Chapter 6

Small-x Behaviour of $xg_1^{NS}(x, Q^2)$ Structure Function

This chapter encompasses the calculation of spin-dependent non-singlet structure function $xg_1^{NS}(x, Q^2)$ by means of solving DGLAP equation with QCD corrections up to next-next-to-leading order. Using the two ansatz, discussed in the chapter 3, developed by combining the features of perturbative Quantum Chromodynamics and Regge theory, as the initial input we have solved the DGLAP equations. The solutions, along with the ansatz allow us to obtain some analytic expressions which represent the joint Bjorken x and Q^2 dependence of $xg_1^{NS}(x, Q^2)$ structure function. The expressions are studied phenomenologically in comparison with experimental data taken from SMC, E143, HERMES, COMPASS and JLab experiments. In addition, our results are compared with some other strong analysis. We have achieved at a great phenomenological success, which signifies the capability of the expressions in describing the small-x behaviour of this non-singlet structure function and their usefulness in determining the structure functions with a reasonable precision.

6.1 Introduction

Proper understanding of the spin structure of nucleon and associated sum rules is expected to offer an important opportunity to investigate Quantum Chromodynamics(QCD) as a theory of strong interaction and hence these observables have been the active frontiers in recent years [62, 141–146]. Many successful experimental programs of polarized deep-inelastic lepton-nucleon scattering in combination with remarkable theoretical efforts have been devoted in order to elucidate the internal spin structure of the nucleon. Polarized deep inelastic lepton scattering experiment have been carried out at SLAC, CERN, DESY and Jefferson Laboratory(JLab)[62]. With the advent of dedicated experimental facilities, recent experiments were able to determine the spin structure functions as well as different sum rules over a wide range of x and Q^2 with ever increasing precision. Simultaneously, tremendous progress is observed in the field of theoretical investigation in determining and understanding the shape of quarks and gluon spin distribution functions with perturbative QCD, non-perturbative QCD, chiral perturbation theory[147], lattice QCD[148], anti-de Sitter/conformal field theory (AdS/CFT)[149], etc., along with different reliable theoretical models. In addition, recently available several dedicated phenomenological works[131, 150–158] in extracting polarized parton distribution function(PPDF) as well as spin structure functions from different experiments within NLO QCD analysis have also significant contributions towards the understanding of spin structure of the nucleon.

In Quantum Chromodynamics, the spin structure function $g_1(x,Q^2)$ is described as Mellin convolutions between parton distribution functions $(\Delta q_i, \Delta g)$ and the Wilson coefficients C_i [159]

$$g_1(x,Q^2) = \frac{1}{2n_f}\sum_{i=1}^{n} e_i^2[C_{NS}\otimes\Delta q_{NS} + C_S\otimes\Delta q_S + 2n_f C_g\otimes\Delta g], \tag{6.1}$$

which consists of three parts, non-singlet $g_1^{NS}(x,Q^2) = \frac{1}{2n_f}\sum_{i=1}^{n} e_i^2[C_{NS}\otimes\Delta q_{NS}]$, singlet $g_1^{S}(x,Q^2) = \frac{1}{2n_f}\sum_{i=1}^{n} e_i^2[C_{S}\otimes\Delta q_{S}]$ and gluon $\Delta G(x,Q^2) = \frac{1}{2n_f}\sum_{i=1}^{n} e_i^2[2n_f C_g\otimes\Delta g]$. The Q^2 distribution of these spin dependent non-singlet, singlet and gluon distribution functions are governed by a set of integro-differential equations, the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations which are given by [24]

$$Q^2\frac{\partial x g_1^{NS}(x,Q^2)}{\partial \ln Q^2} = \frac{\alpha(Q^2)}{2\pi}P_{qq}^{NS}(x,Q^2)\otimes x g_1^{NS}(x,Q^2), \tag{6.2}$$

$$Q^2\frac{\partial\begin{pmatrix} g_1^S(x,Q^2)\\ \Delta G(x,Q^2)\end{pmatrix}}{\partial \ln Q^2} = \begin{pmatrix} P_{qq}^S(x,Q^2) & 2n_f P_{qg}^S(x,Q^2)\\ P_{gq}^S(x,Q^2) & P_{gg}^S(x,Q^2)\end{pmatrix}\otimes\begin{pmatrix} g_1^S(x,Q^2)\\ \Delta G(x,Q^2)\end{pmatrix}. \tag{6.3}$$

Here P_i are the polarized splitting functions [24, 32]. These equations are valid to all orders in the strong coupling constant $\frac{\alpha(Q^2)}{2\pi}$.

