Chapter 4

Small-x Behaviour of $F_2^{NS}(x,Q^2)$ Structure Function

In this chapter I present the full calculation of non-singlet structure function $F_2^{NS}(x,Q^2)$ by means of solving DGLAP equation with QCD corrections up to next-next-to-leading order. Using the two ansatz, discussed in the previous chapter, developed by combining the features of perturbative Quantum Chromodynamics and Regge theory, as the initial input we have solved the DGLAP equations. The solutions, along with the ansatz allow us to obtain some analytic expressions which represent the joint Bjorken x and Q^2 dependence of $F_2^{NS}(x,Q^2)$ structure function. The expressions are studied phenomenologically in comparison with experimental results taken from New Muon Collaboration (NMC) and the results of NNPDF parameterizations. A great phenomenological success is achieved in this regards, which signifies the capability of the expressions in describing the small-x behaviour of the non-singlet structure function and their usefulness in determining the structure functions with a reasonable precision.

4.1 Introduction

The structure function $F_2^{NS}(x,Q^2)$ is the non-singlet part of $F_2(x,Q^2)$ structure function originated in the unpolarized charged lepton DIS and it is given by the difference of proton and neutron structure functions as $F_2^{NS} = F_2^p - F_2^n$[59]. The non-singlet structure function $F_2^{NS}(x,Q^2)$ provides a very good mean to investigate QCD as a theory of strong interaction. Besides being interesting in themselves, the non-singlet structure functions are not marred by the presence of the sea quark and gluon densities about which we have very poor information in particular in the small-x region and hence theoretical analysis by means of them are comparatively technically simpler. Therefore they are regarded as a starting ground for a theoretical description of DIS structure functions.

The Gottfried sum rule[34, 35], associated with $F_2^{NS}(x, Q^2)$ is also an important observable of QCD. The determination of the Gottfried sum rule requires knowledge of $F_2^{NS}(x, Q^2)$ structure functions over the entire region of $x \in (0; 1)$. However, the experimentally accessible x range for DIS is limited for the available data and therefore one should extrapolate results to $x = 0$ and $x = 1$. The extrapolation to $x \to 0$, where F_2^{NS} structure functions grow strongly, is much more important than the extrapolation to $x \to 1$, where structure functions vanish. Again, it is known that maximum contribution (about 90%) to the Gottfried sum rule come from the small $x(\leq 0.1)$ region. Because of the large contribution to the Gottfried sum rule from small x, the small x region is particularly important. Therefore this chapter is an attempt to have the small-x behaviour of $F_2^{NS}(x, Q^2)$ structure function by means of solving the DGLAP equation using the two Regge ansatz discussed in chapter 3 as the initial input.

The Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equation[24] which describe the Q^2 behavior of unpolarised non-singlet structure function $F_2^{NS}(x, Q^2)$ in perturbative Quantum Chromodynamics (QCD) formalism is given by

$$\frac{\partial F_2^{NS}(x, Q^2)}{\partial \ln Q^2} = \int_x^1 \frac{d\omega}{\omega} F_2^{NS}(\frac{x}{\omega}, Q^2) P(\omega). \tag{4.1}$$

Where, $P(\omega)$ is the splitting function associated with $F_2^{NS}(x, Q^2)$ structure function, which is defined up to NNLO by[31]

$$P(\omega) = \frac{\alpha(Q^2)}{2\pi} P^{(0)}(\omega) + \left(\frac{\alpha(Q^2)}{2\pi}\right)^2 P^{(1)}(\omega) + \left(\frac{\alpha(Q^2)}{2\pi}\right)^3 P^{(2)}(\omega). \tag{4.2}$$

Here, $P^{(0)}(\omega)$, $P^{(1)}(\omega)$ and $P^{(2)}(\omega)$ are the corresponding leading order(LO), next-to-leading order (NLO) and next-next-to-leading order(NNLO) corrections to the splitting functions. Splitting functions are given in Appendices.

Again, in LO, NLO and NNLO, the running coupling constant $\frac{\alpha(Q^2)}{2\pi}$ has the forms[23],

$$\left(\frac{\alpha(t)}{2\pi}\right)_{LO} = \frac{2}{\beta_0 t}, \tag{4.3}$$

$$\left(\frac{\alpha(t)}{2\pi}\right)_{NLO} = \frac{2}{\beta_0 t}\left[1 - \frac{\beta_1 \ln t}{\beta_0^2 t}\right], \tag{4.4}$$

and

$$\left(\frac{\alpha(t)}{2\pi}\right)_{NNLO} = \frac{2}{\beta_0 t}\left[1 - \frac{\beta_1 \ln t}{\beta_0^2 t} + \frac{1}{\beta_0^2 t^2}\left[\left(\frac{\beta_1}{\beta_0}\right)^2 (\ln^2 t - \ln t + 1) + \frac{\beta_2}{\beta_0}\right]\right], \tag{4.5}$$

where $\beta_0 = 11 - \frac{2}{3}N_F$, $\beta_1 = 102 - \frac{38}{3}N_F$ and $\beta_2 = \frac{2857}{6} - \frac{6673}{18}N_F + \frac{325}{54}N_F^2$ are the one-loop, two-loop and three-loop corrections to the QCD β-function. Here the running coupling constant is expressed in terms of the variable t, which is defined by $t = \ln(\frac{Q^2}{\Lambda^2})$.

