Chapter 3

On the Solution of DGLAP Evolution Equation

Along with a qualitative analysis of the available methods to solve DGLAP equation, in this chapter we have allude the usefulness of two Q^2 dependent Regge ansatz in solving DGLAP equation in order to have the small-x behaviour of both the spin independent and spin dependent non-singlet structure functions. By means of fitting analysis, we have investigated the compatibility of the two ansatz with the available experimental data and then studied the possible role played by them in evolving the non-singlet structure functions in accord with DGLAP equation.

3.1 Introduction

It is widely believed that QCD is the correct theory of strong interaction. In QCD, the structure functions are governed by a set of integro-differential equations, the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi(DGLAP) evolution equations[24]. The DGLAP equation is a renormalisation group equation for the quarks and gluon inside hadron. It is one of the fundamental equations of perturbative quantum chromodynamics (pQCD), being central to all theoretical predictions for lepton-hadron colliders. The DGLAP evolutions are given in terms of a perturbative expansion of splitting functions (P_{ij}) which describe the probability of a parent parton i producing a daughter parton j with momentum fraction z by the emission of a parton with momentum fraction $1-z$. For the flavor non-singlet ($q_{NS} = q_i - \bar{q}_j$), flavor-singlet($q_s = q_i + \bar{q}_i$) and gluon distributions(g), the DGLAP evolution equations read as follows:

$$\frac{dq^{NS}(x,Q^2)}{d\ln Q^2} = \frac{\alpha(Q^2)}{2\pi}\int_x^1 \frac{dy}{y} q^{NS}(y,Q^2) P_{qq}(\frac{x}{y}), \tag{3.1}$$

$$\frac{d}{d\ln Q^2}\begin{pmatrix} q^s(x,Q^2) \\ g(x,Q^2) \end{pmatrix} = \frac{\alpha(Q^2)}{2\pi}\int_x^1 \frac{dy}{y}\begin{pmatrix} P_{qq}(\frac{x}{y}) & P_{qg}(\frac{x}{y}) \\ P_{gq}(\frac{x}{y}) & P_{gg}(\frac{x}{y}) \end{pmatrix}\begin{pmatrix} q^s(y,Q^2) \\ g(y,Q^2) \end{pmatrix}. \tag{3.2}$$

Solutions of DGLAP equations give the Q^2 evolution of both the parton distribution functions as well as various structure functions. Although QCD predicts the Q^2 dependence of structure functions in accord with the DGLAP equations but they have limitations on absolute prediction of structure functions. DGLAP equations cannot predict the initial values from which the evolution starts, they can only predict the evolution of structure functions with Q^2, once an initial distribution is given. Further, due to its complicated mathematical structure, an exact analytic determination of the structure functions is currently out of reach and one needs to apply approximated methods to arrive on predictions from the DGLAP equation. Accordingly several approximate numerical as well as semi-analytical methods for the solution of DGLAP equation have been discussed considerably over the past years [108–113]. In literature there are essentially two main classes of approaches in order to have solutions of DGLAP equations: those that solve the equation directly in x-space and those that solve it for Mellin transformations of structure functions and invert the transformation back to x-space. The approaches based on Mellin transformation method have been achieved much interest because under Mellin transformation the integro-differential DGLAP equation turns into a continuum of independent matrix differential equations, one for each value of moments(N), which in turn makes the evolution more efficient numerically. However, in this regard as the Mellin transformation of both the splitting functions and the initial input is required, which may not be possible for all functions, especially if higher-order corrections are included in the equations, therefore it is not possible to have exact solution to DGLAP equation in moment space beyond leading order. In contrast to Mellin space, the x-space method is more flexible, since the inputs are only required in x-space; however it is generally considered to less efficient numerically, because of the need to carry out the convolution in DGLAP equations. Taking into account the advantage of being greater flexibility, despite the difficulty in obtaining high accuracy, the x-space methods have been serving as the basis of many widely used programs HOPPET[114], QCDNUM[108], CANDIA[113] etc., and being

incorporated by the CTEQ[98], MRST/MSTW(see [107] and references therein) collaborations. In addition, several numerical and semi-analytical methods have been developed[21, 100–103, 107, 109–113, 115, 138] and achieved significant phenomenological success.

3.2 Methods of Solution of DGLAP Evolution Equation

There exist numerous techniques to solve DGLAP equations. Among them most popular techniques are the Laguerre polynomial method, Mellin transformation method and Brute force method.

The Laguerre Polynomial method for numerical solution of the DGLAP evolution equation is based on the expansion of the structure functions and splitting functions in the basis of orthogonal Laguerre Polynmials[116, 117].

In this method, initially an evolution function $E_{NS}(x,t)$ is defined which describes the evolution of the structure functions from $t=0$ to t as

$$F_{NS}(x,t) = \int_x^1 \frac{d\omega}{\omega} E_{NS}(\frac{x}{\omega},t) F_{NS}(\omega, t=0), \tag{3.3}$$

which satisfies

$$\frac{\partial}{\partial t} E_{NS}(x,t) = \int_x^1 \frac{d\omega}{\omega} P_{NS}(\frac{x}{\omega},t) E_{NS}(\omega, t=0). \tag{3.4}$$

This integro-differential equation has the similar form as the original DGLAP equation. The advantage of introducing an evolution function is that it should be the delta function at $t=0$: $E_{NS}(x, t=0) = \delta(1-x)$ because of its definition in Eq.(3.3).

