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List of Publications

(A) Publications in Peer Reviewed Journals

- [1] Rapid Parameter Estimation of Three Parameter Nonlinear Growth Models, *International Journal of Mathematical Archive*, 4(2), 2013, 274-282.
- [2] Parameter Estimation of Weibull Growth Models in Forestry, *International Journal of Mathematical Trends and Technology*, 8(3), 2014, 157-163.
- [3] A Study on Kinetic Models for Analysing the Bacterial Growth Rate, *American International Journal of Research in Science, Technology, Engineering & Mathematics*, 8(1), 2014, 68-72.
- [4] A Comparative Study of Nonlinear Growth Models on Teak (*Tectona Grandis*) in India, *Journal of Plant Science and Research*, 4(1), 2017, 11-21.
- [5] A Comparative Study of Chapman Richards Growth Model and It's Limiting Cases for Growth of Babul (*Acacia Nilotica*) Trees in India, *Indian Journal of Plant Sciences*, 6(2), 2017, 37-51.
- [6] An Approach to Estimate the Parameters of Von Bertalanffy Growth Model, *Indian Forester*, 144(1), 2018, 41-53.
- [7] A Brief Study on Chapman Richards Growth Function, *Far East Journal of Applied Mathematics* (Accepted).

(B) Publications in Papers Presented in International/National Conferences

- [1] Fitting of Certain Growth Models and its Applications in Forestry Research, presented at the International Conference on Recent Advances in Mathematical Statistics and Its Applications in Applied Sciences held at Department of Statistics, Gauhati University during December 31, 2012 to January 2, 2013.
- [2] Forestry Implications of Certain Four Parameter Growth Models and Their Mathematical Properties. presented at National Seminar on Contemporary Issues of Mathematics and Statistics (NSCIMS), at Gargaon College, Sivasagar, Assam during 22nd and 23rd March 2013.
- [3] Nonlinear Mathematical Models in Forest Growth. presented at the National Conference on Pure And Applied Mathematics, 2013, at Royal Group of Institutions, Guwahati, Assam during 9th - 10th May 2013.
- [4] A Comparative Study on Bacterial Growth Rate Using Kinetic Models. Presented at recent trends in mathematics and its applications (RTMA-2013), at Digboi College, Digboi, Assam during 7th and 8th October 2013.

List of abbreviations and symbols

SAS	Statistical Analysis System
ADF	Augmented Dickey-Fuller
P-P	Probability Plot
Q-Q	Quantile- Quantile
JB	Jarque Bera
DW	Durbin Watson
LB	Ljung-Box
ACF	Autocorrelation Function
PACF	Partial Autocorrelation Functions
NLIN	Non-Linear
NLS	Nonlinear Least-Squares
DBH	Diameter at Breast Height
χ^2	Chi-Square
RMSE	Root Mean Square Error
R^2	Coefficient of determination
R_a^2	Adjusted coefficient of determination
$R_{prediction}^2$	Approximate R^2 for prediction