Substituting the respective splitting functions along with the corresponding running coupling constant in (4.1), the DGLAP evolution equations in LO, NLO and NNLO become

$$\frac{\partial F_2^{NS}(x,t)}{\partial t} = \left(\frac{\alpha(t)}{2\pi}\right)_{LO}\left[\frac{2}{3}\{3 + 4ln(1-x)\}F_2^{NS}(x,t) + I_1(x,t)\right], \tag{4.6}$$

$$\frac{\partial F_2^{NS}(x,t)}{\partial t} = \left(\frac{\alpha(t)}{2\pi}\right)_{NLO}\left[\frac{2}{3}\{3 + 4ln(1-x)\}F_2^{NS}(x,t) + I_1(x,t)\right] + \left(\frac{\alpha(t)}{2\pi}\right)^2_{NLO} I_2(x,t), \tag{4.7}$$

and

$$\frac{\partial F_2^{NS}(x,t)}{\partial t} = \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}\left[\frac{2}{3}\{3 + 4ln(1-x)\}F_2^{NS}(x,t) + I_1(x,t)\right] + \left(\frac{\alpha(t)}{2\pi}\right)^2_{NNLO} I_2(x,t) + \left(\frac{\alpha(t)}{2\pi}\right)^3_{NNLO} I_3(x,t) \tag{4.8}$$

respectively. Here Λ is the QCD cut-off parameter and the integral functions are given by

$$I_1(x,t)=\int_x^1\frac{d\omega}{1-\omega}\left\{\frac{1+\omega^2}{\omega}F_2^{NS}\left(\frac{x}{\omega},t\right)-2F_2^{NS}(x,t)\right\}, \tag{4.9}$$

$$I_2(x,t)=\int_x^1\frac{d\omega}{\omega}P^{(1)}(\omega)F_2^{NS}\left(\frac{x}{\omega},t\right) \tag{4.10}$$

and

$$I_3(x,t)=\int_x^1\frac{d\omega}{\omega}P^{(2)}(\omega)F_2^{NS}\left(\frac{x}{\omega},t\right). \tag{4.11}$$

The DGLAP equations up to NNLO ((4.6)-(4.8)) can be solved analytically using the ansatz $F_2^{NS}(x,t)=A(t)x^{0.5}$ and $F_2^{NS}(x,t)=Bx^{(1-at)}$ as the initial inputs and I have discussed bellow in detailed.

4.2 Solution of DGLAP Evolution Equations with the Initial Input $F_2^{NS}(x,t)=A(t)x^{0.5}$

On substitution of

$$F_2^{NS}(x,t)=F_2^{NS}(x,t)=A(t)x^{0.5} \tag{4.12}$$

and hence

$$F_2^{NS}(\frac{x}{\omega},t)=F_2^{NS}(\frac{x}{\omega},t)=A(t)x^{0.5}\omega^{-0.5}=F_2^{NS}(x,t)\omega^{-0.5} \tag{4.13}$$

in the equations (4.6), (4.7) and (4.8), we obatin

$$\frac{\partial F_2^{NS}(x,t)}{\partial t}=\left(\frac{\alpha(t)}{2\pi}\right)_{LO}\left[\frac{2}{3}\{3+4ln(1-x)\}+\frac{4}{3}\int_x^1\frac{d\omega}{1-\omega}\left\{\frac{1+\omega^2}{\omega}\omega^{-0.5}-2\right\}\right]F_2^{NS}(x,t), \tag{4.14}$$

$$\frac{\partial F_2^{NS}(x,t)}{\partial t}=\left(\frac{\alpha(t)}{2\pi}\right)_{NLO}\left[\frac{2}{3}\{3+4ln(1-x)\}+\frac{4}{3}\int_x^1\frac{d\omega}{1-\omega}\left\{\frac{1+\omega^2}{\omega}\omega^{-0.5}-2\right\}\right]F_2^{NS}(x,t)+\left(\frac{\alpha(t)}{2\pi}\right)^2_{NLO}\int_x^1\frac{d\omega}{\omega}P^{(1)}(\omega)\omega^{-0.5}F_2^{NS}(x,t) \tag{4.15}$$

and

$$\frac{\partial F_2^{NS}(x,t)}{\partial t} = \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}\left[\frac{2}{3}\{3+4ln(1-x)\}+\frac{4}{3}\int_x^1\frac{d\omega}{1-\omega}\left\{\frac{1+\omega^2}{\omega}\omega^{-0.5)}\right.\right.$$
$$\left.\left.-2\right\}\right]F_2^{NS}(x,t)+\left(\frac{\alpha(t)}{2\pi}\right)^2_{NNLO}\int_x^1\frac{d\omega}{\omega}P^{(1)}(\omega)\omega^{-0.5)}F_2^{NS}(x,t)$$
$$+\left(\frac{\alpha(t)}{2\pi}\right)^3_{NNLO}\int_x^1\frac{d\omega}{\omega}P^{(2)}(\omega)\omega^{-0.5}F_2^{NS}(x,t) \quad (4.16)$$

respectively. These equations can be rearranged to have three ordinary differential equations in terms of $F_2^{NS}(x,t)$,

$$\frac{\partial F_2^{NS}(x,t)}{\partial t} = \frac{\alpha(t)}{2\pi}U(x)F_2^{NS}(x,t), \quad (4.17)$$