Here the functions are expanded in terms of the polynomials: $P_{NS}(e^{-\prime x}) = \sum_n P^n_{NS} L_n(\prime x)$ and $E_{NS}(e^{-\prime x}, t) = \sum_n E^n_{NS}(t) L_n(x')$, where P^n_{NS} and $E^n_{NS}(t)$ are the expansion coefficients. The coefficient F^n for a function $F(x)$ is given by $F^n = \int_0^1 L_n(\prime x) F(x)$, and it could be calculated analytically for a simple function. If the two functions on the right-hand side of Eq.(3.4) are expanded, it becomes an integration of two Laguerre polynomials. Using the formula $\int_0^{x'} d\omega' L_n(x'-\omega') L_m(\omega') = L_{n+m}(x') - L_{n+m+1}(x')$ for this integration, we obtain

$$\frac{d}{dt} E_{NS}(t) = \sum_{m=0}^{n} (P^{n-m}_{NS} - P^{n-m-1}_{NS}) E_m(t). \tag{3.5}$$

At $t = 0$ all the expansion coefficients are one, as the evolution function is a delta function. Therefore, the solution of this equation gives a summation of the form:

$$E^m_{NS}(t) = e^{P^0_{NS}t} \sum_{k=0}^{m} \frac{t^k}{k!} B^k_m, B^{K+1}_m = \sum_{i=k}^{m-1} (P^{m-i}_{NS} - P^{m-i-1}_{NS}) B^k_i. \tag{3.6}$$

This recursion relation is calculated with the relations $B^0_i = 1$, $B^1_i = \sum_{j=1}^{i}(P^j_{NS} - P^{j-1}_{NS})$ and $B^k_0 = B^k_1 = ... = B^k_{k-1} = 0$. After all, the evolution is calculated by simple summation:

$$F_{NS}(x,t) = \sum_{n=0} N_{Lag} \sum_{m=0}^{n} \left[E_{n-m}(t) - E_{n-m-1}(t)\right] L_n(-\ln x) F^m_{NS}(t=0). \tag{3.7}$$

In this way, the integro-differential equation turns into a simple summation of Laguerre expansion coefficients, so that this method is regarded as a significant and very efficient numerical method for the numerical solution of the equation. However the accuracy of this technique is limited and it is quite accurate up to x-values not smaller than $x \approx 10^3$. on the other hand for small x the convergence of the expansion decreases. Therefore his method results no longer practical for the solution of DGLAP equation within smaller-x region.

The Mellin transformation method is one of the popular evolution methods[118]. The reason behind popularity of the method is its ability to resolve the right hand side of Eq.(3.1) into a simple product of to moments, namely the moments of distribution function and the moments of splitting functions. In order to have solution in this way, the moments of both the splitting function and distribution function are required. Usually, the moments are well known and assuming a simple model for the distribution functions at certain small Q^2 such that its moments can be calculated easily, the analytical solution of the equation can be obtained in moments space. Furthermore, the computation time is fairly short. These are the reasons why this method has been used as a popular method. For example, it is used for the χ^2 analysis of experimental data for obtaining polarized PDFs[119], whereas the brute-force method is employed in Ref. [120].

The Mellin transformation and inversion are defined by

$$\hat{F}_{NS}(s,t) = \int_0^1 dx x^{s-1} F(x,t), F(x,t) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} ds x^{-s} \hat{F}(s,t). \tag{3.8}$$

The Mellin inversion is a complex integral which consists of an arbitrary real constant c, which should be chosen such that absolutely convergency is achieved in the integral $\int_0^1 dxF(x)x^{c-1}$. Under this transformation, the integro-differential equations become very simple. For example, the nonsinglet evolution equation becomes

$$\frac{\partial}{\partial t}\hat{F}_{NS}(s,t) = \hat{P}_{NS}(s)\hat{F}_{NS}(s,t). \tag{3.9}$$

Its solution is simply given by

$$\hat{F}_{NS}(s,t) = e^{P_{NS}(s)t}\hat{F}_{NS}(s,t=0). \tag{3.10}$$

The moments of the distribution function $\hat{F}_{NS}(s,t=0)$, which is initially considered at a certain $Q^2 = Q_0^2$, can be evaluated and using the well known moments of the splitting functions $\hat{P}_{NS}(s)$, the solution of Eq.(7.14) can easily be obtained in the moment space. However, in order to have the distribution in x space, an inverse Mellin transformation is required. In this regard, one important point to be noted is that, the numerical Mellin inversion is relatively CPU time consumig, which happen even if the analytical expressions of the moments of the initial conditions are well known[121]. Moreover, as discussed in [122], since x variable is associated with the invariant energy W^2 of the virtual photon-hadron scattering process by $W^2 = (1x)/x, x \to 0$ is the infinite energy limit and thus can never experimentally be reached. As a consequence of this all moments are plagued by an a priori infinite uncertainty, which can be reduced by means of assumptions implying that any use of the evolution equations for moments is model dependent.

