Chapter 9

Higher Twist Effects in Non-Singlet Structure Functions and Sum Rules

In this chapter the higher twist corrections to the non-singlet structure functions and sum rules associated with them are studied. Here, possible improvement in the accuracy of our results for the non-singlet structure functions and sum rules due to the inclusion of relevant higher twist terms is investigated. Based on a simple model we have extracted the higher twist contributions to the non-singlet structure functions and sum rules in NNLO perturbative orders and then incorporated them with our results. Our NNLO results along with higher twist corrections are observed to be compatible with experimental data.

9.1 Introduction

The behaviour of the deep inelastic structure functions can be analyzed with the perturbative QCD. A method used for this analysis is the operator product expansion method(OPE)[51]. The OPE is successful in describing the contributions from different quark-gluon operators to hadronic tensor and helps in ordering them according to their twist. In accord with OPE, the DIS structure functions and sum rules consist of two parts, the leading twist(LT) and the higher twist(HT) contributions:

$$F_i(x, Q^2) = F_i^{LT}(x, Q^2) + \frac{H_i(x, Q^2)}{Q^2}, \tag{9.1}$$

where i labels the type of the structure function ($F_i = F_2, F_3, g_1$). The leading twist term is associated with the single particle properties of quarks and gluons inside the nucleon and is responsible for the scaling of DIS structure function via perturbative QCD $\alpha_s(Q^2)$ corrections. The higher twist terms reflect instead the strength of multi-parton interactions (qq and qg). Since such interactions spoil factorization one has to consider their impact on the parton distribution functions extracted in the analysis of low-Q^2 data. Because of the non-perturbative origin it is difficult to quantify the magnitude and shape of the higher twist terms from first principles and current models can only provide a qualitative description for such contributions, which must then be determined phenomenologically from data.

The higher twist terms are governed by the terms contributing at different orders of $1/Q^2$:

$$\frac{H_i(x,Q^2)}{Q^2} = \frac{h_1(x)}{Q^2} + \frac{h_2(x)}{Q^4} + ..., \tag{9.2}$$

the leading term in this expansion is known as twist-two, the sub-leading ones twist-three, etcetera. The higher twist terms are suppressed by terms of order $1/Q^2$, $1/Q^4$..., respectively.

The currently available experimental data on deep inelastic structure functions covers a large kinematical regime with high precision measurements. This provides an interesting challenge for theoretical physics when it comes to describing this data in the low-Q^2 domain. pQCD predictions, even with higher order corrections up to NNLO and NNNLO observed to be not sufficient for a precise description of deep inelastic structure function data, which in turn reveals that the discrepancy among data and pQCD predictions are not primarily the sub-leading terms in powers of α_s, but corrections which are proportional to the reciprocal value of the photon virtuality Q^2, viz. higher-twist terms.

The extraction of higher twist terms from the data is a longstanding problem, as recognized from the very first developments of a pQCD phenomenology [247, 248]. Existing information about higher twist terms in lepton-nucleon structure functions is scarce and somewhat controversial. Early analysis [249,250] suggested a significant HT contribution to the longitudinal structure function F_L. The subsequent studies with both charged leptons [251–253] and neutrinos [140] raised the question of a possible dependence on order of QCD calculation used for the leading twist. The common wisdom is generally that HTs only affect the region of $Q^2 \sim 1 - 3GeV^2$ and can be neglected in the extraction of the leading twist.

