6.6: Estimation of parameters along with the summary of statistical analysis to DBH growth data from Warangal state.

Growth Models		Methods	A	B/b / b_1 / β	k	m / d	χ^2	RMSE	R^2 (in %)	R_a^2	$R_{prediction}^2$ (in %)
Monomolecular		A	76.000	.9849	.1567	---	.265	1.225	99.09	.98	98.65
		B	63.5455	.9991	.2106	---	.349	1.491	98.66	.97	97.54
		A	75.3216	.9779	.1567	---	.257	1.207	99.11	.98	98.66
		D	64.9623	1.0144	.2106	---	.323	1.386	99.84	.98	98.12
		E	70.2822	.9891	.1781	---	.262	1.254	99.05	.98	98.49
		F	79.0094	.9712	.1436	---	.268	1.199	99.13	.99	98.69
Gompertz		A	59.95030	2.3591	.3829	---	.563	1.613	98.43	.97	97.87
		B	51.6012	2.2763	.4702	---	.674	2.040	97.48	.95	95.08
		C	59.0827	2.2666	.3829	---	.455	1.502	98.64	.97	98.07
		D	53.1861	2.3629	.4702	---	.529	1.790	98.06	.96	96.69
		E	66.2514	2.1091	.3025	---	.631	1.528	98.59	.97	98.15
		F	60.7303	2.1568	.3525	---	.487	1.456	98.72	.97	98.21
Logistic		A	54.9474	6.7804	.6390	---	1.064	2.107	97.32	.95	96.53
		B	47.1306	5.9734	.7656	---	1.188	2.669	95.69	.91	91.33
		C	55.8389	6.5998	.6390	---	1.003	2.249	96.94	.94	95.68
		D	49.6432	6.5966	.7656	---	.882	2.266	96.89	.94	94.87
		E	46.9508	6.7822	.8536	---	1.254	2.777	95.34	.91	91.21
		F	47.2798	6.7439	.8417	---	1.188	2.700	95.59	.91	91.81
Weibull	4	A	34.0000	.0000	.0000	.9613	34.059	12.86	0.00	-1.0	-50.53
	3	A	73.6872	---	.1801	0.9535	.253	1.2335	99.08	.99	98.55
	2	A	70.8236	---	.1788	---	.291	1.274	99.01	.98	98.51
VB	4	A	109.1104	65.9357	.0942	.0909	.731	1.754	98.14	.96	97.07
		B	109.6865	57.2476	.0920	.1202	.585	1.467	98.69	.97	98.24
	3	A	120	5.6667	.0771	---	.694	1.735	98.18	.97	97.09
		B	118.817	5.6442	.0739	---	.559	1.463	98.71	.90	98.26
	2	A	69.3280		.1901	---	.330	1.412	98.79	.99	98.21
		B	69.3407	---	.1864	---	.281	1.292	98.99	.99	98.45
CR	4	A	140.6016	.9764	.0562	.9000	.649	1.711	98.23	.96	97.11
		B	140.8934	.9716	.0549	.9211	.523	1.407	98.80	.98	98.34
	3	A	3170.62	---	.0008	.7745	1.796	3.581	92.25	.88	84.79
		B	55.5152	---	.3071	1.1267	.704	2.026	97.52	.96	95.75

Table 6.7: 95% Confidence intervals of the parameters of surviving growth models.

Data	Model	Method	A		B/b / b_1 / β		k		m / d	
			Lower Limit	Upper Limit	Lower Limit	Upper Limit	Lower Limit	Upper Limit	Lower Limit	Upper Limit
Hoshangabad_height	Monomolecular	A	16.489	27.787	0.916	1.151	0.098	0.334	---	---
		C	17.014	26.456	0.912	1.103	0.113	0.319	---	---
		D	16.284	21.569	0.952	1.233	0.191	0.420	---	---
		E	16.423	21.817	0.947	1.207	0.185	0.403	---	---
		F	17.117	23.446	0.939	1.144	0.159	0.350	---	---
	Weibull 3	A	15.347	23.644	---	---	0.186	0.275	0.844	1.343
	Chapman Richards 3	B	16.810	24.679	---	---	0.108	0.369	0.754	1.320
Hoshangabad_DBH	Monomolecula	A	49.66	53.161	1.135	1.198	0.212	0.249	---	---

	r		3							
		B	49.101	52.143	1.159	1.219	0.222	0.256	---	---
		C	50.047	52.436	1.151	1.194	0.218	0.243	---	---
		D	49.035	51.989	1.157	1.215	0.222	0.256	---	---
		E	51.181	53.883	1.133	1.175	0.205	0.231	---	---
		F	50.673	52.964	1.144	1.183	0.213	0.236	---	---
	Gompertz	F	42.358	47.226	2.772	4.027	0.402	0.559	---	---
	Weibull 4	A	46.640	51.084	49.996	58.646	0.159	0.219	1.023	1.268
	Von Bertalanffy 4	A	50.390	51.368	36.187	47.247	0.239	0.257	0.059	0.122
		B	49.729	50.735	39.985	53.582	0.239	0.258	0.024	0.093
Warangal_height	Monomolecular	A	27.052	31.574	1.003	1.118	0.244	0.358	---	---
		B	27.278	32.242	0.986	1.096	0.230	0.346	---	---
		C	27.473	31.229	1.009	1.103	0.253	0.348	---	---
		D	27.475	32.220	0.992	1.098	0.233	0.343	---	---
		E	26.905	30.509	1.018	1.124	0.267	0.369	---	---
		F	27.362	30.922	1.014	1.108	0.260	0.353	---	---
	Gompertz	F	23.311	28.614	1.859	2.960	0.405	0.736	---	---
	Weibull 4	A	23.024	40.374	19.658	52.465	0.226	0.478	0.418	1.259
	Weibull 3	A	24.703	29.471	---	---	0.244	0.293	1.039	1.307
	Weibull 2	A	30.915	32.150	---	---	0.240	0.257	---	---
Warangal_DBH	Monomolecular	F	38.482	119.537	0.892	1.051	0.013	0.274	---	---
	Weibull 3	A	20.880	126.494	---	---	0.065	0.295	0.590	1.317
	Von Bertalanffy 2	A	55.887	82.769	---	---	0.129	0.251	---	---
		B	56.614	82.067	---	---	0.130	0.243	---	---

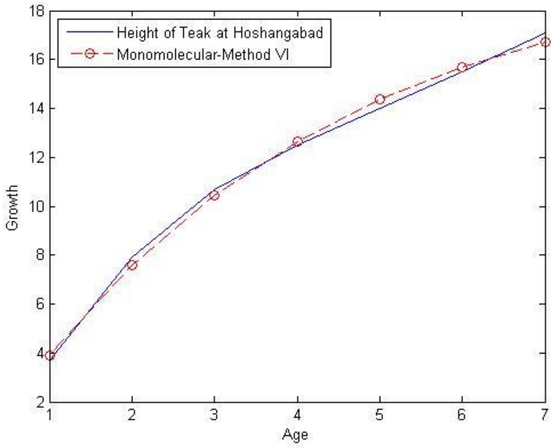


Figure 6.9: Observed data along with the top two results for height growth data of Hoshangabad.

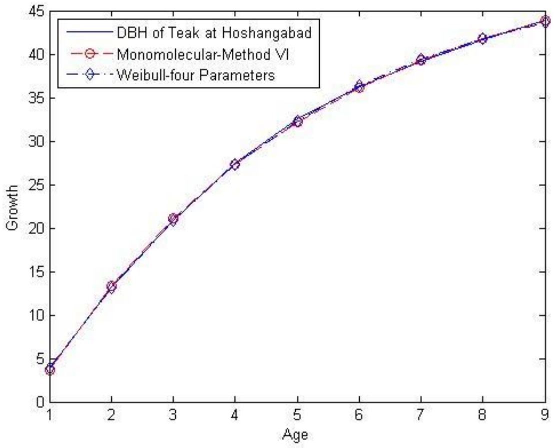


Figure 6.10: Observed data along with the top two results for DBH growth data of Hoshangabad.