The Brute-force method[122,123] is the simplest method in order to have numerical solution of integro-differential equation. For more complicated equations consisting of higher twist terms[124], which could not be easily handled by other methods, such as Mellin transformation as well as Laguerre-polynomial methods, the Brute-force method is suitable, although it being seemed to be too simple. Furthermore, a computer code is so simple that the possibility of a program mistake is small, which means the code could be used for checking other numerical methods.

In the brute-force method, the two variables t and x are divided into small steps, and then the differentiation and integration are defined by

$$\frac{\partial F_{NS}(x,t)}{\partial t} \Rightarrow \frac{F(x_i,t_{j+1}) - F(x_i,t_j)}{\Delta t_j}, \int dxF(x,t) \Rightarrow \sum_{k=1}^{N_x} \Delta x_k F(x_k,t_j), \tag{3.11}$$

where Δt_j and Δx_k are the steps at the positions j and k, and they are given by $\Delta t_j = t_{j+1} - t_j$ and $\Delta x_k = x_k - x_{k-1}$. The numbers of t and x steps are denoted N_t and N_x, respectively. Applying these equations to Eq.(3.1), we write the non-singlet evolution from t_j to t_{j+1} as

$$F_{NS}(x_i, t_{j+1}) = F_{NS}(x_i, t_j) + \Delta t_j \sum_{k=1}^{N_x} \frac{\Delta x_k}{x_k} P_{qq}(x_i/x_k) F_{NS}(x_k, t_j). \tag{3.12}$$

If the initial distribution F_{NS} is considered at $t_1 = 0$, the next one $F_{NS}(x, t_2)$ can be determined by the above equation. Proceeding in this way, step by step, upto $N_t 1$ times, the final distribution at t_{N_t} can be obtained. However, accuracy of the results demands a large number of steps N_t and N_x.

In addition to these three, some other numerical as well as semi analytical methods to solve DGLAP evolution equations are available in literature, such as Matrix approach method, Taylor expansion method, Regge theory method etc. and predict the evolution of various structure functions with considerable phenomenological success.

3.3 A Regge Inspired Approach to Solve the DGLAP Equation

Due to the unavailability of exact analytical way of solving the DGLAP equations, in current analysis this set of equations are solved numerically by using an initial input distribution for the structure function at a fixed Q^2, in terms of some free parameters, the parameters are so adjusted that the parametrization best fit the existing data. However, the consideration of a specific parametrization with large number of parameters is potentially a source of bias, i.e. systematic error which is very difficult to control. Furthermore, when a parametrization is fitted to the data, it is very hard to obtain a determination not only of the best fitting parameters, but also of their errors. Therefore, explorations of the possibility of obtaining accurate solutions of DGLAP evolution equations without an initial input or with initial input, consisting of less number of parameters are always interesting. Under this motivation, this thesis is devoted to the exploration of a semi-analytic approach of solving DGLAP equation for non-singlet structure functions using two Regge inspired model with less number of parameters. Here particular emphasis is given to the non-singlet structure functions because they are considered as the starting ground for theoretical description

of DIS structure functions. Besides being interesting in themselves, another significant advantage is that QCD analysis by means of non-singlet structure functions is comparatively technically simpler.

In order to perform a fit, one must start with a particular ansatz for the structure functions at some reference Q_0^2. In most of the existing fitting analysis, including those in the experimental papers it has been performed by assuming a simple power behavior based on Regge theory. Regge theory predicts the x dependence of the structure functions at fixed Q^2 and at small x. In Regge theory the x dependency of the non-singlet structure functions F_i^{NS}, $i = 2, 3$ (i.e., $F_{i=2}^{NS} = F_2^{NS}$ and $F_{i=3}^{NS} = xF_3$) are described with a power law, $F_i^{NS}(x) = B_i^{NS} x^{\lambda_i^{NS}}$, for fixed Q^2[13,59,127,128]. Besides being x dependency, the structure functions, in accordance with QCD predictions, are dependent on Q^2 also. The Bjorken Scalling violation or the Q^2 dependence of the structure functions is one of the significant predictions of Quantum Chromodynamics and recent experiments also reveal the evidence of Q^2 dependency of the structure functions even at small-x. Therefore in order to have Q^2 behavior of non-singlet structure functions we have to modify the Regge predictions by incorporating Q^2 dependency either to the exponents (λ_i^{NS}) only or to the coefficients (B_i^{NS}) or both. Here, we have preferred to investigate the possibility of first two cases i.e., firstly, the coefficients (B_i^{NS}) are Q^2 dependent with constant exponents and next, the exponents (λ_i^{NS}) are Q^2 dependent with constant coefficients.

3.3.1 Regge Ansatz with Q^2 Dependent Coefficient and Constant Intercept

There are many phenomenological models, developed within the Regge approach for Deep Inelastic Scattering and structure functions. The simple Regge pole exchange model predicts that, towards smaller values of x the non-singlet unpolarized structure functions $F_2^{NS}(x, Q^2)$, $xF_3(x, Q^2)$ and $xg_1(x, Q^2)$ behave as