$$\frac{\partial F_2^{NS}(x,t)}{\partial t} = \left[\left(\frac{\alpha(t)}{2\pi}\right)_{NLO}U(x)+\left(\frac{\alpha(t)}{2\pi}\right)^2_{NLO}V(x)\right]F_2^{NS}(x,t), \quad (4.18)$$

and

$$\frac{\partial F_2^{NS}(x,t)}{\partial t} = \left[\left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}U(x)+\left(\frac{\alpha(t)}{2\pi}\right)^2_{NNLO}V(x)\right.$$
$$\left.+\left(\frac{\alpha(t)}{2\pi}\right)^3_{NNLO}W(x)\right]F_2^{NS}(x,t) \quad (4.19)$$

which can be easily solved to have

$$F_2^{NS}(x,t)\Big|_{LO} = C_1\exp\left[U(x)\int\left(\frac{\alpha(t)}{2\pi}\right)_{LO}dt\right], \quad (4.20)$$

$$F_2^{NS}(x,t)\Big|_{NLO} = C_2\exp\left[U(x)\int\left(\frac{\alpha(t)}{2\pi}\right)_{NLO}dt+V(x)\int\left(\frac{\alpha(t)}{2\pi}\right)^2_{NLO}dt\right], \quad (4.21)$$

and

$$F_2^{NS}(x,t)\Big|_{NNLO} = C_3\exp\left[U(x)\int\left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}dt+V(x)\int\left(\frac{\alpha(t)}{2\pi}\right)^2_{NNLO}dt\right.$$
$$\left.+W(x)\int\left(\frac{\alpha(t)}{2\pi}\right)^3_{NNLO}dt\right]. \quad (4.22)$$

respectively. Here,

$$U(x) = \frac{2}{3}\{3 + 4ln(1-x)\} + \frac{4}{3}\int_x^1 \frac{d\omega}{1-\omega}\left\{\frac{1+\omega^2}{\omega}\omega^{-0.5} - 2\right\}, \tag{4.23}$$

$$V(x) = \int_x^1 \frac{d\omega}{\omega} P^{(1)}(\omega)\omega^{-0.5}, \tag{4.24}$$

$$W(x) = \int_x^1 \frac{d\omega}{\omega} P^{(2)}(\omega)\omega^{-0.5}, \tag{4.25}$$

and C_1, C_2, C_3 are the constants originated due to integration .

Now at a fixed value of $x = x_0$, the t dependence of the structure function $F_2^{NS}(x,t)$ in LO is given by

$$F_2^{NS}(x_0,t)\Bigg|_{LO} = C_1 \exp\left[U(x_0)\int\left(\frac{\alpha(t)}{2\pi}\right)_{LO} dt\right]. \tag{4.26}$$

Again the value of the structure function at $x = x_0$ and $t = t_0$ in accord with (4.26) is

$$F_2^{NS}(x_0,t_0)\Bigg|_{LO} = C_1 \exp\left[U(x_0)\int\left(\frac{\alpha(t)}{2\pi}\right)_{LO} dt\right]\Bigg|_{t=t_0}. \tag{4.27}$$

Dividing (4.26) by (4.27) and rearranging a bit we obtain the t evolution of $F_2^{NS}(x,t)$ in accord with the LO DGLAP equation with respect to the point $F_2^{NS}(x_0,t_0)$ as

$$F_2^{NS}(x_0,t)\Bigg|_{LO} = F_2^{NS}(x_0,t_0)\exp\left[U(x_0)\int_{t_0}^{t}\left(\frac{\alpha(t)}{2\pi}\right)_{LO} dt\right]. \tag{4.28}$$

Again in accord with our preassumption (4.12), the t dependence of $F_2^{NS}(x,t)$ at a particular value of $x = x_0$ is given by

$$F_2^{NS}(x_0,t) = A(t)x_0^{0.5}. \tag{4.29}$$

Dividing (4.12) by (4.29), we have the following relation

$$F_2^{NS}(x,t) = F_2^{NS}(x_0,t)\left(\frac{x}{x_0}\right)^{0.5}, \tag{4.30}$$

which describes both t and x dependence of $F_2^{NS}(x,t)$ structure function in terms of the t dependent function $F_2^{NS}(x_0,t)$.

Now combining (4.28) and (4.30) we obtain the relation,

$$F_2^{NS}(x,t)\Big|_{LO} = F_2^{NS}(x_0,t_0)\exp\left[U(x_0)\int_{t_0}^{t}\left(\frac{\alpha(t)}{2\pi}\right)_{LO}dt\right]\left(\frac{x}{x_0}\right)^{0.5}, \tag{4.31}$$

which describes both t and x dependence of $F_2^{NS}(x,t)$ structure function in LO in terms of the input point $F_2^{NS}(x_0,t_0)$.

Proceeding in the similar way we can obtain the expressions representing both x and t dependence of $F_2^{NS}(x,t)$ structure function in terms of an input point $F_2^{NS}(x_0,t_0)$ in NLO and NNLO as

$$\begin{aligned} F_2^{NS}(x,t)\Big|_{NLO} &= F_2^{NS}(x_0,t_0)\exp\left[U(x_0)\int_{t_0}^{t}\left(\frac{\alpha(t)}{2\pi}\right)_{NLO}dt\right. \\ &\left. +V(x_0)\int_{t_0}^{t}\left(\frac{\alpha(t)}{2\pi}\right)^2_{NLO}dt\right]\left(\frac{x}{x_0}\right)^{0.5} \end{aligned} \tag{4.32}$$

and

$$\begin{aligned} F_2^{NS}(x,t)\Big|_{NNLO} &= F_2^{NS}(x_0,t_0)\exp\left[U(x_0)\int_{t_0}^{t}\left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}dt\right. \\ &+V(x_0)\int_{t_0}^{t}\left(\frac{\alpha(t)}{2\pi}\right)^2_{NNLO}dt \\ &\left. +W(x_0)\int_{t_0}^{t}\left(\frac{\alpha(t)}{2\pi}\right)^3_{NNLO}dt\right]\left(\frac{x}{x_0}\right)^{0.5} \end{aligned} \tag{4.33}$$

respectively.