The higher twist terms are presently poorly known and currently is a subject of both theoretical and phenomenological studies. A better understanding of HT terms, in particular their role in describing low Q^2 and high x DIS data is important and provides valuable information on quark gluon correlations inside the nucleon. The importance of highertwist (HT) contribution to structure functions was pointed from the very beginning of QCD comparison with experimental data[247] on structure functions. Despite a fast progress in theoretical QCD calculations of power corrections to non-singlet structure functions and sum rules [254, 255] (for reviews and references see [256]), the shape of HT (order $1/Q^2$) contributions is measured only for F_2 SF [257] and is still only estimated for xF_3[258]. Several reports are available on the determination of the higher twist contributions in the deeply-inelastic structure functions $F_2^{ep,ed}(x, Q^2$, (see [259] for details). Higher twist contributions were also studied in deep-inelastic neutrino scattering in Ref. [260–263]. Also in the case of polarized deeply inelastic scattering higher twist corrections are present in general. Since the polarized structure functions are measured through an asymmetry, the effect of higher twist contributions in the denominator function has to be known in detail. In [264] no significant higher twist contributions were found. Other authors claim contributions in the low x region[265], which is also the region of very low values of Q^2.

The non-singlet structure functions as well as associated sum rules obtained in the previous chapters in this thesis by means of incorporating the ansatz $F_i = Ax^{1-bt}$ as the initial input to DGLAP equation are the results of pQCD effect with higher order corrections up to NNLO. Although our results are capable of describing the available experimental data with considerable phenomenological success, in the following sections we report on better description of the data by our results along with higher twist corrections. We have incorporated relevant higher twist terms, proposed in different theoretical as well as phenomenological analysis to our results and analysed their effect on possible improvement in accuracy of our results in describing available experimental data.

The usual approach in analyses whose main aim is the extraction of leading twist PDFs is either to parametrize the higher twist contributions by a phenomenological form and fit the parameters to the experimental data[197, 266], or to extract the Q^2 dependence by fitting it in individual bins in x [267–271]. Such an approach effectively includes contributions from multiparton correlations (the true higher twist contributions) along with other power corrections that are not yet part of the theoretical treatment of DIS at low Q^2. These include $O(1/Q2)$ contributions such as jet mass corrections [272] and soft gluon resummation [273], as well as contributions which are of higher order in α_s but whose logarithmic Q^2 behavior mimics terms $\propto \frac{1}{Q^2}$ at low virtuality[271, 274].

9.2 Higher Twist in Non-Singlet Structure Functions

In order to estimate the higher twist contribution to the non-singlet structure functions, we have performed an analysis based on a simple model. Here the first higher twist term is extracted and to do so we have parameterised the non-singlet structure functions as

$$F_i^{data}(x_i, Q^2) = F_i^{LT}(x_i, Q^2) + \frac{h_1(x_i)}{Q^2}. \tag{9.3}$$

Here leading twist(LT) term corresponds to the pQCD contribution to structure functions and the constants $h_1(x_i)$ (one per x - bin) parameterize the x dependence of higher twist contributions. For the leading twist term, we have utilised the results for the non-singlet structure functions obtained in our previous chapters. Incorporating our results for non-singlet structure functions in NNLO as the LT terms we have extracted the difference, $F_i^{data}(x_i, Q^2) - F_i^{LT}(x_i, Q^2)$ from their corresponding experimental data and then fitted with $h_1(x_i)/Q^2$. From the best fitting values, we have determined the higher twist contribution terms h_i per x-bin. In this analysis we have performed our fitting analysis within the kinematical region $0.0125 \le x \le 0.5$ and $1 \le Q^2 \le 20GeV^2$. In this analysis we have extracted the higher twist contribution to the F_2^{NS} and xF_3 structure functions only. Due to unavailability of g_1^{NS} data at different Q^2 we could not include the g_1^{NS} structure function. The higher twist effects in F_2^{NS} and xF_3 are presented in the subsection 9.2.1 and 9.2.2 respectively bellow.