$$\frac{1}{x} F_2^{NS}(x) = B_2^{NS} x^{-\lambda_2^{NS}}, \tag{3.13}$$

$$xF_3(x) = B_3^{NS} x.x^{-\lambda_3^{NS}} \tag{3.14}$$

and

$$xg_1^{NS}(x) = B_g^{NS} x.x^{-\lambda_3^{NS}} \tag{3.15}$$

respectively, with the exponents $\lambda_i^{NS} = \alpha_{A_2}(0)$. According to Regge theory, $F_2^{NS}(x)$, $xF_3(x)$ and xg_1^{NS} are governed by the A_2 Regge trajectory with the intercept $\alpha_{A_2}(0)$. For $\alpha_{A_2}(0) \approx 0.5$, the behaviors (3.13) and (3.15) are stable and considerable phenomenological success is observed in this regards. As a consequence, the small x behavior of the unpolarized non-singlet structure functions can be expressed as

$$F_2^{NS}(x) = B_2^{NS} x^{0.5}, \tag{3.16}$$

$$xF_3(x) = B_3^{NS} x^{0.5} \tag{3.17}$$

and

$$xg_1^{NS}(x) = B_g^{NS} x.x^{0.5}. \tag{3.18}$$

Now, in accordance with QCD, we should expect all the dependence on Q^2 to be in $B^{NS}_{(i=2,3,g)}$, so that the Regge predictions, (7.13), (7.13) and (3.18) for x dependence do not change. Therefore, incorporating the Q^2 behavior of the structure functions in terms of the functions $B^{NS}_{(i=2,3,g)}(Q^2)$, we have the QCD modified Regge like model for both x as well as Q^2 dependent non-singlet structure functions at small x as

$$F_2^{NS}(x, Q^2) = B_2^{NS}(Q^2) x^{0.5}, \tag{3.19}$$

$$xF_3(x, Q^2) = B_3^{NS}(Q^2) x^{0.5} \tag{3.20}$$

and

$$xg_1^{NS}(x) = B_g^{NS} x.x^{0.5}. \tag{3.21}$$

The non-singlet structure functions in this form does not contain any fitting parameter. We just need to evaluate the Q^2 dependent function $B_i(Q^2)$ and it can be obtained by means of solving the respective DGLAP evolution equations using these ansatz as the initial input, which is discussed briefly in the section 3.5 and in detailed in the chapters 4, 5, and 6.

3.3.2 Regge Ansatz with Q^2 Dependent Intercept and Constant Coefficient

Instead of being constant, there are several predictions on the Q^2 dependency of the Regge intercept. There were predictions [129, 130] that the exponent would be larger at high values of Q^2 and these types of predictions were born out from two different equations of perturbative QCD: the DGLAP equation and BFKL equation. Although this Regge model seems to legitimate as far the early data are concerned, which were mostly taken at moderate Q^2 ($\approx 10GeV^2$) and x values of around $x \geq 0.01$ but the recent measurement of F_i^{NS} for available small-x in the interval $0.0001 < x < 0.01$ can be described with a single Regge type exchange $F_i^{NS} = Ax^{\alpha}$, in which the intercept has a smooth Q^2 dependence and varies like x^{α} with $-0.5 \leq \alpha \leq 0$. In the case of g_1^{NS} similar behaviour was predicted with valon model[131] and a variation from -0.13 to -0.3 was obtained within the interval of Q^2 from $2GeV^2$ to $10GeV^2$. On the other hand, Ref.[132] predicts a behaviour of the type, $g_1^{NS} \simeq \left(\frac{Q^2}{x^2}\right)^{\Delta_{NS}/2}$, with $\Delta_{NS} = 0.42$ in which the asymptotic scaling of g_1^{NS} depends on only one variable $\frac{Q^2}{x^2}$. In addition there are several studies on Q^2 dependency of the intercepts of the non-singlet structure functions[133].

In this section we have investigated the possibility of a simple Regge ansatz of the type $F_i^{NS} = Ax^{-bt}$ with Q^2 dependent intercept in order to describe the small-x behaviour of the structure functions. The underlying idea behind the assumption of this type of model is as follows: HERA measurements[134, 135] suggest that the behavior of F_2 structure function at low-x is consistent with a dependence $F_2(x, Q^2) = Cx^{-\lambda(Q^2)}$, where the coefficient A is independent of Q^2 and the exponent, defined by $\lambda(Q^2) = a \ln\left(\frac{Q^2}{\Lambda^2}\right) = at$, is observed to rise linearly with $\ln Q^2$. Here Λ is the QCD cut off parameter and $t = \ln\left(\frac{Q^2}{\Lambda^2}\right)$. Thus we see that the rise of the un-polarized structure function ($F_2(x, Q^2)$) is much steeper than that predicted by Regge theory and gets steeper and steeper as Q^2 increases. Since this observation it has been the challenging issue to resolve whether the Regge intercepts for $F_2(x, Q^2)$ structure function as well as it's non-singlet, singlet and gluon parts, along with the spin structure functions are Q^2 dependent or not. Further, before the observation at HERA, there are several predictions on the Q^2 dependency of the Regge intercept[129, 130]. These predictions as well as experimental observations at HERA motivated us to consider the possibility that the Regge behaved non-singlet part, $\frac{1}{x}F_2^{NS}$ of $F_2(x, Q^2)$ structure function is also