4.3 Solution of DGLAP Evolution Equations with the Initial Input $F_2^{NS}(x,t) = Bx^{(1-bt)}$

Now considering the ansatz, $F_2^{NS}(x,t) = Bx^{(1-bt)}$ as the initial input we obtain the DGLAP equations in LO, NLO and NNLO as

$$\begin{aligned} \frac{\partial F_2^{NS}(x,t)}{\partial t} = \left(\frac{\alpha(t)}{2\pi}\right)_{LO}&\left[\frac{2}{3}\{3+4ln(1-x)\}+\frac{4}{3}\int_x^1\frac{d\omega}{1-\omega}\left\{\frac{1+\omega^2}{\omega}\omega^{(bt-1)}\right.\right. \\ &\left.\left. -2\right\}\right]F_2^{NS}(x,t), \end{aligned} \tag{4.34}$$

$$\frac{\partial F_2^{NS}(x,t)}{\partial t} = \left(\frac{\alpha(t)}{2\pi}\right)_{NLO}\left[\frac{2}{3}\{3+4ln(1-x)\}+\frac{4}{3}\int_x^1\frac{d\omega}{1-\omega}\left\{\frac{1+\omega^2}{\omega}\omega^{(bt-1)} -2\right\}\right]F_2^{NS}(x,t)+\left(\frac{\alpha(t)}{2\pi}\right)^2_{NLO}\int_x^1\frac{d\omega}{\omega}P^{(1)}(\omega)\omega^{(bt-1)}F_2^{NS}(x,t) \quad (4.35)$$

and

$$\frac{\partial F_2^{NS}(x,t)}{\partial t} = \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}\left[\frac{2}{3}\{3+4ln(1-x)\}+\frac{4}{3}\int_x^1\frac{d\omega}{1-\omega}\left\{\frac{1+\omega^2}{\omega}\omega^{(bt-1)} -2\right\}\right]F_2^{NS}(x,t)+\left(\frac{\alpha(t)}{2\pi}\right)^2_{NNLO}\int_x^1\frac{d\omega}{\omega}P^{(1)}(\omega)\omega^{(bt-1)}F_2^{NS}(x,t) + \left(\frac{\alpha(t)}{2\pi}\right)^3_{NNLO}\int_x^1\frac{d\omega}{\omega}P^{(2)}(\omega)\omega^{(bt-1)}F_2^{NS}(x,t) \quad (4.36)$$

respectively, which can be easily solved to have

$$F_2^{NS}(x,t)\Big|_{LO} = C_1\exp\left[\int\left(\frac{\alpha(t)}{2\pi}\right)_{LO}U(x,t)dt\right], \quad (4.37)$$

$$F_2^{NS}(x,t)\Big|_{NLO} = C_2\exp\left[\int\left(\frac{\alpha(t)}{2\pi}\right)_{NLO}U(x,t)dt+\int\left(\frac{\alpha(t)}{2\pi}\right)^2_{NLO}V(x,t)dt\right] \quad (4.38)$$

and

$$F_2^{NS}(x,t)\Big|_{NNLO} = C_3\exp\left[\int\left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}U(x,t)dt+\int\left(\frac{\alpha(t)}{2\pi}\right)^2_{NNLO}V(x,t)dt + \int\left(\frac{\alpha(t)}{2\pi}\right)^3_{NNLO}W(x,t)dt\right] \quad (4.39)$$

respectively. Here

$$U(x,t) = \frac{2}{3}\{3+4ln(1-x)\}+\frac{4}{3}\int_x^1\frac{d\omega}{1-\omega}\left\{\frac{1+\omega^2}{\omega}\omega^{(bt-1)}-2\right\}, \quad (4.40)$$

$$V(x,t) = \int_x^1\frac{d\omega}{\omega}P^{(1)}(\omega)\omega^{(bt-1)}, \quad (4.41)$$

$$W(x,t) = \int_x^1 \frac{d\omega}{\omega} P^{(2)}(\omega)\omega^{(bt-1)} \tag{4.42}$$

and C_1, C_2 and C_3 are the constants originated due to integration.
At a fixed value of $x = x_0$, the t dependence of the structure function in LO is given by

$$F_2^{NS}(x_0,t) = C_1 \exp\left[\int \left(\frac{\alpha(t)}{2\pi}\right)_{LO} U(x_0,t)dt\right]. \tag{4.43}$$

Again the value of the structure function at $x = x_0$ and $t = t_0$ in accord with (4.43) is given by

$$F_2^{NS}(x_0,t_0) = C_1 \exp\left[\int \frac{\alpha(t)}{2\pi} U(x_0,t)dt\right]\Bigg|_{t=t_0}. \tag{4.44}$$