In this chapter we have concentrated on the non-singlet part of the polarized nucleon structure function. Here we have investigated the small-x behaviour of xg_1^{NS} structure function. The investigation is based on the solution of the DGLAP evolution equation in LO, NLO and NNLO using the two ansatz $xg_1^{NS}(x,Q^2) = A(Q^2)x^0.5$ and $xg_1^{NS}(x,Q^2) = Ax^{(1-at)}$ as the initial inputs. We have performed a phenomenological analysis of these solutions in comparison with different experimental measurements[71, 73–75] as well as the predictions due to different models [131, 160–162] and achieved at a very good phenomenological success. The phenomenological success achieved in this regard reflects, on one hand the acceptability of the Regge ansatz in describing the small x behavior of the non-singlet part of spin structure function and on the other hand, the usefulness of the Regge ansatz in evolving the spin structure function, $g_1^{NS}(x,Q^2)$ in accord with DGLAP equation with a considerable precision within smaller x region.

For simplicity, defining $xg_1^{NS}(x,Q^2) = g^{NS}(x,Q^2)$, the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equation which describe the Q^2 behavior of polarised non-singlet structure function $xg_1^{NS}(x,Q^2)$ in perturbative Quantum Chromodynamics (QCD) formalism is given by

$$\frac{\partial g^{NS}(x,t)}{\partial t} = \frac{\alpha(t)}{2\pi}\int_x^1 \frac{d\omega}{\omega} g^{NS}\left(\frac{x}{\omega},t\right)P_{qq}^{NS}(\omega), \tag{6.4}$$

in terms of the variable $t = \ln(\frac{Q^2}{\Lambda^2})$. Here the splitting function, $P_{qq}^{NS}(\omega)$ is defined up to next-next-to-leading order by

$$P_{qq}^{NS}(\omega) = \frac{\alpha(t)}{2\pi}P^{(0)}(\omega) + \left(\frac{\alpha(t)}{2\pi}\right)^2 P^1(\omega) + \left(\frac{\alpha(Q^2)}{2\pi}\right)^3 P_i^2(\omega), \tag{6.5}$$

where, $P^{(0)}(\omega)$, $P^{(1)}(\omega)$ and $P^{(2)}(\omega)$ are the corresponding LO, NLO and NNLO corrections to the splitting functions[24, 32]. Splitting functions are given in Appendices.

Again, in LO, NLO and NNLO, the running coupling constant $\frac{\alpha(Q^2)}{2\pi}$ has the forms[23],

$$\left(\frac{\alpha(t)}{2\pi}\right)_{LO} = \frac{2}{\beta_0 t}, \tag{6.6}$$

$$\left(\frac{\alpha(t)}{2\pi}\right)_{NLO} = \frac{2}{\beta_0 t}\left[1 - \frac{\beta_1 \ln t}{\beta_0^2 t}\right], \tag{6.7}$$

and

$$\left(\frac{\alpha(t)}{2\pi}\right)_{NNLO} = \frac{2}{\beta_0 t}\left[1 - \frac{\beta_1 \ln t}{\beta_0^2 t} + \frac{1}{\beta_0^2 t^2}\left[\left(\frac{\beta_1}{\beta_0}\right)^2 (\ln^2 t - \ln t + 1) + \frac{\beta_2}{\beta_0}\right]\right] \qquad (6.8)$$

respectively, where $\beta_0 = 11 - \frac{2}{3}N_F$, $\beta_1 = 102 - \frac{38}{3}N_F$ and $\beta_2 = \frac{2857}{6} - \frac{6673}{18}N_F + \frac{325}{54}N_F^2$ are the one-loop, two-loop and three-loop corrections to the QCD β-function. Here the running coupling constant is expressed in terms of the variable t, which is defined by $t = \ln(\frac{Q^2}{\Lambda^2})$. Substituting the respective splitting functions along with the corresponding running coupling constant in (6.4) the DGLAP evolution equations in LO, NLO and NNLO become

$$\frac{\partial g^{NS}(x,t)}{\partial t} = \left(\frac{\alpha(t)}{2\pi}\right)_{LO}\left[\frac{2}{3}\{3 + 4ln(1-x)\}g^{NS}(x,t) + I_1(x,t)\right], \qquad (6.9)$$