satisfy a functional behaviour, $F_2^{NS}(x,Q^2) = Ax^{-b\ln(\frac{Q^2}{\Lambda^2})} = Ax^{-bt}$ similar to $F_2(x,Q^2)$. Again as the non-singlet structure functions, $\frac{1}{x}F_2^{NS}$ and $F_3(x,t)$ and $g_1^{NS}(x,t)$ are Regge behaved[131, 136], therefore their x dependency will be similar within smaller-x region. Further, in QCD the Q^2 behaviour of these structure functions are governed by the same DGLAP equation. Therefore the x and Q^2 dependency for all the non-singlet structure functions are similar and in accord with F_2, and hence F_2^{NS} here we assume that the Q^2 dependency of the Regge behaved structure function F_i^{NS} is dominated only by the intercept and it satisfies a relation of the type

$$F_i^{NS}(x,t) = A_i x^{1-b_i t}, \tag{3.22}$$

where A_i and b_i are arbitrary constants, which are to be determined by fitting expressions with respective available experimental results. Here for simplicity, F_i^{NS} is defined to represent all of $F_2^{NS}(x,t)$, $xF_3^{NS}(x,t)$ and $xg_1^{NS}(x,t)$ structure functions.

3.4 Fitting Analysis of Our Models

The Regge like ansatz for F_i^{NS} structure functions in the form of Eqs.(3.19-3.21) does not consists of any parameters to be fitted. It will be seen in the following section as well as next three chapters that the unknown Q^2 dependent coefficient can be obtained by means of solving the DGLAP equation with the ansatz as the initial input. However the Regge like ansatz for F_i^{NS} structure function in the form of Eq. (3.22) consists of two parameters. This parametrization can be expressed in a different form in terms of only one parameter b by eliminating the parameter A, as A has no effect on the structure function in our approach, which is done as follows: The value of the F_i^{NS} structure function at any point (x_0,t_0) in the (x,t) coordinate system is given by

$$F_i^{NS}(x_0,t_0) = A_i x_0^{(1-b_i t_0)}. \tag{3.23}$$

Dividing (3.22) by (3.23) and rearranging a bit we get

$$F_i^{NS}(x,t) = F_i^{NS}(x_0,t_0) x^{(1-b_i t)} x_0^{-(1-b_i t_0)}. \tag{3.24}$$

This reduced form of the structure function consists of only one fitting parameter, the parameter b_i and a known input point $F_i^{NS}(x_0,t_0)$, which can be taken from the available experimental data. If the input point is more accurate and precise, we can

	Input point	Value of b	$\frac{\chi^2}{d.o.f}$	Kinematical region
F_2^{NS}	0.010348 ± 0.006208	0.118 ± 0.028	0.85	$x < 0.05$ and $Q^2 \leq 20$
xF_3	0.3298 ± 0.02605	0.0744 ± 0.0136	1.98	$x < 0.05$ and $Q^2 \leq 20$
xg_1^{NS}	$0.0133075 \pm 0.0.001938$	0.0759 ± 0.0107	1.41	$x < 0.05$ and $Q^2 \leq 20$

Table 3.1: Summary of best fitting results for different structure functions.

expect batter fitting. There are not any specific reason in choosing the input point. Any one of the data points at a certain value of $x = x_0$ and $t = t_0$ can be considered as the input point. Off course, the sensitivity of different inputs will be different. However instead of choosing the input point on the basis of their sensitivity, in our manuscript we have incorporated a suitable condition in determining the input point. We have considered that particular point from the most recent measurements as the input point in which experimental errors are minimum. Under this condition we have selected the points, given in the Table:3.1, for different structure functions as the initial input point and then fitted the expressions with all the available experimental data. We have observed that the above parametrization fit best for the values of b_i which are collected in Table. 1 along with the corresponding $\frac{\chi^2}{d.o.f.}$. In this analysis, we have considered the QCD cut-off parameter λ to be fixed and the considered values are 0.323 GeV^2, 0.337 GeV^2 and 0.300 GeV^2 for F_2^{NS}, xF_3 and xg_1^{NS} respectively. Best fitted results are depicted in Figures 3.1, 3.2 and 3.3 for F_2^{NS}, xF_3 and xg_1^{NS} respectively along with the available experimental data. In addition, we have shown the band due to the uncertainty assoiated with input and the fitting parameter b. The figures reflect a very good consistency between the ansatz and the experimental data.

As far the Figures are concerned, we see that the Regge ansatz Eq.(3.24) (with Q^2 dependent intercept) for the non-singlet structure functions are compatible with their respective experimental data within kinematical region of our consideration.

3.5 Solution of DGLAP Equation for F_i^{NS} with the Regge Ansatz

We now investigate how the two analytic ansatz help in solving the DGLAP evolution equations in order to have the Q^2 behavior of non-singlet structure functions. For simplicity, as an example, here we would like to discuss only the solution of LO DGLAP equation for F_2^{NS} structure function. When the two ansatz are introduced to the LO DGLAP evolution equation (3.1)