Dividing (4.43) by (4.44) and rearranging a bit we obtain the t dependence of $F_2^{NS}(x,t)$ in accord with LO DGLAP evolution equation with respect to the point $F_2^{NS}(x_0,t_0)$ as

$$F_2^{NS}(x_0,t) = F_2^{NS}(x_0,t_0) \exp\left[\int_{t_0}^{t} \left(\frac{\alpha(t)}{2\pi}\right)_{LO} U(x_0,t)dt\right]. \tag{4.45}$$

Again, as both t and x dependence of $F_2^{NS}(x,t)$ is assumed to satisfy

$$F_2^{NS}(x,t) = B.x^{(1-bt)} \tag{4.46}$$

relation, and at any fixed $x = x_0$ we have

$$F_2^{NS}(x_0,t) = B.x_0^{(1-bt)}, \tag{4.47}$$

which represents the t dependence of the structure function at any fixed value of $x = x_0$. Dividing (4.46) by (4.47) we have the following relation

$$F_2^{NS}(x,t) = F_2^{NS}(x_0,t)\left(\frac{x}{x_0}\right)^{(1-bt)}, \tag{4.48}$$

which gives both t and x dependence of $F_2^{NS}(x,t)$ structure function in terms of the t dependent function $F_2^{NS}(x_0,t)$ at fixed $x = x_0$.

Now combining (4.45) and (4.48) we obtain the expression representing both x and t dependence of $F_2^{NS}(x,t)$ structure function in terms of an input point $F_2^{NS}(x_0,t_0)$ in LO as

$$F_2^{NS}(x,t)\Bigg|_{LO} = F_2^{NS}(x_0,t_0)\exp\left[\int_{t_0}^{t}\left(\frac{\alpha(t)}{2\pi}\right)_{LO} U(x_0,t)dt\right]\left(\frac{x}{x_0}\right)^{(1-bt)}. \tag{4.49}$$

Similarly we may have the joint x and t dependence of $F_2^{NS}(x,t)$ structure function in NLO and NNLO as

$$\begin{aligned}F_2^{NS}(x,t)\Bigg|_{NLO} = F_2^{NS}(x_0,t_0)\exp\Bigg[&\int_{t_0}^{t}\left(\frac{\alpha(t)}{2\pi}\right)_{NLO} U(x_0,t)dt\\ &+\int_{t_0}^{t}\left(\frac{\alpha(t)}{2\pi}\right)^2_{NLO} V(x_0,t)dt\Bigg]\left(\frac{x}{x_0}\right)^{(1-bt)}\end{aligned} \tag{4.50}$$

and

$$\begin{aligned}F_2^{NS}(x,t)\Bigg|_{NNLO} = F_2^{NS}(x_0,t_0)\exp\Bigg[&\int_{t_0}^{t}\left(\frac{\alpha(t)}{2\pi}\right)_{NNLO} U(x_0,t)dt\\ &+\int_{t_0}^{t}\left(\frac{\alpha(t)}{2\pi}\right)^2_{NNLO} V(x_0,t)dt\\ &+\int_{t_0}^{t}\left(\frac{\alpha(t)}{2\pi}\right)^3_{NNLO} W(x_0,t)dt\Bigg]\left(\frac{x}{x_0}\right)^{(1-bt)}.\end{aligned} \tag{4.51}$$

4.4 Results and Discussion

The equations (4.31)-(4.33) and (4.49)-(4.51) are the analytic expressions representing both x and Q^2 dependence of $F_2^{NS}(x,Q^2)$ structure function jointly, obtained by means of solving the DGLAP equations in LO, NLO and NNLO incorporating the Regge ansatz, $F_2^{NS}(x,Q^2) = A(Q^2)x^{0.5}$ and $F_2^{NS}(x,Q^2) = Bx^{1-bt}$ as the initial inputs respectively. These expressions are consisting of an input point $F_2^{NS}(x_0,t_0)$, which can be taken from the available experimental data. If the input point is more accurate and precise, we can expect batter results. There are not any specific reason in choosing the input point. Any one of the data points at a certain value of $x = x_0$ and $t = t_0$ can be considered as the input point. Off course, the sensitivity of different inputs will be

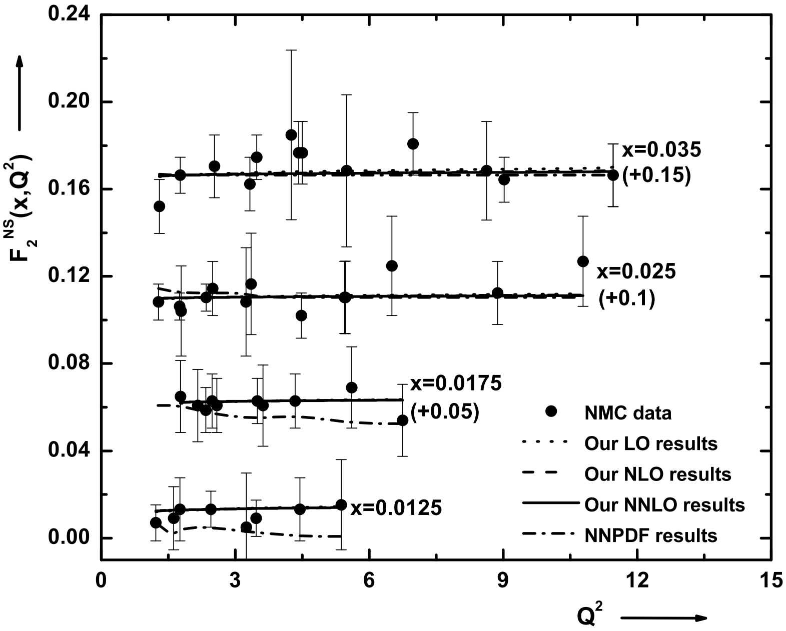

Figure 4.1: Q^2 evolution of $F_2^{NS}(x, Q^2)$ structure functions in accord with (4.31)-(4.33) in comparison with NMC[63] and NNPDF[101] results. For clarity, the points are offset by the amount given in parenthesis. (Q^2's are taken in the unit of GeV^2).