$$\frac{\partial g^{NS}(x,t)}{\partial t} = \left(\frac{\alpha(t)}{2\pi}\right)_{NLO}\left[\frac{2}{3}\{3 + 4ln(1-x)\}g^{NS}(x,t) + I_1(x,t)\right] + \left(\frac{\alpha(t)}{2\pi}\right)^2_{NLO} I_2(x,t), \qquad (6.10)$$

and

$$\frac{\partial g^{NS}(x,t)}{\partial t} = \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}\left[\frac{2}{3}\{3 + 4ln(1-x)\}g^{NS}(x,t) + I_1(x,t)\right] + \left(\frac{\alpha(t)}{2\pi}\right)^2_{NNLO} I_2(x,t) + \left(\frac{\alpha(t)}{2\pi}\right)^3_{NNLO} I_3(x,t) \qquad (6.11)$$

respectively. Here the integral functions are given by

$$I_1(x,t) = \int_x^1 \frac{d\omega}{1-\omega}\left\{\frac{(1+\omega^2)}{\omega}g^{NS}\left(\frac{x}{\omega},t\right) - 2g^{NS}(x,t)\right\}, \qquad (6.12)$$

$$I_2(x,t) = \int_x^1 \frac{d\omega}{\omega}P^{(2)}(\omega)g^{NS}\left(\frac{x}{\omega},t\right) \qquad (6.13)$$

and

$$I_3(x,t) = \int_x^1 \frac{d\omega}{\omega}P^{(3)}(\omega)g^{NS}\left(\frac{x}{\omega},t\right). \qquad (6.14)$$

We now solve the DGLAP equations up to NNLO ((6.9)-(6.11)) analytically using the ansatz $xg_1^{NS}(x,t) = A(t)x^{0.5}$ and $xg_1^{NS}(x,t) = Bx^{(1-at)}$ as the initial inputs. Here in both the case we have discussed in detailed the LO solution and then the same formalism is extended to have corresponding NLO and NNLO solutions.

6.2 Solution of DGLAP Evolution Equations with the Initial Input $xg_1^{NS}(x,t) = A(t)x^{0.5}$

If we consider that the non-singlet part of the spin structure function satisfies the following Regge ansatz:

$$g_1^{NS}(x,t) = g^{NS}(x,t) = A(t)x^{0.5}, \tag{6.15}$$

then the t dependence of $xg_1^{NS}(x,t)$ structure function at a particular value of $x = x_0$ is given by

$$g^{NS}(x_0,t) = A(t)x_0^{0.5}. \tag{6.16}$$

Dividing (6.15) by (6.16) we have the following relation

$$g^{NS}(x,t) = g^{NS}(x_0,t)\left(\frac{x}{x_0}\right)^{0.5}, \tag{6.17}$$

which gives both t and x dependence of $g^{NS}(x,t)$ structure function in terms of the t dependent function $g^{NS}(x_0,t)$ at fixed $x = x_0$. The t dependent function, $g^{NS}(x_0,t)$ can be obtained from the DGLAP equation.

Substituting $g^{NS}(x,t) = A(t)x^{0.5}$ and $g^{NS}(\frac{x}{\omega},t) = \omega^{-0.5}g^{NS}(x,t)$ in equation, (6.9), we obtain

$$\frac{\partial g^{NS}(x,t)}{\partial t} = \left(\frac{\alpha(t)}{2\pi}\right)_{LO}\left[\frac{2}{3}\{3+4ln(1-x)\} + \frac{4}{3}\int_x^1 \frac{d\omega}{1-\omega}\left\{\frac{1+\omega^2}{\omega}\omega^{-0.5} - 2\right\}\right]g^{NS}(x,t), \tag{6.18}$$

which can be rearranged to have an ordinary differential equation and can be solved easily to have

$$g^{NS}(x,t)\bigg|_{LO} = C_1 \exp\left[U(x)\int\left(\frac{\alpha(t)}{2\pi}\right)_{LO} dt\right]. \tag{6.19}$$

Here

$$U(x) = \frac{2}{3}\{3 + 4ln(1-x)\} + \frac{4}{3}\int_x^1 \frac{d\omega}{1-\omega}\left\{\frac{1+\omega^2}{\omega}\omega^{-0.5} - 2\right\} \tag{6.20}$$

and C is the constant of integration.