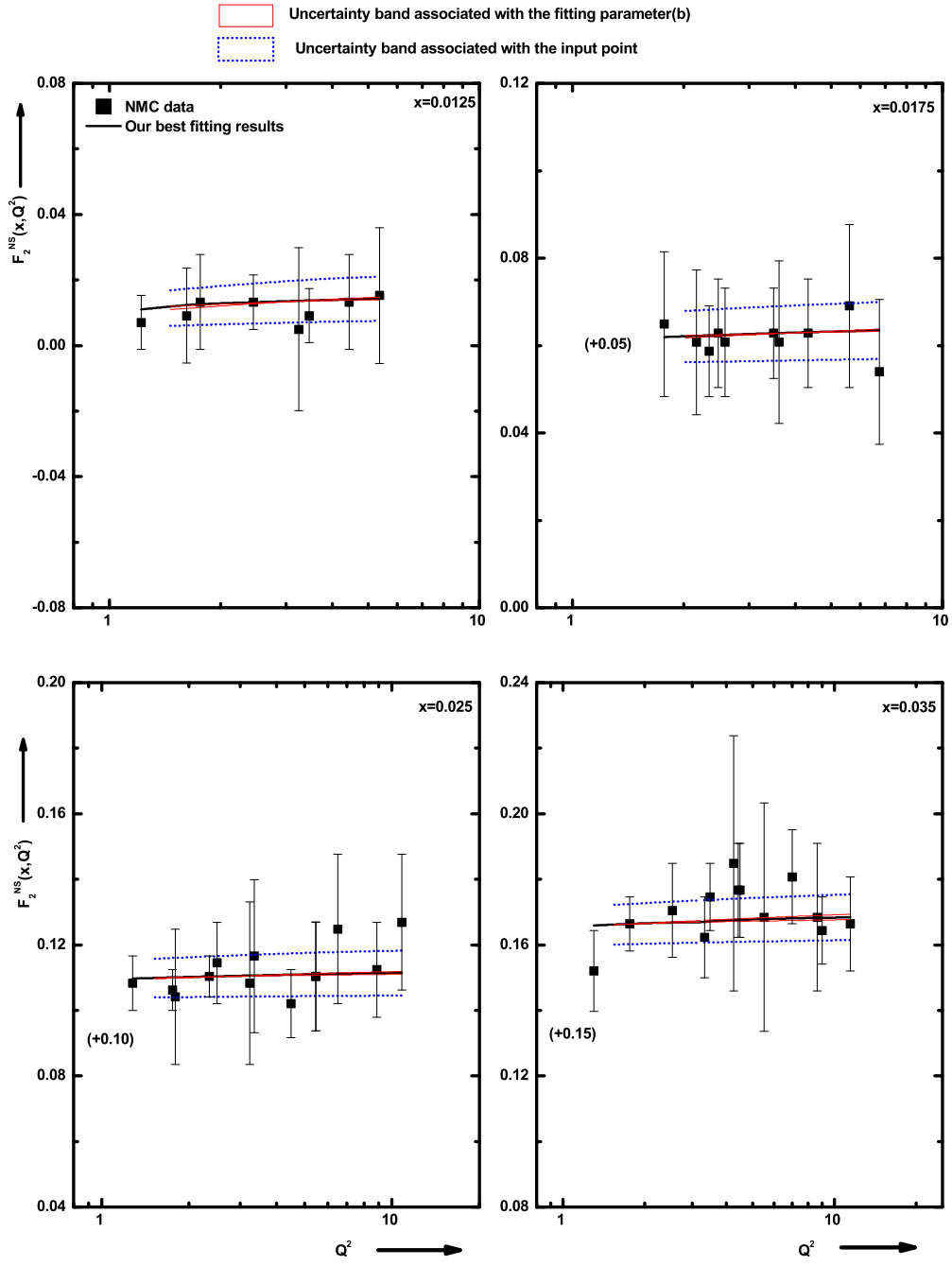

Figure 3.1: Our best fit results of Eq.(3.24) for $F_2^{NS}(x,Q^2)$ structure functions to NMC[63] results. For clarity, the points are offset by the amount given in parenthesis.(Q^2's are taken in the unit of GeV^2).