different. However instead of choosing the input point on the basis of their sensitivity, in our manuscript we have incorporated a suitable condition in determining the input point. We have considered that particular point from the most recent measurements as the input point in which experimental errors are minimum. Under this condition we have selected the point $F_2^{NS}(x_0, t_0) = 0.010348 \pm 0.006208$ at $x_0 = 0.025$ and $Q^2 = 2.34686 GeV^2$ from the experimental results of NMC[63]. Here we have considered the central value of the input point. Further the expressions (4.49)-(4.51) consists of the additional parameter b which has the value $b = 0.118 \pm 0.028$ for $F_2^{NS}(x, Q^2)$ as obtained in Chapter 3.

With the input point $F_2^{NS}(x_0, t_0) = 0.010348$, substituting the respective expressions in LO, NLO and NNLO for running coupling constant, $\frac{\alpha_s(t)}{2\pi}$ and performing the corresponding integrations, we have obtained both x as well as Q^2 evolution of $F_2^{NS}(x, Q^2)$ structure function in accord with the equations (4.31), (4.32) and (4.33) respectively. The Q^2 evolution results at fixed value of x are depicted in Fig. 4.1 in comparison with the experimental data taken from NMC[63] and with the results of NNPDF collaboration[101]. In Fig. 4.2, the x evolution of $F_2^{NS}(x, Q^2)$ for fixed values of Q^2 are depicted along with NMC and NNPDF results. In all figures, as indicated ,

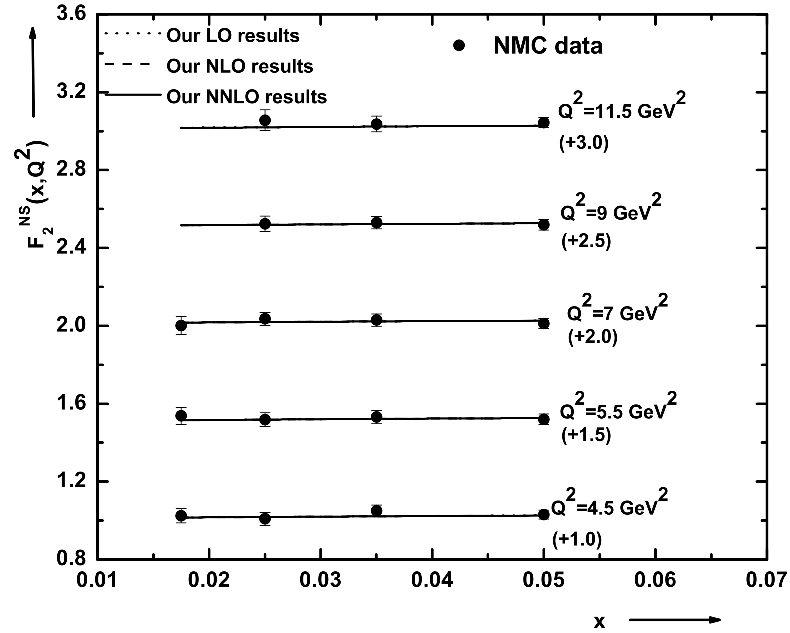

Figure 4.2: x evolution of $F_2^{NS}(x, Q^2)$ structure functions in accord with (4.31)-(4.33) in comparison with NMC[63] results. For clarity, the points are offset by the amount given in parenthesis.

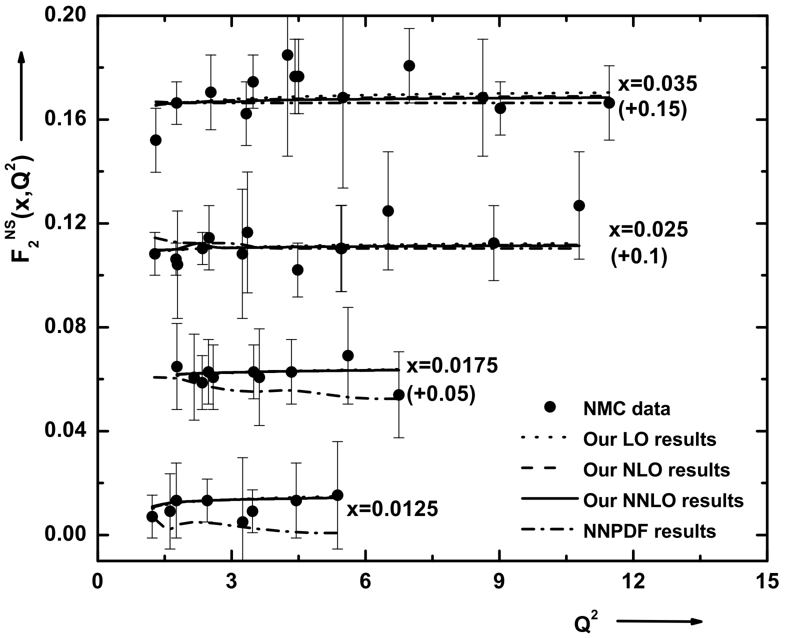

Figure 4.3: Q^2(in the unit of GeV^2) evolution of $F_2^{NS}(x, Q^2)$ structure functions in accord with (4.49)-(4.51) in comparison with NMC[63] results. For clarity, the points are offset by the amount given in parenthesis. (Q^2's are taken in the unit of GeV^2).