Now at a fixed value of $x = x_0$, the t dependence of the structure function $g_1^{NS}(x, t)$ in LO is given by

$$g^{NS}(x_0, t)\Bigg|_{LO} = C_1 \exp\left[U(x_0)\int \left(\frac{\alpha(t)}{2\pi}\right)_{LO} dt\right]. \tag{6.21}$$

Again the value of the structure function at $x = x_0$ and $t = t_0$ in accord with (6.21) is

$$g^{NS}(x_0, t_0)\Bigg|_{LO} = C_1 \exp\left[U(x_0)\int \left(\frac{\alpha(t)}{2\pi}\right)_{LO} dt\right]\Bigg|_{t=t_0}. \tag{6.22}$$

Dividing (6.21) by (6.22) and rearranging a bit we obtain the t evolution of $g^{NS}(x, t)$ in accord with the LO DGLAP equation with respect to the point $g^{NS}(x_0, t_0)$ as

$$g^{NS}(x_0, t)\Bigg|_{LO} = g^{NS}(x_0, t_0) \exp\left[U(x_0)\int_{t_0}^{t} \left(\frac{\alpha(t)}{2\pi}\right)_{LO} dt\right]. \tag{6.23}$$

Now substituting $g^{NS}(x_0, t)\Big|_{LO}$ from (6.23) in (6.17), we have a relation representing both x and t dependence of structure function in LO, in terms of the input point $g^{NS}(x_0, t_0)$ given by

$$g^{NS}(x, t)\Bigg|_{LO} = g^{NS}(x_0, t_0) \exp\left[\int_{t_0}^{t} \left(\frac{\alpha(t)}{2\pi}\right)_{LO} U(x_0, t) dt\right]\left(\frac{x}{x_0}\right)^{0.5}. \tag{6.24}$$

Proceeding in the similar way we can obtain the relation for $g^{NS}(x, t)$ structure function in NLO and NNLO as

$$g^{NS}(x, t)\Bigg|_{NLO} = g^{NS}(x_0, t_0) \exp\left[U(x_0)\int_{t_0}^{t} \left(\frac{\alpha(t)}{2\pi}\right)_{NLO} dt \right.$$
$$\left. + V(x_0)\int_{t_0}^{t} \left(\frac{\alpha(t)}{2\pi}\right)^2_{NLO} dt\right]\left(\frac{x}{x_0}\right)^{0.5} \tag{6.25}$$

and

$$g^{NS}(x,t)\Big|_{NNLO} = g^{NS}(x_0,t_0)\exp\left[U(x_0)\int_{t_0}^{t}\left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}dt + V(x_0)\int_{t_0}^{t}\left(\frac{\alpha(t)}{2\pi}\right)^2_{NNLO}dt + W(x_0)\int_{t_0}^{t}\left(\frac{\alpha(t)}{2\pi}\right)^3_{NNLO}dt\right]\left(\frac{x}{x_0}\right)^{0.5} \quad (6.26)$$

respectively, where

$$V(x) = \int_x^1 \frac{d\omega}{\omega}P^{(1)}(\omega)\omega^{-0.5} \quad (6.27)$$

and

$$W(x) = \int_x^1 \frac{d\omega}{\omega}P^{(2)}(\omega)\omega^{-0.5}. \quad (6.28)$$

6.3 Solution of DGLAP Evolution Equations with the Initial Input $xg_1^{NS}(x,t) = Bx^{(1-bt)}$

Now we considered that the non-singlet part of the spin structure function satisfies the following Regge ansatz:

$$g_1^{NS}(x,t) = Ax^{(-bt)} \quad (6.29)$$

and hence we have

$$xg_1^{NS}(x,t) = g^{NS}(x,t) = A.x^{(1-bt)}. \quad (6.30)$$

The t dependence of $xg_1^{NS}(x,t)$ structure function at a particular value of $x = x_0$ is givent by

$$g^{NS}(x_0,t) = A.x_0^{(1-bt)}. \quad (6.31)$$

Dividing (6.30) by (6.31) we have the following relation

$$g^{NS}(x,t) = g^{NS}(x_0,t)\left(\frac{x}{x_0}\right)^{(1-bt)}, \quad (6.32)$$

which gives both t and x dependence of $g^{NS}(x,t)$ structure function in terms of the t dependent function $g^{NS}(x_0,t)$ at fixed $x = x_0$. The t dependent function, $g^{NS}(x_0,t)$ can be obtained from the DGLAP equation.