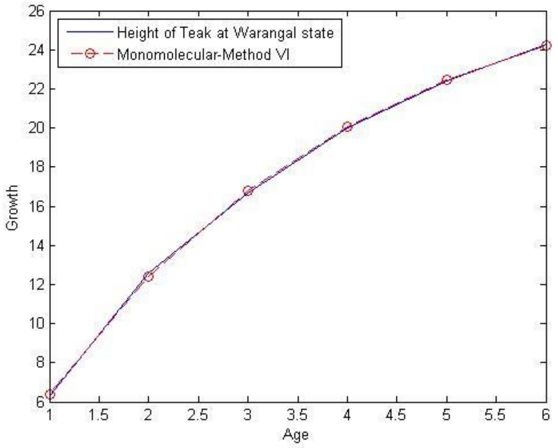


Figure 6.11: Observed data along with the top two results for height growth data of Warangal.

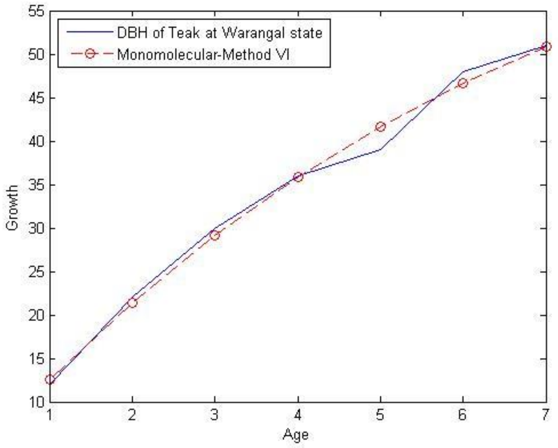


Figure 6.12: Observed data along with the top two results for DBH growth data of Warangal.

The eleven different forms of six growth models are fitted to Top height and maximum diameter growth of Babul tree in India. The parameters of these models are estimated using a total of thirty one methods of estimation. The best fit results are determined from Chapter 2 to Chapter 5. This chapter is performing a comparative study among those results to find out the best fit model among the forms of the growth models.

6.3.5 Comparison between the results for top height growth of Babul tree in India

The best fit results from each chapter for top height growth of Babul tree are demonstrated in **Table 6.8**. By observing and analyzing the results it can be concluded that the Monomolecular growth model with method E and method F produces the best results for top height growth of Babul as the values of RMSE, $R^2_{prediction}$ and R^2 (0.02, 99.99 and 99.99 respectively) are better than the remaining results. The two results are plotted in order to illustrate their differences (**Figure 6.13**). Both results produce a very similar result for top height growth data of babul tree in India.

Table 6.8: Collection of best fit growth results from each chapter for top height growth data of Babul trees.

Age (Year)		Observed Data	Monomolecular (Method A)	Monomolecular (Method C)	Monomolecular (Method D)	**Monomolecular (Method E)**	**Monomolecular (Method F)**	Four parameters Weibull Model	von Bertalanffy with three parameters (Method B)	Chapman Richard with four parameters (Method B)
5		8.14	8.14	8.14	8.16	**8.14**	**8.14**	8.145	8.17	8.13
10		12.19	12.21	12.19	12.18	**12.19**	**12.19**	12.181	12.16	12.15
15		14.93	14.93	14.91	14.89	**14.91**	**14.91**	14.914	14.88	14.86
20		16.70	16.76	16.74	16.73	**16.74**	**16.74**	16.743	16.73	16.72
25		17.98	17.98	17.96	17.98	**17.96**	**17.96**	17.957	17.99	18.01
Parameters	A		20.46	20.44	20.58	**20.47**	**20.47**	20.28	20.66	20.98
	β		0.899	0.898	0.893	**0.897**	**0.897**	17.84	2.29	0.93
	k		0.400	0.400	0.391	**0.398**	**0.398**	0.39	0.39	0.36
	m		---	---	---	**---**	**---**	1.04	---	0.89
na ly si	χ^2		.0002	.0001	.0002	**.0001**	**.0001**	Not applicable	0.000	Not Applicable

	$RMSE$	0.026	0.021	0.024	**0.020**	**0.020**	0.023	0.030	0.042
	$R^2\ (in\ \%)$	99.99	99.99	99.99	**99.99**	**99.99**	99.99	99.99	99.99
	R_a^2	0.99	0.99	0.99	**0.99**	**0.99**	0.99	0.99	0.99
	$R_{prediction}^2\ (in\ \%)$	99.99	99.99	99.99	**99.99**	**99.99**	99.99	99.99	99.98

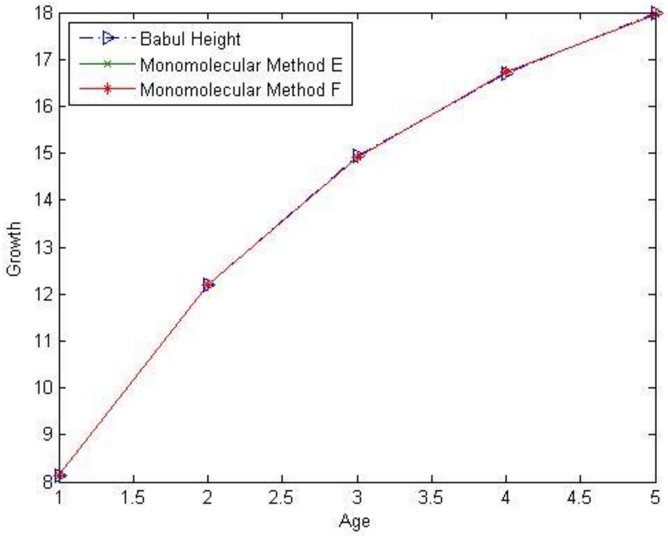


Figure 6.13: Plot of best fit results (Monomolecular growth model with method E and F) for top height growth data of Babul tree in India.

6.3.6 Comparison between the results for maximum diameter growth of Babul tree in India

The best fit results in each chapter for maximum diameter growth of Babul tree are demonstrated in **Table 6.9**. By observing and analyzing the results it can be concluded that the Weibull growth model with four parameters produces the best results for maximum diameter growth of Babul as the value of RMSE, $R_{prediction}^2$ and R^2 (0.086, 99.98 and 99.99 respectively) are better than the remaining results. The result is plotted along with the observed data in order to illustrate their differences (**Figure 6.14**). The result produces a very similar result for maximum diameter growth data of babul tree in India.

Table 6.9: Collection of best fit growth results from each chapter for maximum diameter growth data of Babul trees.