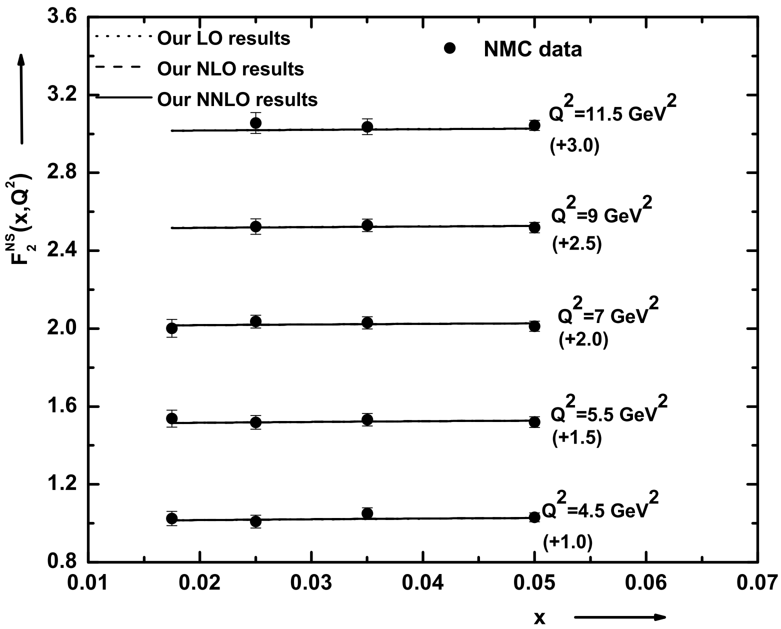

Figure 4.4: x evolution of $F_2^{NS}(x, Q^2)$ structure functions in accord with (4.49)-(4.51) in comparison with NMC[63] and NNPDF[101] results. For clarity, the points are offset by the amount given in parenthesis.

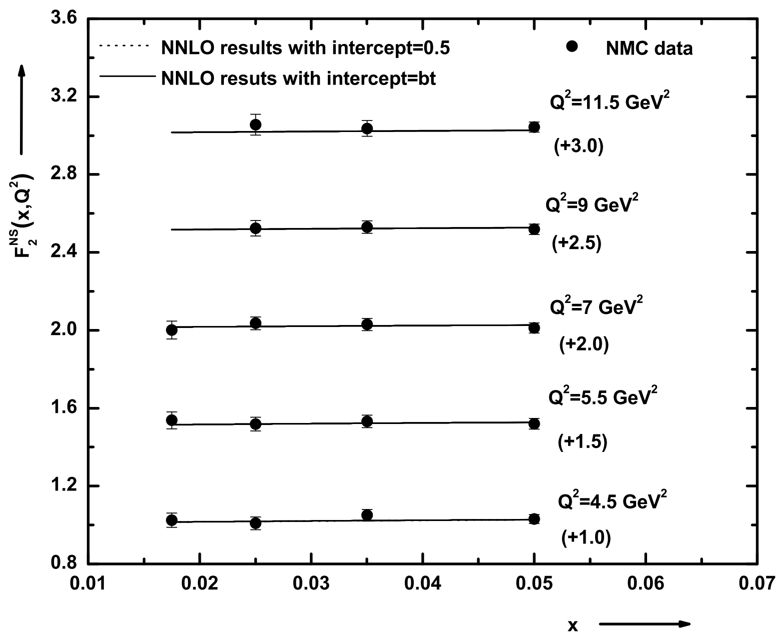

Figure 4.5: Q^2 evolution of $F_2^{NS}(x, Q^2)$ structure functions in accord with (4.33) and (4.51) in comparison with NMC[63] and NNPDF[101] results. For clarity, the points are offset by the amount given in parenthesis. (Q^2's are taken in the unit of GeV^2).

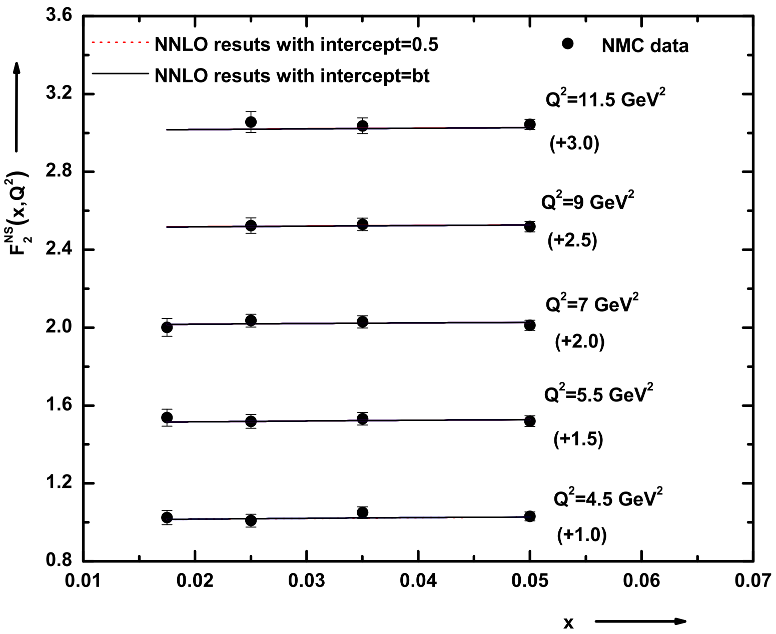

Figure 4.6: x evolution of $F_2^{NS}(x,Q^2)$ structure functions in accord with (4.33) and (4.51) in comparison with NMC[63] results. For clarity, the points are offset by the amount given in parenthesis.