Substituting $g^{NS}(x,t) = Ax^{1-bt}$ and $g^{NS}(\frac{x}{\omega},t) = \omega^{-(1-bt)}g^{NS}(x,t)$ in equation (6.9) and rearranging a bit we can convert the LO DGLAP equation into an ordinary differential equation which can be easily solved to have

$$g^{NS}(x,t)\Big|_{LO} = C\exp\left[\int\left(\frac{\alpha(t)}{2\pi}\right)_{LO} U(x,t)dt\right]. \tag{6.33}$$

Here

$$U(x,t) = \frac{2}{3}\{3+4ln(1-x)\} + \frac{4}{3}\int_x^1 \frac{d\omega}{1-\omega}\left\{\frac{1+\omega^2}{\omega}\omega^{-(1-bt)} - 2\right\}, \tag{6.34}$$

and C is the constant of integration.

At a fixed value of $x = x_0$, the t dependence of the structure function in LO is given by

$$g^{NS}(x_0,t) = C\exp\left[\int\left(\frac{\alpha(t)}{2\pi}\right)_{LO} U(x_0,t)dt\right]. \tag{6.35}$$

Again the value of the structure function at $x = x_0$ and $t = t_0$ in accord with (6.35) is given by

$$g^{NS}(x_0,t_0) = C\exp\left[\int\frac{\alpha(t)}{2\pi}U(x_0,t)dt\right]\Bigg|_{t=t_0}. \tag{6.36}$$

Dividing (6.35) by (6.36) and rearranging a bit we obtain the t dependence of $g^{NS}(x_0,t)$ in accord with LO DGLAP evolution equation with respect to the point $g^{NS}(x_0,t_0)$ as

$$g^{NS}(x_0,t) = g^{NS}(x_0,t_0)\exp\left[\int_{t_0}^{t}\left(\frac{\alpha(t)}{2\pi}\right)_{LO} U(x_0,t)dt\right]. \tag{6.37}$$

Now substituting $g^{NS}(x_0,t)\Big|_{LO}$ from (6.37) in (6.32), we have a relation representing both x and t dependence of structure function in LO, in terms of the input point $g^{NS}(x_0,t_0)$ given by

$$g^{NS}(x,t)\Big|_{LO} = g^{NS}(x_0,t_0)\exp\left[\int_{t_0}^{t}\left(\frac{\alpha(t)}{2\pi}\right)_{LO} U(x_0,t)dt\right]\left(\frac{x}{x_0}\right)^{(1-bt)}. \tag{6.38}$$

Proceeding in the similar way we can obtain the relation for $g^{NS}(x,t)$ structure function in NLO and NNLO as

$$g^{NS}(x,t)\Big|_{NLO} = g^{NS}(x_0,t_0)\exp\left[\int_{t_0}^{t}\left(\frac{\alpha(t)}{2\pi}\right)_{NLO} U(x_0,t)dt + \int_{t_0}^{t}\left(\frac{\alpha(t)}{2\pi}\right)^2_{NLO} V(x_0,t)dt\right]\left(\frac{x}{x_0}\right)^{(1-bt)}, \tag{6.39}$$

and

$$g^{NS}(x,t)\Big|_{NNLO} = g^{NS}(x_0,t_0)\exp\left[\int_{t_0}^{t}\left(\frac{\alpha(t)}{2\pi}\right)_{NNLO} U(x_0,t)dt + \int_{t_0}^{t}\left(\frac{\alpha(t)}{2\pi}\right)^2_{NNLO} V(x_0,t)dt + \int_{t_0}^{t}\left(\frac{\alpha(t)}{2\pi}\right)^3_{NNLO} W(x_0,t)dt\right]\left(\frac{x}{x_0}\right)^{(1-bt)}, \tag{6.40}$$

respectively, where

$$V(x,t) = \int_x^1 \frac{d\omega}{\omega} P^{(1)}(\omega)\omega^{-(1-bt)} \tag{6.41}$$

and

$$W(x,t) = \int_x^1 \frac{d\omega}{\omega} P^{(2)}(\omega)\omega^{-(1-bt)}. \tag{6.42}$$