9.2.1 Higher Twist Effect in F_2^{NS} Structure Function

As discussed above, the simple parametrization

$$F_2^{data}(x_i, Q^2) = (F_2^{NS})^{LT}(x_i, Q^2) + \frac{h_1(x_i)}{Q^2}, \tag{9.4}$$

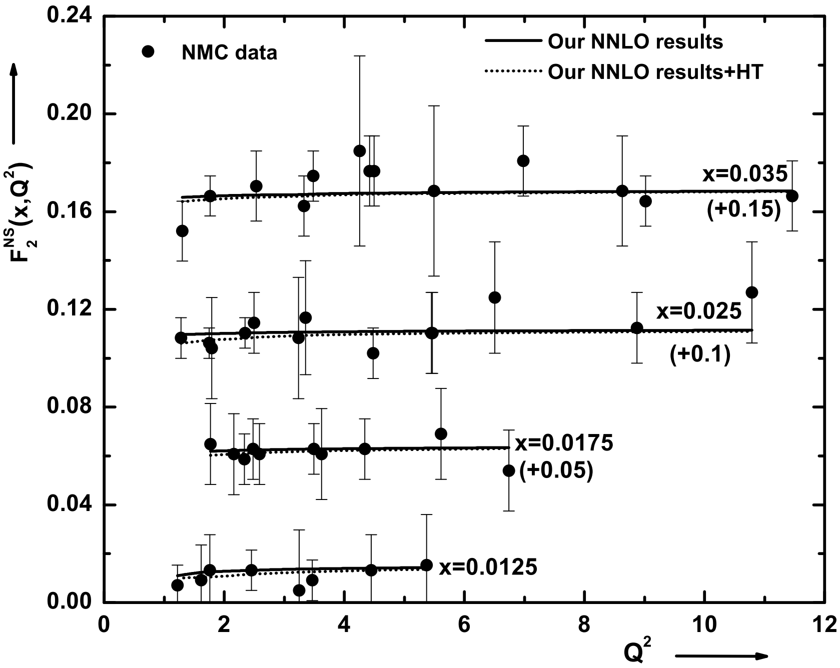

Figure 9.1: Higher twist corrections to F_2^{NS} structure function at NNLO. (Q^2's are taken in the unit of GeV^2).

for the F_2^{NS} structure function incorporating higher twist contributions in terms of the parameter $h_1(x_i)$ is fitted to the NMC data for the x-bins $x_i = 0.0125, 0.0175, 0.025, 0.035$. Here we have used the NNLO results (4.51) for the term $(F_2^{NS})^{LT}(x_i, Q^2)$. Best fitted values of h_1 at different values of x for the F_2^{NS} structure functions are presented in Table 9.1 and Fig. 9.1 along with the $\frac{\chi^2}{d.o.f.}$ value.

x_i	h_1^{NNLO}
0.0125	-0.00397 ± 0.0025
0.0175	-0.00283 ± 0.0029
0.025	-0.0045 ± 0.0026
0.035	-0.0022 ± 0.0052
$\frac{\chi^2}{d.o.f.}$	0.85

Table 9.1: Higher Twist corrections to F_2^{NS} structure functions at NNLO.

In Fig. 9.1 we have presented the best fitting results of (9.4) for F_2^{NS} in comparison with NMC experimental data. Here both the NNLO results, with HT and without HT are shown. Significant higher twist contribution to F_2^{NS} structure function is observed in the low-x, low-Q^2 region. We observe that our expressions along with the

HT corrections provide better description of NMC data than without HT within our kinematical region of consideration.

9.2.2 Higher Twist Effect in xF_3^{NS} Structure Function

In a similar way, the parametrization

$$xF_3^{data}(x_i, Q^2) = xF_2^{LT}(x_i, Q^2) + \frac{h_1(x_i)}{Q^2}, \tag{9.5}$$

is used for the xF_3^{NS} structure function with higher twist contributions in terms of the parameter $h_1(x_i)$. Incorporating the NNLO result (5.51) as the LT term, we have fitted the parametrization with 9.5 with the CCFR, NuTeV, CHORUS and CDHSW data for the x-bins $x_i = 0.0125, 0.015, 0.0175, 0.025, 0.035, 0.045$. Best fitted values of h_1 at different values of x for the xF_3^{NS} structure functions are presented in Table 9.2 and Fig. 9.2 along with the $\frac{\chi^2}{d.o.f.}$ value.

x_i	h_1^{NNLO}
0.0125	0.064 ± 0.0258
0.015	0.00504 ± 0.00804
0.0175	0.0189 ± 0.034
0.025	0.00797 ± 0.0368
0.035	-0.0118 ± 0.0295
0.045	-0.0429 ± 0.0306
$\frac{\chi^2}{d.o.f.}$	1.03

Table 9.2: Higher Twist corrections to xF_3 structure functions at NNLO.