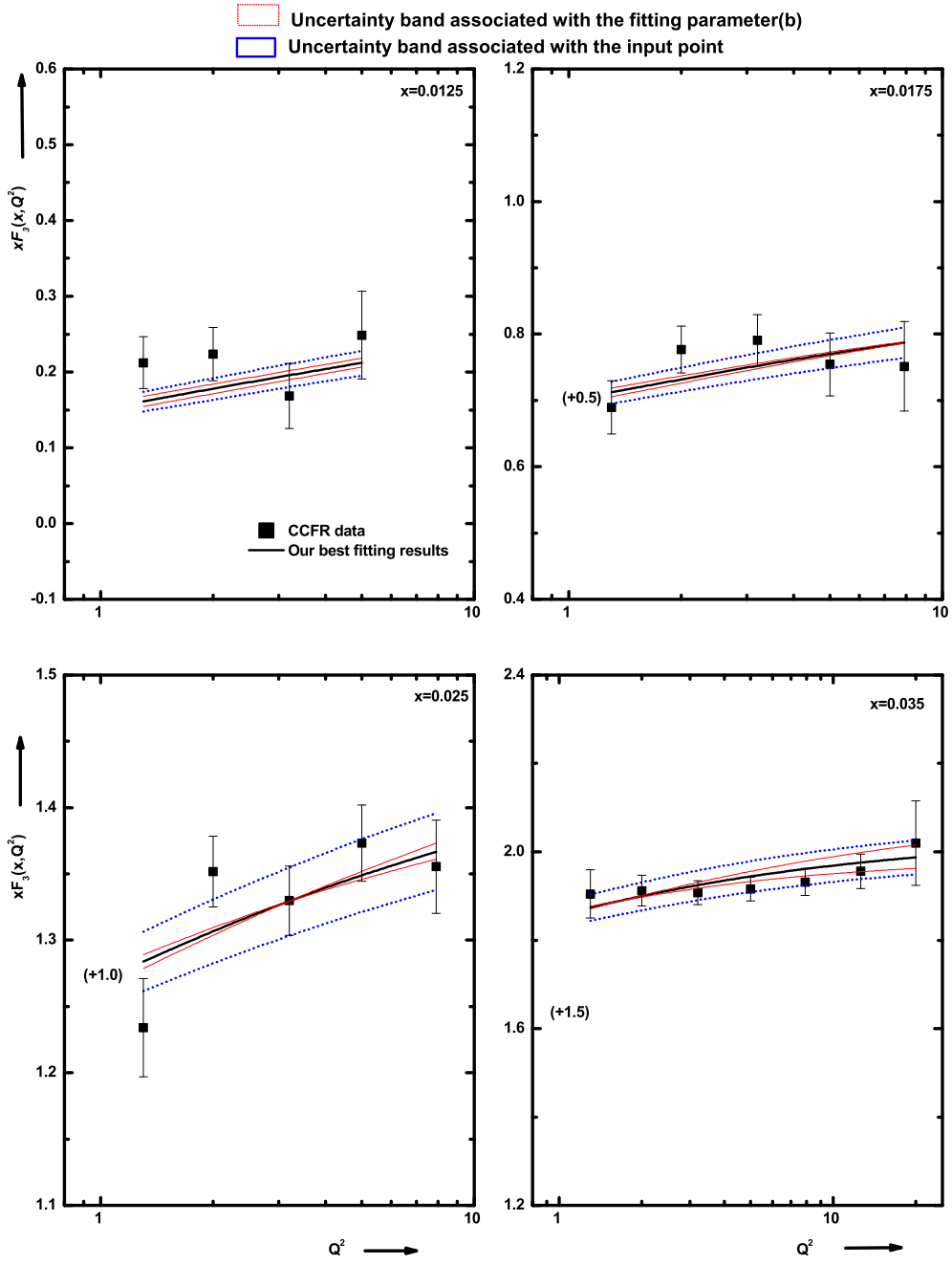

Figure 3.2: Our best fit results of Eq.(3.24) for $xF_3(x,Q^2)$ structure functions to CCFR[66] results. For clarity, the points are offset by the amount given in parenthesis. (Q^2's are taken in the unit of GeV^2).

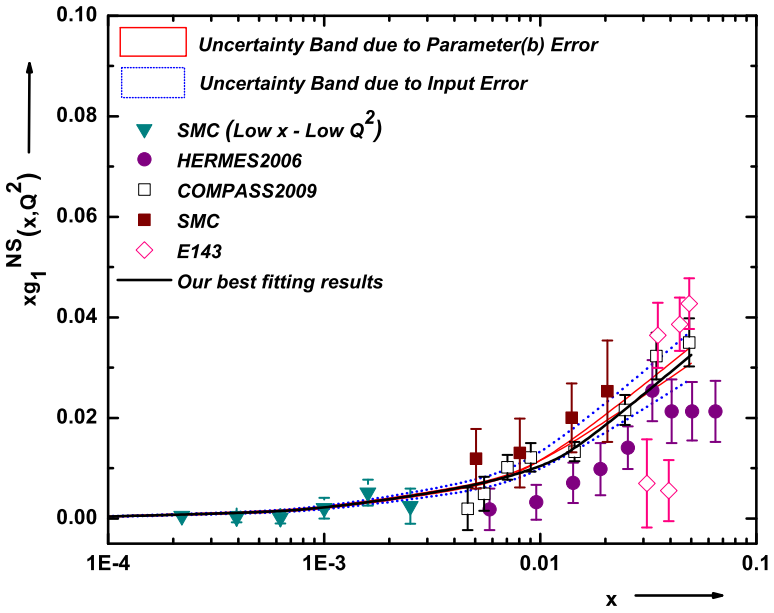

Figure 3.3: Our best fit results of Eq.(3.24) for $xg_1^{NS}(x,Q^2)$ structure functions to the experimental data taken from SMC[74], HERMES[73], COMPASS[71] and E143[75]. Here the results are plotted against x.

$$\frac{\partial F_2^{NS}(x,t)}{\partial t} = \left(\frac{\alpha(t)}{2\pi}\right)_{LO}\left[\frac{2}{3}\{3+4ln(1-x)\}F_2^{NS}(x,t) + I_1(x,t)\right], \tag{3.25}$$

and rearrange a bit, we obtain

$$\frac{\partial F_2^{NS}(x,t)}{\partial t} = \left(\frac{\alpha(t)}{2\pi}\right)_{LO}\left[\frac{2}{3}\{3+4ln(1-x)\} + \frac{4}{3}\int_x^1 \frac{d\omega}{1-\omega} \left\{\frac{1+\omega^2}{\omega}\omega^{(-0.5)} - 2\right\}\right] F_2^{NS}(x,t), \tag{3.26}$$