Age (Year)	Observed Data	Monomolecular (Method C)	Monomolecular (Method D)	Monomolecular (Method E)	Monomolecular (Method F)	**Four parameters Weibull Model**	von Bertalanffy with two parameters (Method B)	Chapman Richard with four parameters (Method B)
5	12.19	12.18	12.17	12.18	12.17	**12.22**	12.22	12.19
10	20.83	20.85	20.87	20.85	20.86	**20.72**	20.86	20.83
15	26.92	26.98	27.00	26.98	26.99	**27.06**	26.96	26.92
20	31.49	31.32	31.32	31.32	31.32	**31.43**	31.28	31.30
25	34.29	34.39	34.37	34.39	34.38	**34.29**	34.32	34.47
Parameters	A	41.80	41.65	41.83	41.72	**38.7870**	41.6625	43.1172
	β	1.002	1.004	1.002	1.003	**35.4811**	---	1.0263
	k	0.346	0.349	0.3458	0.348	**0.2894**	0.3473	0.3079
	m	---	---	---	---	**1.2213**	---	0.90
Analysis	χ^2	0.001	0.001	0.001	0.001	Not applicable	0.002	Not Applicable
	$RMSE$	0.093	0.092	0.093	0.092	**0.086**	0.100	0.118
	R^2 $(in\ \%)$	99.99	99.99	99.99	99.99	**99.99**	99.98	99.98
	R_a^2	0.99	0.99	0.99	0.99	**0.99**	0.99	0.99
	$R^2_{prediction}$ $(in\ \%)$	99.97	99.97	99.97	99.97	**99.98**	99.97	99.94

The eleven different forms of six growth models are also fitted to Top height, mean diameter at breast height and cumulative basal area production from the Bowmont Norway spruce thinning experiment using thirty one methods of estimation. The best fit results are determined from Chapter 2 to Chapter 5. This chapter is performing a comparative study among those results to find out the best fit model among the forms of the growth models.

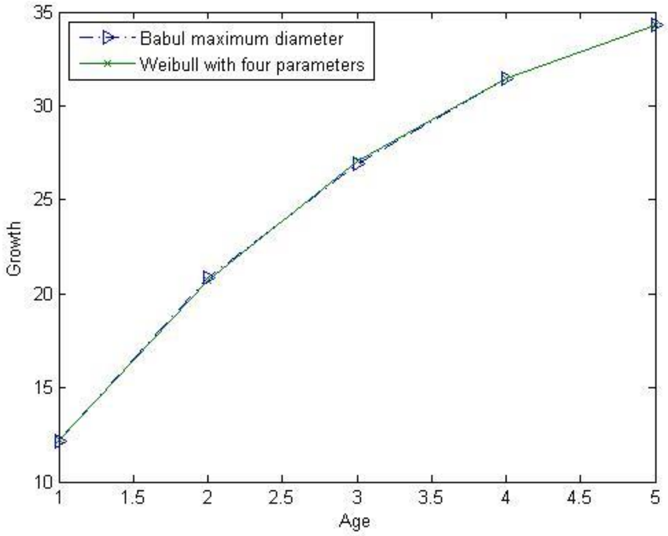


Figure 6.14: Plot of the best fit result (Weibull growth model with four parameters) along with the observed data for maximum diameter growth data of Babul tree in India.

6.3.7 Comparison between the results for top height growth data from Norway experiment

The best fit results from each chapter for top height growth are presented in **Table 6.10**. By observing and analyzing the results it can be concluded that the Gompertz growth model with method F produces the best result for top height growth data from the Bowmont Norway spruce thinning as the values of RMSE, $R^2_{prediction}$ and R^2 (0.104, 99.92 and 99.93 respectively) are better than the remaining results. The fitted value is plotted along with the observed data in order to illustrate their differences (**Figure 6.15**). The result forms a very similar result for top height data from Bowmont Norway spruce thinning experiment.

Table 6.10: Collection of best fit growth results from each chapter for top height growth data from the Bowmont Norway spruce thinning experiment.

Age (Year)	Observed Data	**Gompertz (Method F)**	Four parameters Weibull Model	von Bertalanffy with three parameters (Method B)	Chapman Richard with four parameters (Method B)
20	7.30	**7.33**	7.36	7.18	7.16
25	9.00	**9.05**	9.02	9.1	9.09
30	10.90	**10.79**	10.76	10.88	10.87
35	12.60	**12.47**	12.47	12.53	12.51
40	13.90	**14.06**	14.07	14.06	14.03
45	15.40	**15.53**	15.54	15.47	15.44
50	16.90	**16.86**	16.87	16.79	16.76
55	18.20	**18.04**	18.05	18	17.98
60	19.00	**19.09**	19.08	19.13	19.11
65	20.00	**20.00**	19.98	20.18	20.18
Parameters	A	**25.01**	24.6476	33.4387	35.6324
	β	**1.48**	18.5303	5.1069	0.8827
	k	**.19**	0.0693	0.0759	0.0643
	m	**---**	1.2992	---	0.9124
Analysis	χ^2	**0.007**	0.009	0.01	0.01195
	$RMSE$	**.104**	0.109	0.12	0.1307
	R^2 $(in\ \%)$	**99.93**	99.93	99.91	99.90
	R_a^2	**0.99**	0.99	0.99	0.99
	$R_{prediction}^2$ $(in\ \%)$	**99.92**	99.91	99.87	99.86

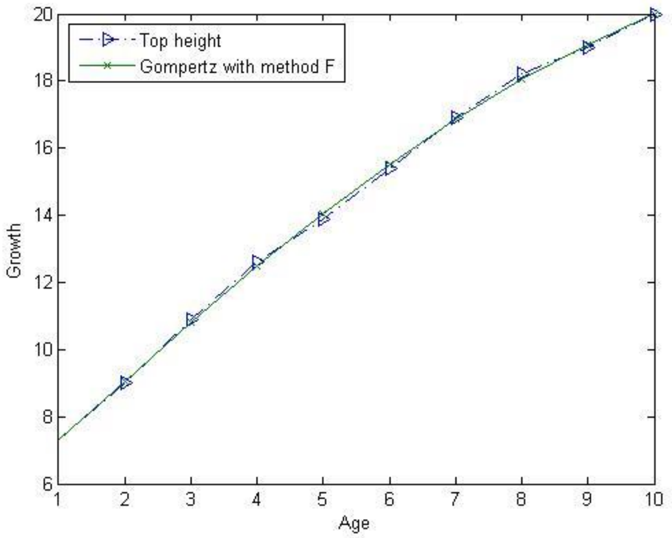


Figure 6.15: Plot of the best fit result (Gompertz growth model with method F) along with the observed data for top height growth data from the Bowmont Norway spruce thinning experiment.

6.3.8 Comparison between the results for DBH growth data from Norway experiment

The best fit results from each chapter for mean diameter breast height data growth are presented in the **Table 6.11**. By observing and analyzing the results it can be concluded that the Logistic growth model with method F produces the best result for mean diameter breast height data from the Bowmont Norway spruce thinning as the value of RMSE, $R^2_{prediction}$ and R^2 (0.164, 99.89 and 99.93 respectively) are better than the remaining results. The result is plotted along with the observed data in order to illustrate their differences (**Figure 6.16**). The result creates a very similar plot for mean diameter breast height data from Bowmont Norway spruce thinning experiment.

Table 6.11: Collection of best fit growth results from each chapter for mean diameter breast height data from the Bowmont Norway spruce thinning experiment.

Age (Year)	Observed Data	**Logistic (Method F)**	Four parameters Weibull Model	von Bertalanffy with three parameters (Method B)	Chapman Richard with four parameters (Method B)
20	8.40	**8.41**	8.42	7.59	7.09
25	10.40	**10.33**	10.24	10.26	10.08
30	12.35	**12.46**	12.44	12.79	12.77
35	14.74	**14.74**	14.81	15.19	15.23
40	17.13	**17.09**	17.20	17.47	17.51
45	19.50	**19.41**	19.50	19.63	19.63
50	21.49	**21.61**	21.64	21.68	21.63
55	23.82	**23.61**	23.58	23.62	23.48
60	25.55	**25.38**	25.29	25.46	25.24
65	26.50	**26.90**	26.78	27.21	26.89
Parameters	A	**32.7599**	33.15763	59.2153	60.3632
	β	**3.8566**	25.74324	4.7722	0.9676
	k	**0.2873**	0.03980	0.0531	0.0435
	m	**---**	1.54465	$---$	0.8207
Analysis	χ^2	**0.011**	0.013	0.147	0.307
	$RMSE$	**0.164**	0.165	0.424	0.528
	R^2 $(in\ \%)$	**99.93**	99.93	99.52	99.24
	R_a^2	**0.99**	0.99	0.99	0.99
	$R_{prediction}^2$ $(in\ \%)$	**99.89**	99.89	99.38	99.17