6.4 Results and Discussion

The equations (6.24)-(6.26) and (6.38)-(6.40) are the analytic expressions representing both x and Q^2 dependence of $xg_1^{NS}(x,Q^2)$ structure function jointly, obtained by means of solving the DGLAP equations in LO, NLO and NNLO incorporating the Regge ansatz, $xg_1^{NS}(x,t) = A(t)x^{0.5}$ and $xg_1^{NS}(x,t) = Bx^{1-bt}$ as the initial inputs respectively. These expressions are consisting of an input point $xg_1^{NS}(x_0,t_0)$, which can be taken from the available experimental data. If the input point is more accurate and precise, we can expect batter results. There are not any specific reason in choosing the input point. Any one of the data points at a certain value of $x = x_0$ and $t = t_0$ can be considered as the input point. Off course, the sensitivity of different inputs will be

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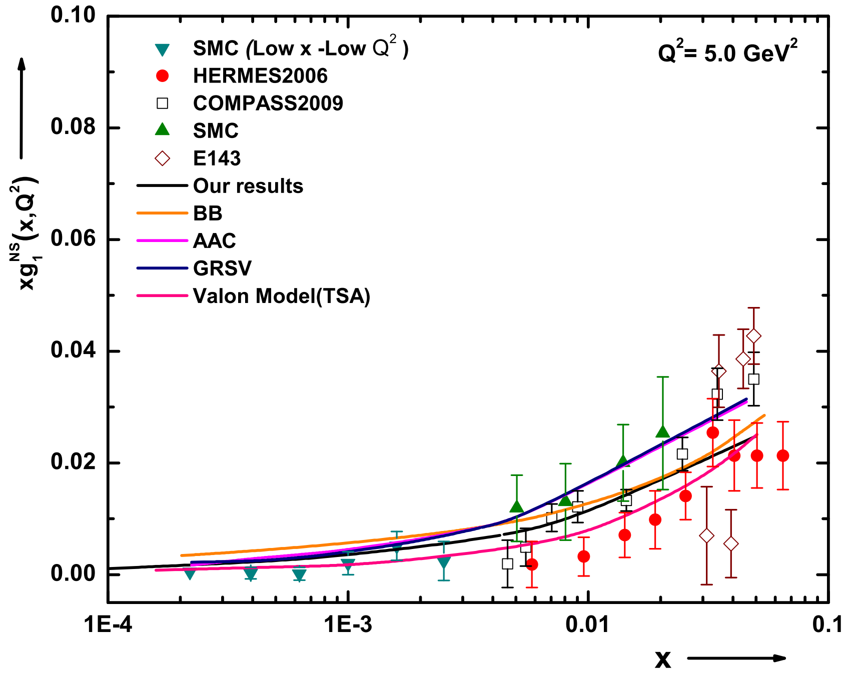

Figure 6.1: xg_1^{NS} structure function in accord with (6.24)-(6.26), compared with the data taken from SMC[74], HERMES[73], COMPASS[71] and E143[75] experiments and the results of TSA[131], AAC[160], BB[161] and GRSV[162] collaborations.

different. However instead of choosing the input point on the basis of their sensitivity, in our manuscript we have incorporated a suitable condition in determining the input point. We have considered that particular point from the most recent measurements as the input point in which experimental errors are minimum. Under this condition we have selected the point $g^{NS}(x_0 = 0.0143955, Q_0^2 = 5GeV^2) = 0.0133075$ at $x_0 = 0.0143955$ and $Q^2 = 5GeV^2$ from COMPASS[71] experimental data. Here we have considered the central value of the input point. Further the expressions (6.38)-(6.40) consists of the additional parameter b which has the value $b = 0.0759 \pm 0.0107$ for xg_1^{NS} as obtained in Chapter 3.

With the input point $g^{NS}(x_0 = 0.0143955, Q_0^2 = 5GeV^2) = 0.0133075$, substituting the respective expressions in LO, NLO and NNLO for running coupling constant, $\frac{\alpha_s(t)}{2\pi}$ and performing the corresponding integrations, we have obtained the x evolution of $xg_1^{NS}(x, Q^2)$ structure function in accord with the equations (6.24), (6.25) and (6.26) respectively. The x evolution results for two fixed value of $Q^2 = 5.0GeV^2$ are depicted in Fig. 6.1. However, as there are not any available experimental results for