$$\frac{\partial F_2^{NS}(x,t)}{\partial t} = \left(\frac{\alpha(t)}{2\pi}\right)_{LO}\left[\frac{2}{3}\{3+4ln(1-x)\} + \frac{4}{3}\int_x^1 \frac{d\omega}{1-\omega} \left\{\frac{1+\omega^2}{\omega}\omega^{(bt-1)} - 2\right\}\right] F_2^{NS}(x,t), \tag{3.27}$$

which have the form ordinary differential equations

$$\frac{\partial F_2^{NS}(x,t)}{\partial t} = \frac{\alpha(t)}{2\pi}U(x)F_i^{NS}(x,t), \tag{3.28}$$

$$\frac{\partial F_2^{NS}(x,t)}{\partial t} = \frac{\alpha(t)}{2\pi} U(x,t) F_i^{NS}(x,t) \tag{3.29}$$

respectively. These two equations can be easily solved to have

$$F_2^{NS}(x,t)\Big|_{LO} = C_1 \exp\left[U(x) \int \left(\frac{\alpha(t)}{2\pi}\right)_{LO} dt\right] \tag{3.30}$$

and

$$F_2^{NS}(x,t)\Big|_{LO} = C_1 \exp\left[\int \left(\frac{\alpha(t)}{2\pi}\right)_{LO} U(x,t) dt\right], \tag{3.31}$$

respectively.

Now at a fixed value of $x = x_0$, the t dependence of the structure function $F_2^{NS}(x,t)$ in accord with (4.20) is given by

$$F_2^{NS}(x_0,t)\Big|_{LO} = C_1 \exp\left[U(x_0) \int \left(\frac{\alpha(t)}{2\pi}\right)_{LO} dt\right]. \tag{3.32}$$

Again the value of the structure function at $x = x_0$ and $t = t_0$ in accord with (4.26) is

$$F_2^{NS}(x_0,t_0)\Big|_{LO} = C_1 \exp\left[U(x_0) \int \left(\frac{\alpha(t)}{2\pi}\right)_{LO} dt\right]\Bigg|_{t=t_0}. \tag{3.33}$$

Dividing (4.26) by (4.27) and rearranging a bit we obtain the t evolution of $F_2^{NS}(x,t)$ in accord with the LO DGLAP equation with respect to the point $F_2^{NS}(x_0,t_0)$ as

$$F_2^{NS}(x_0,t)\Big|_{LO} = F_2^{NS}(x_0,t_0) \exp\left[U(x_0) \int_{t_0}^{t} \left(\frac{\alpha(t)}{2\pi}\right)_{LO} dt\right]. \tag{3.34}$$

Again in accord with our preassumptions (3.19), (3.20) and (3.21), the t dependence of $F_2^{NS}(x,t)$ at a particular value of $x = x_0$ is given by

$$F_2^{NS}(x_0,t) = B(Q^2) x_0^{0.5}. \tag{3.35}$$

Dividing any of (3.19), (3.20) and (3.21) by (4.29), we have the following relation

$$F_2^{NS}(x,t) = F_2^{NS}(x_0,t) \left(\frac{x}{x_0}\right)^{0.5}, \tag{3.36}$$

which describes both t and x dependence of $F_2^{NS}(x,t)$ structure function in terms of the t dependent function $F_2^{NS}(x_0,t)$.

Now combining (4.28) and (4.30) we obtain the relation,

$$F_2^{NS}(x,t)\Big|_{LO} = F_2^{NS}(x_0,t_0)\exp\left[U(x_0)\int_{t_0}^{t}\left(\frac{\alpha(t)}{2\pi}\right)_{LO}dt\right]\left(\frac{x}{x_0}\right)^{0.5}, \tag{3.37}$$

which describes both t and x dependence of $F_2^{NS}(x,t)$ structure function in LO in terms of the input point $F_2^{NS}(x_0,t_0)$.

Proceeding in a similar way, from Eq. (4.37) we can have both t and x dependence of $F_2^{NS}(x,t)$ structure function as

$$F_2^{NS}(x,t)\Big|_{LO} = F_2^{NS}(x_0,t_0)\exp\left[\int_{t_0}^{t}\left(\frac{\alpha(t)}{2\pi}\right)_{LO}U(x_0,t)dt\right]\left(\frac{x}{x_0}\right)^{(1-bt)}. \tag{3.38}$$

As far the equations (4.31) and (4.49) are concerned, they are the analytic expressions representing both x and Q^2 dependence of $F_2^{NS}(x,Q^2)$ structure function jointly, which are obtained by means of solving the DGLAP equations in LO incorporating the Regge ansatz, $F_2^{NS}(x,Q^2) = A(Q^2)x^{0.5}$ and $F_2^{NS}(x,Q^2) = Bx^{1-bt}$ as the initial inputs respectively. These expressions are consisting of an input point $F_2^{NS}(x_0,t_0)$, which can be taken from the available experimental data. Moreover, the Eq. (4.31) does not contain any fitting parameter, however the Eq. (4.49) consists of only one fitting parameter b. Using a suitable input point, $F_2^{NS}(x_0,t_0)$ from experimental data, we can obtain both x and Q^2-evolution of $F_2^{NS}(x,Q^2)$ structure function with the best fitted value of b. The calculation as well as phenomenological studies of un-polarised and polarised structure functions, F_2^{NS}, xF_3 and xg_1^{NS} with pQCD corrections upto NNLO is discussed in detailed in the chapter 4, chapter 5 and chapter 6 respectively. □□