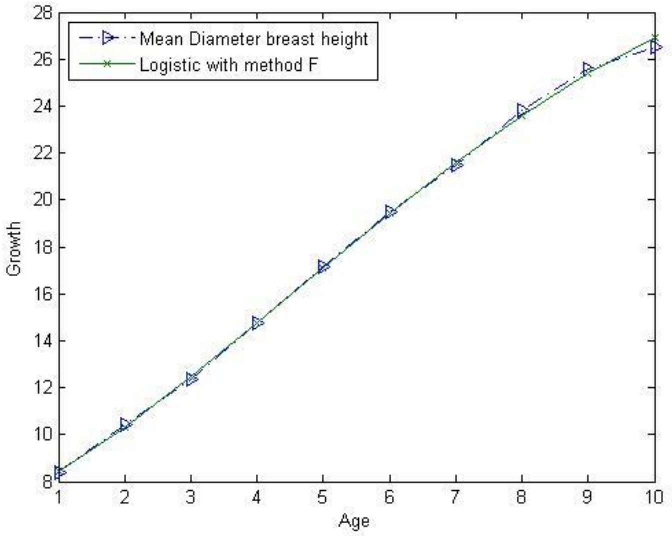


Figure 6.16: Plot of the best fit result (Logistic growth model with method F) along with the observed data for mean diameter breast height from the Bowmont Norway spruce thinning experiment.

6.3.9 Comparison between the results for cumulative basal area production data from Norway experiment

The best fit results from each chapter for cumulative basal area production data growth are presented in the **Table 6.12**. By observing and analyzing the results it can be concluded that the Gompertz growth model with method F produced the best result for cumulative basal area production data from the Bowmont Norway spruce thinning as the value of RMSE, $R^2_{prediction}$ and R^2 (0.748, 99.90 and 99.92 respectively) are better than the remaining results. The result is plotted along with the observed data in order to illustrate their differences (**Figure 6.17**). The result produces a very similar result for cumulative basal area production data from Bowmont Norway spruce thinning experiment.

Table 6.12: Collection of best fit growth results from each chapter for cumulative basal area production data from the Bowmont Norway spruce thinning experiment.

Age (Year)	Observed Data	**Gompertz (Method F)**	von Bertalanffy with three parameters (Method B)	Chapman Richard with four parameters (Method B)
20	37.99	**38.43**	37.03	36.66
25	49.00	**48.63**	48.68	48.52
30	60.41	**59.17**	59.72	59.60
35	68.91	**69.70**	70.16	70.02
40	78.73	**79.91**	80.05	79.85
45	89.83	**89.56**	89.42	89.14
50	98.60	**98.49**	98.28	97.95
55	107.00	**106.62**	106.67	106.31
60	114.80	**113.92**	114.61	114.26
64	119.54	**120.38**	122.13	121.82
Parameters	A	**158.94**	255.7066	284.1495
	β	**1.70**	24.7172	0.9350
	k	**0.18**	0.0548	0.0432
	m	**---**	---	0.9067
Analysis	χ^2	**0.075**	0.139	0.156
	$RMSE$	**0.748**	1.094	1.093
	R^2 $(in\ \%)$	**99.92**	99.83	99.83
	R_a^2	**0.99**	0.99	0.99
	$R_{prediction}^2$ $(in\ \%)$	**99.90**	99.75	99.76

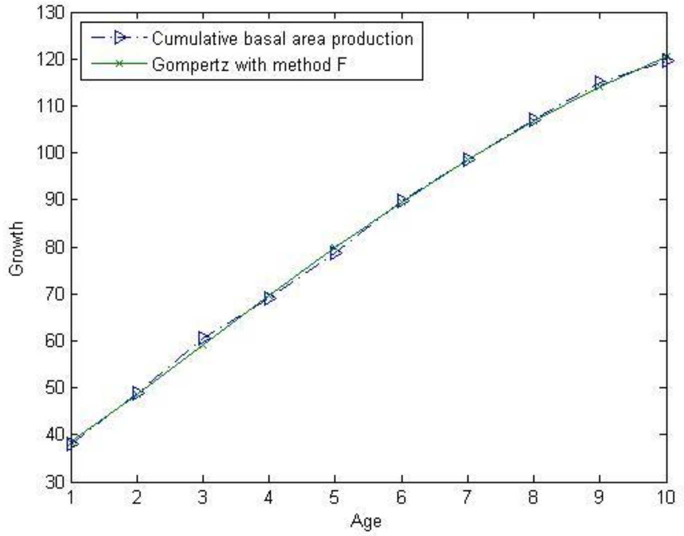


Figure 6.17: Plot of the best fit result (Gompertz growth model with method F) along with the observed data for cumulative basal area production from the Bowmont Norway spruce thinning experiment.

6.3.10 Fitting of GARCH family models

As the data sets used in this thesis are stationary, so the GARCH family models have been used. MATLAB software has been used to fit the GARCH class models. The estimated parameters along with the summary of statistical analysis are presented in the Table **6.13**. Form the results it is observed that, the GARCH(2,2) model is not fitted for any one of the data sets used in this study, due to some negative definite matrix during computation. For the same reason, the GARCH(1,1) model is also not fitted for both Babul tree data sets. Although GARCH family models have been fitted for the remaining data sets but the models does not give better fit than the other growth models discussed in chapter 2 to chapter 5.

Table 6.13: Estimated parameters and the summary of statistical analysis for the GARCH family model

Data	Model	Parameters				χ^2	$RMSE$	R^2	R_a^2
		Constant	GARCH{1}	GARCH{2}	ARCH{1}				
Height growth of Teak from Hoshangabad	GARCH(1,1)	53.164	---	---	0.738764	0.570	1.145	85.86	0.76
	GARCH(2,1)	53.164	---	---	0.738764	0.570	1.145	85.86	0.76
	EGARCH(1,1,1)	2.30259	0.512775	---	1	1.093	1.559	73.78	0.56
	EGARCH(2,1,1)	3.19879	---	0.323643	1	1.868	1.971	58.09	0.30
DBH growth of Teak from Hoshangabad	GARCH(1,1)	100	0.0519038	---	0.948096	2.728	2.795	92.23	0.89
	GARCH(2,1)	100	---	0.0714413	0.928558	2.264	2.721	92.64	0.89
	EGARCH(1,1,1)	3.08603	0.523555	---	1	6.417	4.879	76.33	0.66
	EGARCH(2,1,1)	6.369	---	0.0223956	1	11.58	6.574	57.03	0.40
Height growth of Teak from Warangal	GARCH(1,1)	100	0.0609329	---	0.74275	0.794	1.729	82.75	0.65
	GARCH(2,1)	100	---	0.062402	0.741041	0.806	1.762	82.10	0.64
	EGARCH(1,1,1)	2.30259	0.582642	---	1	1.122	2.083	74.97	0.49
	EGARCH(2,1,1)	3.82594	---	0.309297	1	2.671	3.115	44.05	-.12
DBH growth of Teak from Warangal	GARCH(1,1)	100	0.17792	---	0.82208	4.412	4.935	75.38	0.58
	GARCH(2,1)	100	---	0.174329	0.82567	3.989	4.909	75.62	0.59
	EGARCH(1,1,1)	2.30259	0.663906	---	1	3.842	4.897	75.75	0.59
	EGARCH(2,1,1)	6.928	---	-0.0119932	1	8.255	7.098	49.05	0.15
Top height growth of Babul	GARCH(1,1)		Estimated GARCH model is invalid						
	GARCH(2,1)	100	---	---	0.578363	0.505	1.380	59.61	-0.21
	EGARCH(1,1,1)	2.30259	0.546951	---	1	0.413	1.257	65.50	-0.00
	EGARCH(2,1,1)	2.95489	---	0.416876	1	0.741	1.646	42.57	-0.72