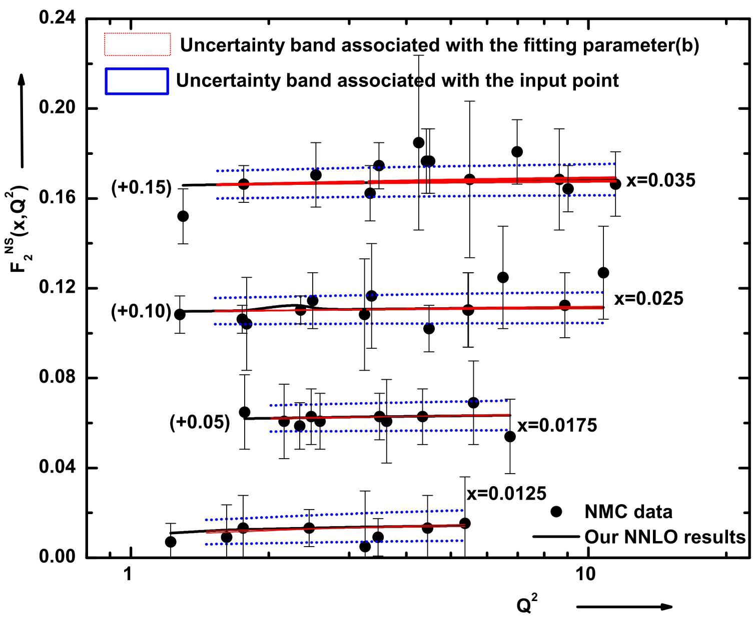

Figure 4.7: Q^2 evolution of $F_2^{NS}(x,Q^2)$ structure functions in accord with NNLO corrections, (4.33) and (4.51) in comparison with NMC[63] results. For clarity, the points are offset by the amount given in parenthesis. (Q^2's are taken in the unit of GeV^2).

the dotted curves represent the LO results, the dashed curves represent NLO results and the solid lines are representing NNLO results. Experimental data are given with vertical upper and lower error bars for total uncertainties of statistical and systematic errors.

Again the results from equations (4.49), (4.50) and (4.51) for Q^2 and x evolution of $F_2^{NS}(x, Q^2)$ structure function with $F_2^{NS}(x_0, t_0) = 0.010348$ and $b = 0.118$ are depicted in Fig. 4.3 and Fig. 4.4 respectively. The experimental results from NMC and the results of NNPDF collaboration are also plotted along with our results. Here, our LO, NLO and NNLO results are represented by the dotted, dashed and solid curves respectively. The solid circles are used to represent the NMC data point and they are along with vertical upper and lower error bars for total uncertainties of statistical and systematic errors.

As far the figures, 4.1-4.4 are concerned, we observe a very good consistency between theoretical and experimental as well as parametrization results within the kinematical region $x < 0.05$ and $Q^2 \leq 20GeV^2$ of our consideration, especially, if the NNLO results are concerned. The most consistent results, the NNLO results for both the inputs along with NMC and NNPDF results are plotted in Figs. 4.5 and 4.6 . It reflects the comparative picture of the results obtained by means of the two ansatz. However within our kinematical region of consideration we do not observe any significant differences among them. This implies that the analytic expressions, we have obtained by means of solving the DGLAP equations with both the ansatz as the initial input, are applicable in describing the small x behaviour of $F_2^{NS}(x, Q^2)$ structure function with a considerable precision.

In addition, we have shown in the Fig. 4.7, the band due to the uncertainty associated with input and the fitting parameter b. Here the uncertainty due to the fitting parameter is considerably less than that of due to input point.

4.5 Summary

We have employed the usefulness of two ansatz as the initial input in order to solve DGLAP equation up to NNLO and obtain Q^2 evolution of the unpolarized non-singlet structure function $F_2^{NS}(x, Q^2)$. The structure function, evolved as the solutions of the DGLAP equations are studied phenomenologically in comparison with the results taken from NMC and NNPDF collaborations. We observe a very good agreement between our theoretical results and other experimental results as well as parametrization, within the kinematical range $x < 0.05$ and $Q^2 = 20GeV^2$ of our consideration. The phenomenological success achieved in this study suggests that the two simple QCD featured Regge behaved ansatz $F_2^{NS}(x,Q^2) = A(Q^2)x^{0.5}$ and $F_2^{NS}(x,Q^2) = Bx^{1-bt}$ are capable of evolving $F_2^{NS}(x,Q^2)$ structure functions with Q^2 in accord with DGLAP equations at small-x. However we could not distinguish the efficiencies among the two models in comparison with experimental data within the kinematical range of our consideration. We hope future experimental measurements at very very small values of Bjorken x will clarify their differences and help us in batter understanding of the structure of nucleon. □□