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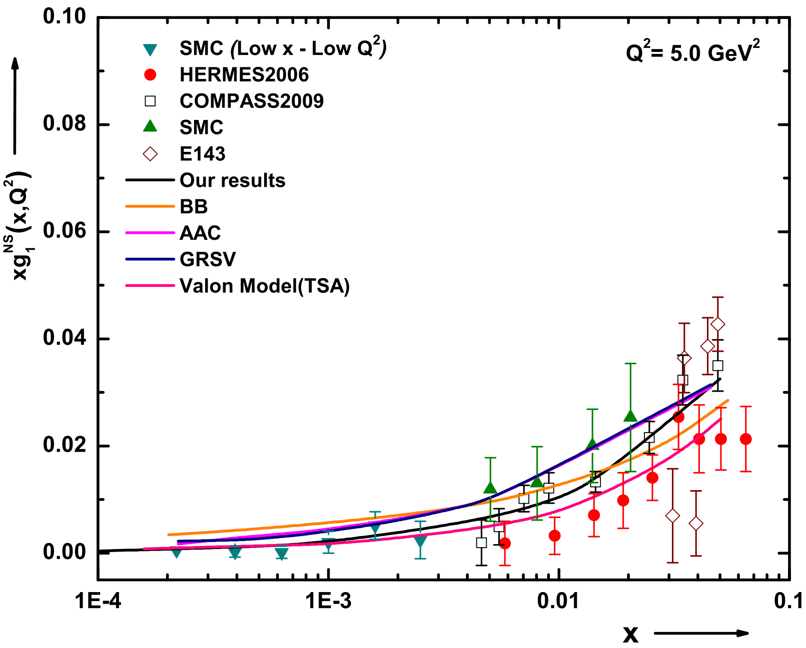

Figure 6.2: xg_1^{NS} structure function in accord with (6.38)-(6.40), compared with the data taken from SMC[74], HERMES[73], COMPASS[71] and E143[75] experiments and the results of TSA[131], AAC[160], BB[161] and GRSV[162] collaborations.

different Q^2, we could not have comparative analysis of our Q^2 evolution results. Our x evolution results are plotted along with the experimental results taken from SMC[74], HERMES[73], COMPASS[71] and E143[75] experiments. In addition to these, we have also included the predictions made by Taghavi-Shahri and Arash(TSA)[131], Asymmetry Analysis Collaboration(AAC)[160], Blumlein and Bottcher(BB)[161] and Gluck, Reya, Startmann and Vogelsang(GRSV)[162] based on various models, in our comparative analysis. We see that $g^{NS}(x, Q^2)$ structure functions evolved with respect to the input point are consistent with those of experimental measurements as well as other models. This implies that the expressions, we have obtained by means of solving the DGLAP equations analytically, are applicable in describing small x behaviour of $xg_1^{NS}(x, Q^2)$ structure function with a considerable precision.

Again the results from equations (6.38),(6.39) and (6.40) for x evolution of $xg_1^{NS}(x, Q^2)$ structure function with $xg_1^{NS}(x_0, Q_0^2) = 0.0133075$ and $b = 0.0759$ are depicted in Fig. 6.2. In this case also as we do not have experimental data point for various Q^2, we could not perform the comparative analysis of our Q^2 evolution results.

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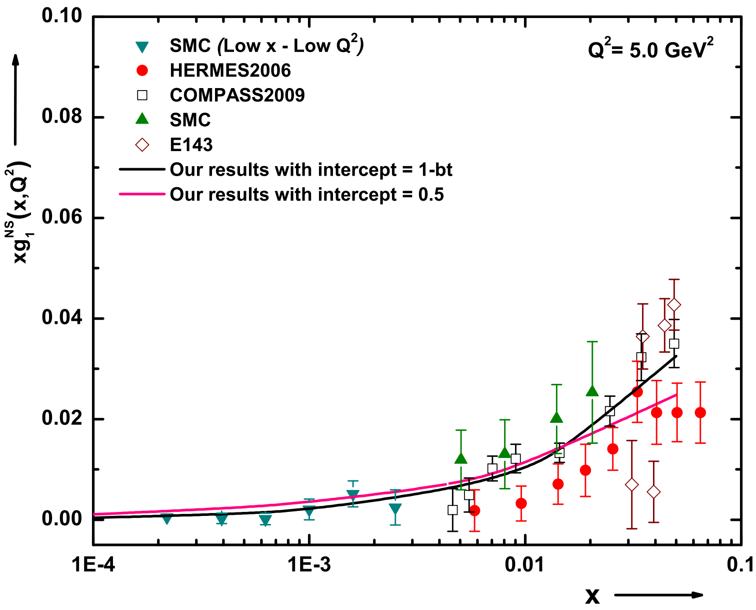

Figure 6.3: xg_1^{NS} structure function in accord with (6.26) and (6.40) and in comparison with the data taken from SMC[74], HERMES[73], COMPASS[71] and E143[75] experiments.