Max diameter growth of Babul	GARCH(1,1)		Estimated GARCH model is invalid						
	GARCH(2,1)	100	---	0.312574	0.687426	1.111	2.784	70.11	0.10
	EGARCH(1,1,1)	2.30259	0.635707	---	1	1.099	2.766	70.51	0.12
	EGARCH(2,1,1)	5.61331	---	0.103061	1	2.520	4.116	34.67	-0.95
Top height growth data from Norway	GARCH(1,1)	37.9483	---	---	.895241	0.340	0.772	95.33	0.94
	GARCH(2,1)	37.9482	---	---	0.895242	0.340	0.772	95.33	0.94
	EGARCH(1,1,1)	0.92705	0.803812	---	1	1.179	1.360	85.49	0.81
	EGARCH(2,1,1)	2.76263	---	0.454982	1	2.989	2.203	61.92	0.49
DBH growth data from Norway	GARCH(1,1)	48.4224	---	----	0.952022	0.544	1.087	96.05	0.95
	GARCH(2,1)	48.4220	---	----	0.952024	0.544	1.087	96.05	0.95
	EGARCH(1,1,1)	1.11364	0.787645	---	1	2.592	2.272	82.70	0.77
	EGARCH(2,1,1)	3.99671	---	0.280508	1	5.900	3.486	59.28	0.46
Basal area data from Norway	GARCH(1,1)	100	0.0996659	---	0.900334	9.332	8.679	85.22	0.82
	GARCH(2,1)	170.641	---	0.0984493	0.901551	7.634	8.236	87.59	0.83
	EGARCH(1,1,1)	1.66479	0.798702	---	1	9.256	9.200	84.51	0.79
	EGARCH(2,1,1)	6.78252	---	0.211797	1	23.487	40.928	59.23	0.46

It is observed that for height growth data of teak tree in Hoshangabad division, Monomolecular growth model along with methods F (value of $R^2_{prediction}$ and R^2 are 99.34 and 99.58 respectively) is found to be more suitable than the remaining growth models whereas Monomolecular growth model with method F ($R^2_{prediction}$ and R^2 values are 99.98 and 99.98 respectively) provides a better fit along with four parameter's Weibull growth model for DBH growth data of teak tree in Hoshangabad division. Also for height growth data of teak tree in Warangal state, Monomolecular growth model along with methods F (value of $R^2_{prediction}$ and R^2 are 99.96 and 99.97 respectively) is found to be more suitable than the remaining growth models whereas Monomolecular growth model with method F ($R^2_{prediction}$ and R^2 values are 99.69 and 99.13 respectively) provides a better fit for DBH growth data of teak tree in Warangal state. It is also observed that, Monomolecular growth model with all its methods of estimation provides a healthy fit for the data sets of teak growth except the DBH growth of Warangal state. In case of DBH growth of Warangal state, the table values of χ^2 for 95% level of significance is found to be smaller than the calculated χ^2 values for two of the methods (Method B and D). Also, three of its method (Method A, C and E) are found failed to attend the 0.99 value of R^2_a.

It is also observed that for top height growth data of babul tree, Monomolecular growth model along with methods E and method F is found to be more suitable than the remaining growth models. For both the methods of estimation, the model produces a very similar result. Whereas Weibull growth model with four parameters provides a better fit for maximum diameter growth data of Babul trees in India. For top height age data and for cumulative basal area production from the Bowmont Norway spruce thinning experiment, the Gompertz growth model with method F produces the best fit whereas the Logistic growth model with method F is found to be more suitable for the mean diameter at breast height data, originated from the Bowmont Norway spruce thinning experiment.

6.3.11 Test of autocorrelation of the best fit results

The **Durbin Watson** (DW) test [34] and **Ljung-Box** (LB) test [33] for autocorrelation have been use to test the autocorrelation for estimated residuals of best fit results. The test statistics have been presented in Table 6.14. The DW values of the estimated residuals are approximately equivalent to 2 for the top height, DBH and cumulative basal area production data from Norway. Therefore the residuals of the estimated models may not be autocorrelated. For the Babul tree data, height growth data of Teak from Hoshangabad, DBH growth data of Teak from Warangal state and height growth data of Teak from Warangal state; the DW test fails to provide any conclusion due to less number of observations. Also for the results for DBH growth data of Teak from Hoshangabad division, the DW test is inconclusive as the value of DW lies between upper and lower bound of Durbin Watson statistics at 5% significance level. But in LB test, the $h-$value is 0 for all the data sets. The $h-$ value 0 based on LB test, which indicates that there is not enough evidence to reject the null hypothesis that the residuals of the returns are not autocorrelated. Therefore, it can be concluded that, the residuals of the estimated models may not be autocorrelated.

Table 6.14: DW and LB test statistics for residual series of the best fit results.

Data Name along with Method	**DW Test**	**LB Test (h-value)**
Monomolecular model with method F for height growth data of Teak from Hoshangabad division	1.5482	0
Monomolecular model with method F for DBH growth data of Teak from Hoshangabad division	1.5902	0
Weibull model with for four parameters for	1.7006	0

DBH growth data of Teak from Hoshangabad division		
Monomolecular model with method F for height growth data of Teak from Warangal state	2.7296	0
Monomolecular model with method F for DBH growth data of Teak from Warangal state	2.7015	0
Monomolecular model with method E and F for top height growth of Babul tree in India	3.4724	0
Weibull model with for four parameters for maximum diameter growth of Babul tree in India	3.1667	0
Gompertz model with method F for top height growth data from Norway	2.0445	0
Logistic model with method F for DBH growth data from Norway	1.9493	0
Gompertz model with method F for cumulative basal area production data from Norway	1.9834	0

6.3.12 Test of Stationarity for residuals of the best fit results

Stationarity for residuals of the best fit results can be checked using autocorrelation, partial autocorrelation function and unit root test [35]. The autocorrelation, partial autocorrelation function of residual series of the best fit results are presented from Figure 6.18 to Figure 6.27. From Figure 6.18 to Figure 6.27, it can be observed that the spikes of autocorrelation, partial autocorrelation function are converse to zero very quickly imply the residual series are stationary.

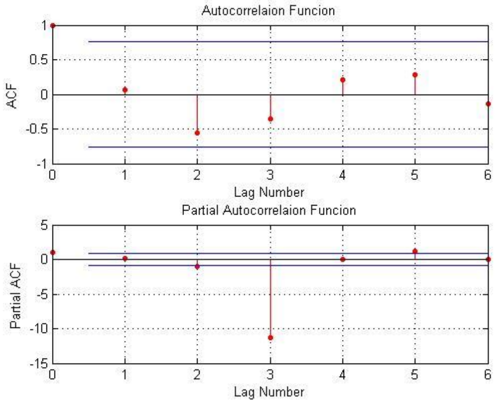


Figure 6.18: ACF and PACF of estimated residuals from Monomolecular model with method F for height growth data of Teak from Hoshangabad division

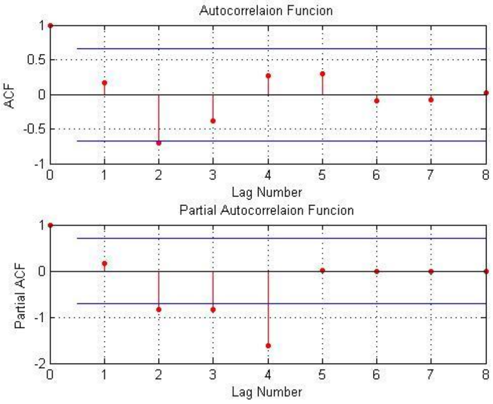


Figure 6.19: ACF and PACF of estimated residuals from Monomolecular model with method F for DBH growth data of Teak from Hoshangabad division

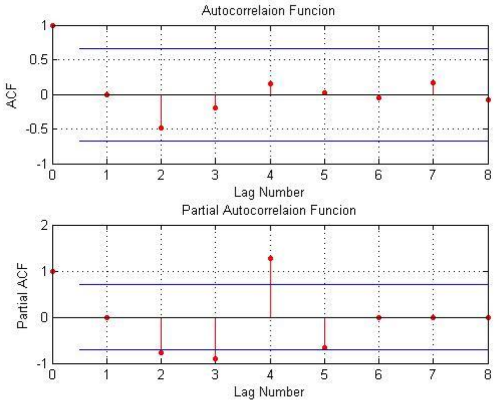


Figure 6.20: ACF and PACF of estimated residuals from Weibull model with for four parameters for DBH growth data of Teak from Hoshangabad division

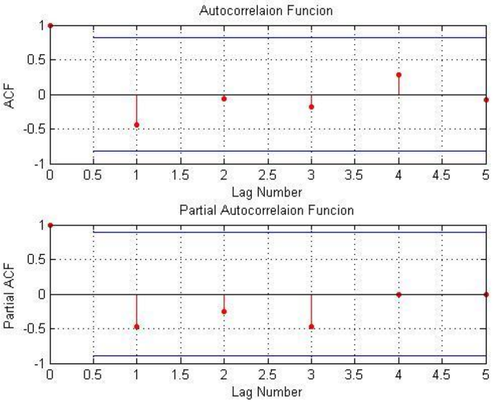


Figure 6.21: ACF and PACF of estimated residuals from Monomolecular model with method F for height growth data of Teak from Warangal state

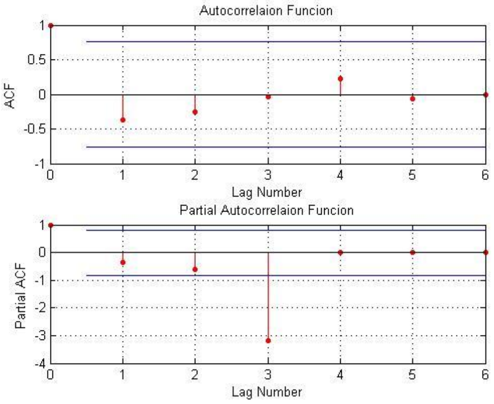


Figure 6.22: ACF and PACF of estimated residuals from Monomolecular model with method F for DBH growth data of Teak from Warangal state

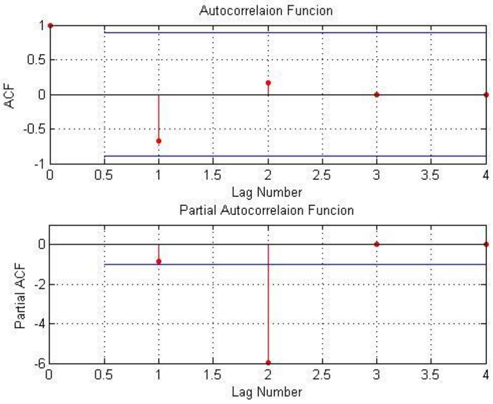


Figure 6.23: ACF and PACF of estimated residuals from Monomolecular model with method E and F for top height growth of Babul tree

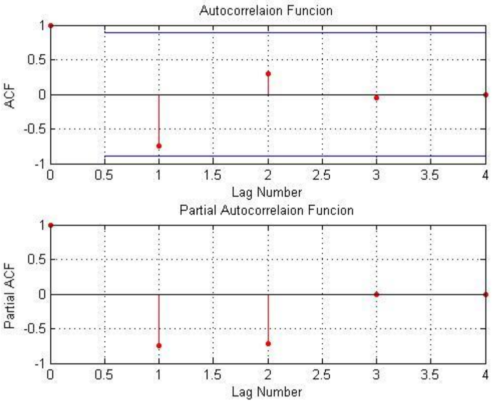


Figure 6.24: ACF and PACF of estimated residuals from Weibull model with four parameters for maximum diameter growth of Babul tree

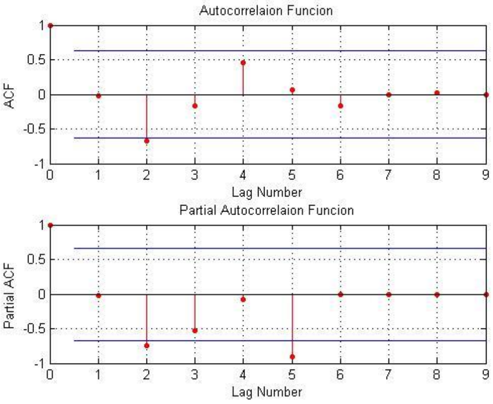


Figure 6.25: ACF and PACF of estimated residuals from Gompertz model with method F for top height growth data from Norway

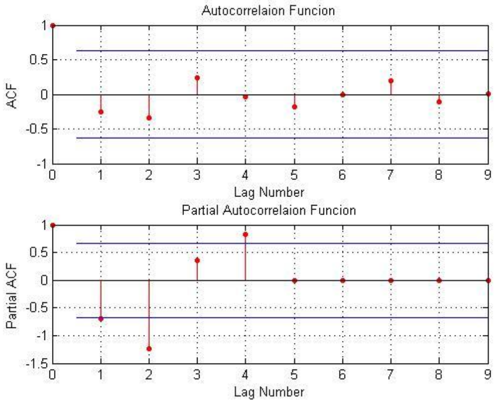


Figure 6.26: ACF and PACF of estimated residuals from Logistic model with method F for DBH growth data from Norway

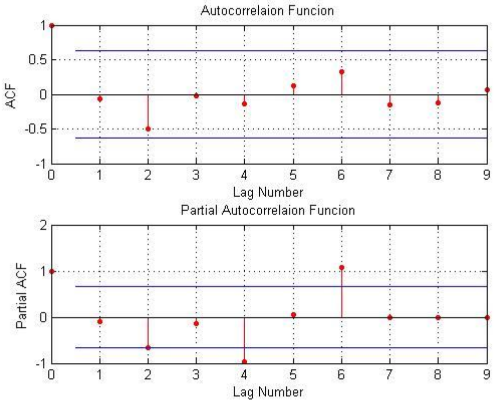


Figure 6.27: ACF and PACF of estimated residuals from Gompertz model with method F for cumulative basal area production data from Norway

Now the Augmented Dickey-Fuller (ADF) unit root test has been used to check the stationarity of the data sets, which are presented in Table 6.15. From the table, it is observed that, for all data sets, the $p-$values are less then 5%. It means that, the null hypothesis H_0, considering the process is a unit root, can be rejected. For top height and maximum diameter growth data of Babul tree, unit root test is not applicable due to less number of observations. Thus the residual series for selected results are stationary.

Table 6.15: $p-$ values of ADF test of residual series of the best fit results.