However, the x evolution results are compared with SMC, E143, HERMES and COMPASS experimental results and with several predictions made in Ref [131, 160–162] based on various model.

Also we have estimated the uncertainty associated with the fitting parameter b and the chosen input point and the respective uncertainty bands are shown in Fig. 6.4 separately. Here the uncertainty due to the fitting parameter is considerably less than that of due to input point. However both the uncertainties are observed to be decreasing as x decreases.

As far the figures, 6.2 - 6.4 are concerned, we observe a very good consistency among our theoretical results and other experimental as well as parametrization results within the kinematical region $x < 0.05$ of our consideration. Our x evolution results for both the inputs along with other experimental results are plotted in Fig. 6.3. It reflects the comparative picture of the results obtained by means of the two ansatz. However within our kinematical region of consideration we do not observe any significant differences among them. This implies that the analytic expressions, we have obtained by means of solving the DGLAP equations with both the ansatz as the initial input, are applicable in describing the small x behaviour of $xg_1^{NS}(x, Q^2)$ structure

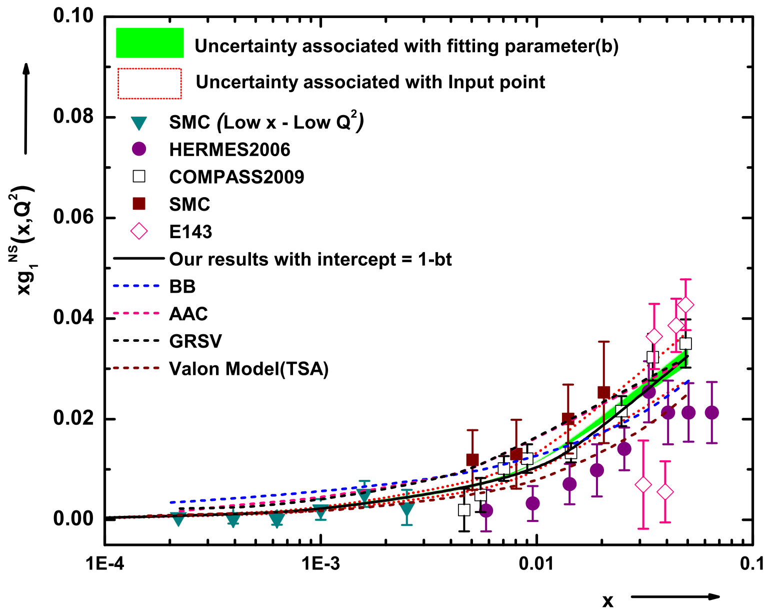

Figure 6.4: xg_1^{NS} structure function in accord with (6.40)and in comparison with different experimental data and theoretical as well as phenomenological analysis, along with the uncertainty band associated with the fitting parameter(b) and the chosen input point.

function with a considerable precision.

6.5 Summary

In this paper we have obtained some expressions for the non-singlet part of spin structure function, $xg_1^{NS}(x, Q^2)$ at small-x by means of analytical solution of DGLAP equation in LO, NLO and NNLO using a Regge like ansatz with Q^2 dependent intercept as the initial input. Both the Regge inspired ansatz in accord with DGLAP equations provides a very good description of the small-x behaviour of $g_1^{NS}(x, Q^2)$, which are consistent with other experimental results. The consistency of the results for $xg_1^{NS}(x, Q^2)$ due to the Regge like models $g^{NS}(x, t) = Ax^{0.5}$ and $g^{NS}(x, t) = Ax^{1-bt}$ with different experimental results taken from SMC[74], HERMES[73], COMPASS[71] and E143[75] and other strong analysis [131,160–162] signifies that the model is applicable in describing the small-x behaviour of $xg_1^{NS}(x, Q^2)$ structure function although it being simple. Moreover, in this method we do not require the knowledge of initial distributions of structure functions at all values of x from 0 to 1. Here, we just require one input point at any fixed x and Q^2 and with respect to that point both the x and

Q^2 evolution of structure functions can be obtained. □□