Data Name along with Method	$\boldsymbol{p-}$ **value**
Monomolecular model with method F for height growth data of Teak from Hoshangabad division	0.0000
Monomolecular model with method F for DBH growth data of Teak from Hoshangabad division	0.0000
Weibull model with for four parameters for DBH growth data of Teak from Hoshangabad division	0.0023
Monomolecular model with method F for height	0.0066

growth data of Teak from Warangal state	
Monomolecular model with method F for DBH growth data of Teak from Warangal state	0.0061
Gompertz model with method F for top height growth data from Norway	0.0009
Logistic model with method F for DBH growth data from Norway	0.0004
Gompertz model with method F for cumulative basal area production data from Norway	0.0271

6.3.13 Normality checking of the best fit results

The normal probability plot (P-P) and quantile- quantile (Q-Q) plots have been used to check the normality of the residuals. The P-P plots of estimated residuals of various best fit results have shown in Figure 6.28 - Figure 6.37. The P-P plots which make a little S-patterned curve rather than a straight line indicate that the residuals may be normal. The Q-Q plots of estimated residuals of various best fit results have shown in Figure 6.38-Figure 6.47. It is also observed that the Q-Q plots deviate badly from a straight line for all the results, indicating that the residual series of the estimated results may be normal.

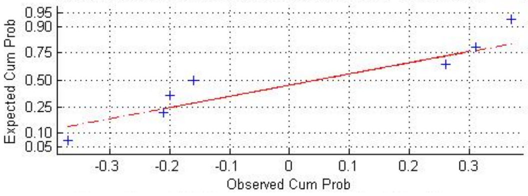


Figure 6.28: P-P plot of estimated results from Monomolecular model with method F for height growth data of Teak from Hoshangabad division

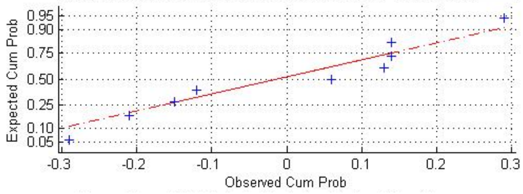
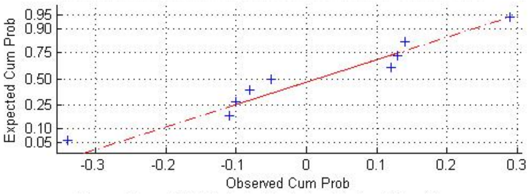


Figure 6.29: P-P plot of estimated results from Monomolecular model with method F for DBH growth data of Teak from Hoshangabad division



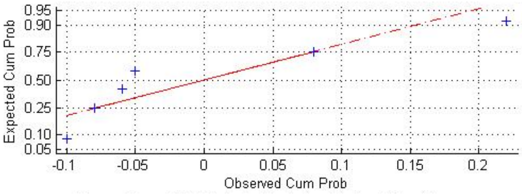
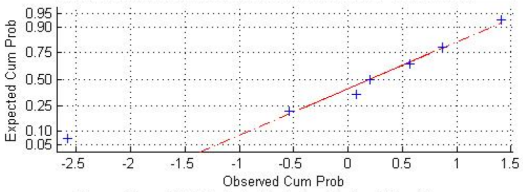


Figure 6.30: P-P plot of estimated results from Weibull model with for four parameters for DBH growth data of Teak from Hoshangabad division

Figure 6.31: P-P plot of estimated results from Monomolecular model with method F for height growth data of Teak from Warangal state



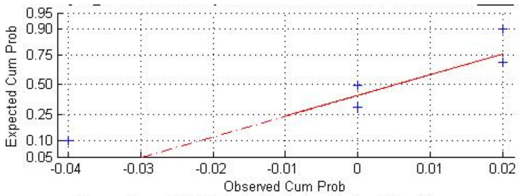
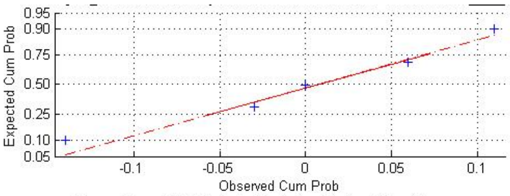


Figure 6.32: P-P plot of estimated results from Monomolecular model with method F for DBH growth data of Teak from Warangal state

Figure 6.33: P-P plot of estimated results from Monomolecular model with method E and F for top height growth of Babul tree



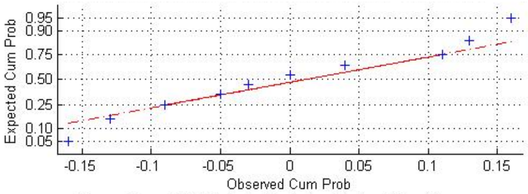
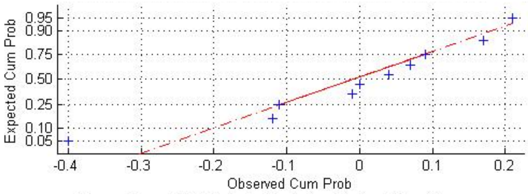


Figure 6.34: P-P plot of estimated results from Weibull model with four parameters for maximum diameter growth of Babul tree

Figure 6.35: P-P plot of estimated results from Gompertz model with method F for top height growth data from Norway



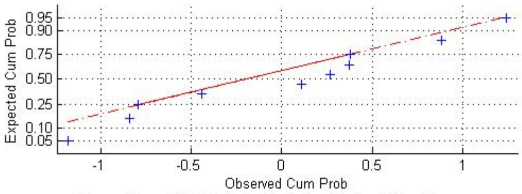


Figure 6.36: P-P plot of estimated results from Logistic model with method F for DBH growth data from Norway

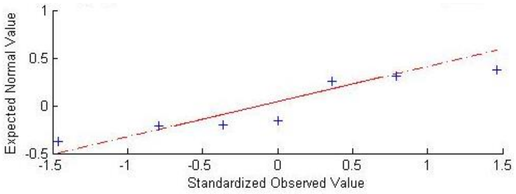


Figure 6.37: P-P plot of estimated results from Gompertz model with method F for cumulative basal area production data from Norway

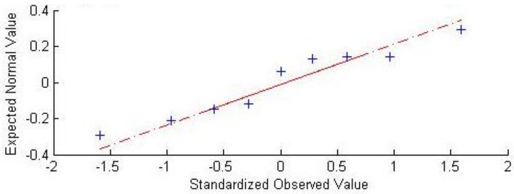


Figure 6.38: Q-Q plot of estimated results from Monomolecular model with method F for height growth data of Teak from Hoshangabad division

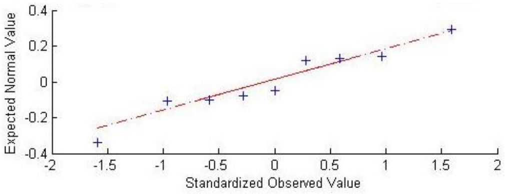


Figure 6.39: Q-Q plot of estimated results from Monomolecular model with method F for DBH growth data of Teak from Hoshangabad division

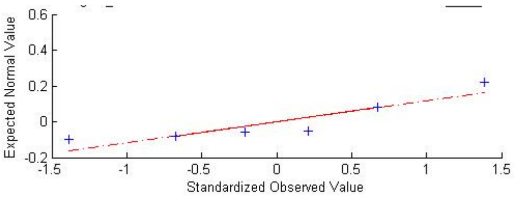


Figure 6.40: Q-Q plot of estimated results from Weibull model with for four parameters for DBH growth data of Teak from Hoshangabad division

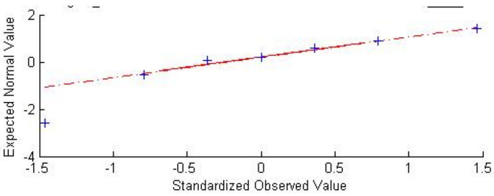


Figure 6.41: Q-Q plot of estimated results from Monomolecular model with method F for height growth data of Teak from Warangal state

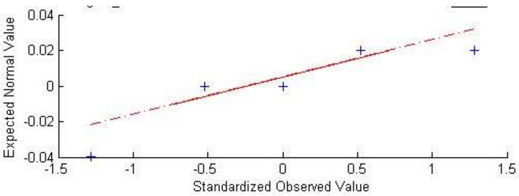


Figure 6.42: Q-Q plot of estimated results from Monomolecular model with method F for DBH growth data of Teak from Warangal state

Figure 6.43: Q-Q plot of estimated results from Monomolecular model with method E and F for top height growth of Babul tree

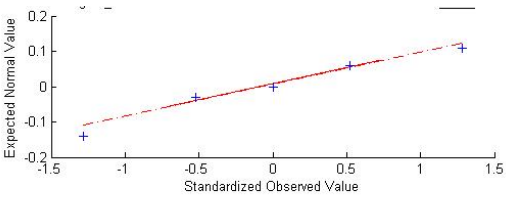


Figure 6.44: Q-Q plot of estimated results from Weibull model with four parameters for maximum diameter growth of Babul tree

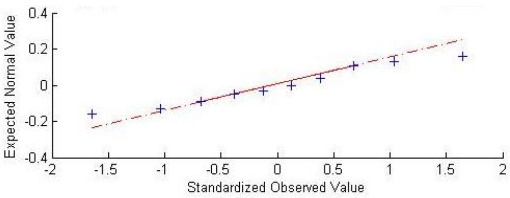


Figure 6.45: Q-Q plot of estimated results from Gompertz model with method F for top height growth data from Norway

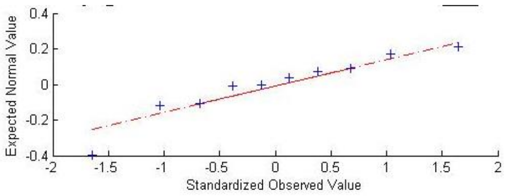


Figure 6.46: Q-Q plot of estimated results from Logistic model with method F for DBH growth data from Norway

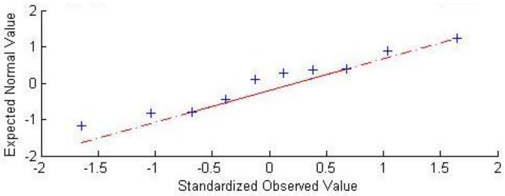


Figure 6.47: Q-Q plot of estimated results from Gompertz model with method F for cumulative basal area production data from Norway

The Jarque Bera (JB) test, a formal test for the residuals, has also been produced to check the normality of the estimated residuals of various best fit results. The values of the JB test statistics are given in Table 6.16. JB test states that if JB statistical value is less than the critical value, then the null hypothesis can't be rejected. In other word, the residuals are normally distributed. From the Table 6.16 it can be observed that, for all the best fit results, the JB statistical value is less than the critical value. Thus, it can be concluded that the results may be normal.

Table 6.16: JB test statistics for the estimated residuals of various best fit results

Data Name	JB Statistics	Critical Value
Height growth data of Teak from Hoshangabad division	0.8794	1.8603
DBH growth data of Teak from Hoshangabad division	0.6637	2.3352

(Monomolecular F)		
DBH growth data of Teak from Hoshangabad division (Weibull 4)	0.1885	2.3352
Height growth data of Teak from Warangal state	1.0983	1.5619
DBH growth data of Teak from Warangal state	1.5529	1.8603
Top height growth of Babul tree in India	0.7465	1.2185
Maximum diameter growth of Babul tree in India	0.3091	1.2185
Top height growth data from Norway experiment	0.6797	2.5239
DBH growth data from Norway experiment	1.9754	2.5239
Cumulative basal area production data from Norway experiment	0.5267	2.5239

6.4 Discussion

There might be more than one model that to be regarded as 'useful'. It means that the data are inadequate and ambivalent concerning some impact or parameterization or structure. It is reasonable that several models would serve almost similarly well in approximating a set of data. There is often considerable uncertainty in the choice of a specific model as the "best" approximating model [6].

In the study by Tewari et al. [81], calibrate three growth models to the height growth of *Acacia Nilotica* in Gujarat state in India. A similar study was also done by Abakar and Ahmed [1] for the growth of the species in Riverine Forests - Blue Nile. The data originated from the Bowmont Norway spruce thinning experiment, sample plot 3661 were used to study by Fekedulegn [21] and Fekedulegn et al. [22] by using nine different growth models. In these works, the parameters were estimated using different methods of estimation, which involve minimizing the objective function using nonlinear optimization technique. In Table 6.17, a comparative study is presented to shows the comparison between the results obtained by the newly introduced method of estimation and the results obtained by existing method of estimation for Bowmont Norway spruce thinning experiment data sets [21]. In his thesis by Fekedulegn [21], the Von Bertalanffy and Chapmen Richard growth model produced the better results with the nonlinear least square technique for the data of Norway experiment. Thus those RMSE of the Von Bertalanffy and Chapmen Richard growth model were used to compare with the best fit results obtained from this study. From Table 6.17, it is clearly observed that the newly introduced methods of estimation present in this thesis can compete with the existing method of estimations.

Table 6.17: Comparison between the results obtained by the newly introduced method of estimation and the results obtained from Fekedulegn [21] for Bowmont Norway spruce thinning experiment data sets

Data Name	**Results from this study**		**Results from Fekedulegn [21]**	
	RMSE	$\boldsymbol{R^2}$	**RMSE**	$\boldsymbol{R^2}$
Top Height	0.10	99%	0.12	99%
DBH	0.16	99%	0.23	99%
Cumulative Basal Area Production	0.75	99%	0.81	99%

According to the results from teak growth, Monomolecular growth model while estimated by Method F provides the better results for all data sets whereas four parameters Weibull growth model offer similar result for DBH growth of Hoshangabad division. By observing all the results of teak growth, it can be concluded that the Monomolecular growth model is more reasonable over the remaining growth models for describing the growth of Teak in India. One may consider any method of estimation (From method A to method F) to estimate the parameters of the Monomolecular growth model but the Method F is more preferable. Similarly, to describe the height growth pattern of Babul tree in India, the Monomolecular growth model (method E or method F) is preferable whereas Weibull growth model with four parameters can be used to describe the growth of maximum diameter of Babul tree. Again to describe the growth pattern of mean diameter at breast height data, originated from the Bowmont Norway spruce thinning experiment; the Logistic growth model with method F is more suitable. Whereas Gompertz growth model with method F can be used to describe the height growth and cumulative basal area production from the Bowmont Norway spruce thinning experiment. Especially, the method A for Gompertz, Logistic, Monomolecular, Von Bertalanffy and Chapmen Richard growth models will be very helpful when a few numbers of observations are available.

6.5 Conclusion

The Chapmen Richards's growth model along with its limiting cases and Weibull growth models are used by various forestry researchers and estimated its parameters using various methods of estimation, almost all of which involve minimizing the objective function using nonlinear optimization method. All the method requires a large amount of computations. In this chapter, an attempt has been made to find the best fit growth model along with the best method of estimation for the teak growth and Babul growth in India based on the available teak data. This study also investigates the same for the top height age, the cumulative basal area production and the mean diameter at breast height data, originated from the Bowmont Norway spruce thinning experiment. Specific selection criterions with six distinct steps are considered to compare the results. GARCH family models are fitted to the data sets to provide a comparative study. A comparative study have also been made between the best fit results from this study and the results obtained by existing method of estimation for Bowmont Norway spruce thinning experiment data sets.

The main objective of this thesis is to introduce some new method of estimations to estimate the parameters of few models. On account of that, total of 31 methods of estimation have been introduced to estimate the parameters for different growth models and check the validity of the methods using few forestry data. In contrast, modern statistical methods require some depth knowledge of mathematics. The newly introduced methods of estimation present in this thesis demand less computation and can compete with the existing method of estimations. For our methods of estimation it is not necessary that data should be equidistance. This is one of the advantages of our methods of estimation. This will provide some simple tools for researchers with limited experience in the application of more complex models. This study will help the researchers in the area of forestry and mathematical modelling.

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