In Fig. 9.2 we have presented the best fitting results of (9.5) for xF_3^{NS} in comparison with CCFR experimental data. Here both the NNLO results, with HT and without HT are shown. Significant higher twist contribution to xF_3^{NS} structure function is observed in the low-x, low-Q^2 region. We observe that our expressions along with the HT corrections provide better description of CCFR data than without HT within our kinematical region of consideration.

9.3 Higher Twist Effect in Sum Rules

In the previous section, the higher twist effects in non-singlet structure functions are estimated by means of a simple model. We now extend the similar formalism in

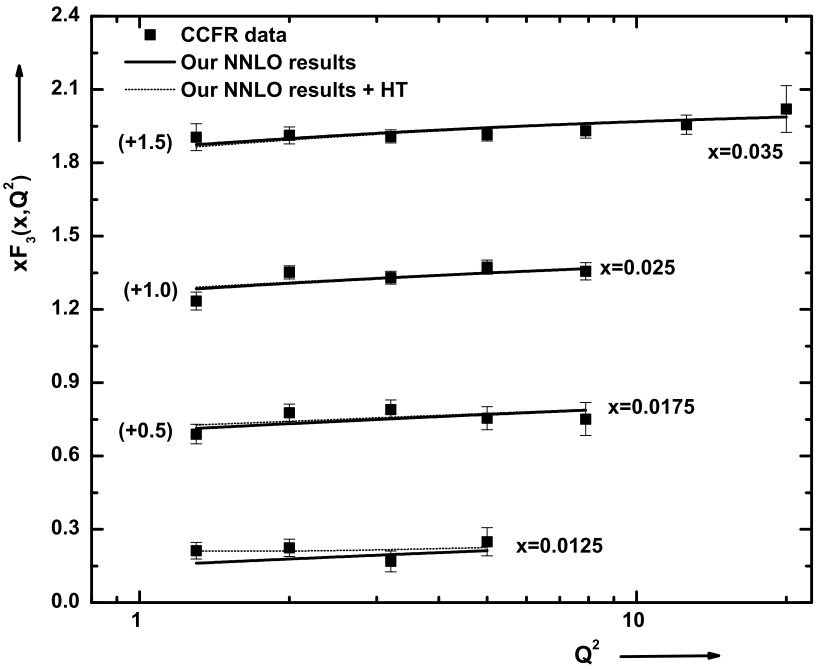

Figure 9.2: Higher twist corrections to xF_3^{NS} structure function at NNLO. (Q^2's are taken in the unit of GeV^2).

order to extract the higher twist contribution to the sum rules associated with the non-singlet structure functions. Here we have parameterized the sum rules as

$$S_i(Q^2) = S_i(Q^2)\Big|_{LT} + \frac{\mu_4}{Q^2}, \tag{9.6}$$

where leading twist(LT) term corresponds to the pQCD contribution to the respective sum rules and μ_4 signifies the contribution from first higher twist term. Our results for the sum rules, obtained in chapter 7 can be utilised as the LT term and then by means of fitting the model (9.6) with the low Q^2 ($0.5 \leq Q^2 \leq 5GeV^2$) experimental data taken from their respective experiments we can estimate the respective higher twist terms. In the following subsections we have presented the results of higher twist effects for Gross-Llewellyn Smith sum rule(GLSSR) and Bjorken sum rule(BSR). Due to unavailability sufficient experimental data, we could not include the Gottfried sum rule in this chapter.

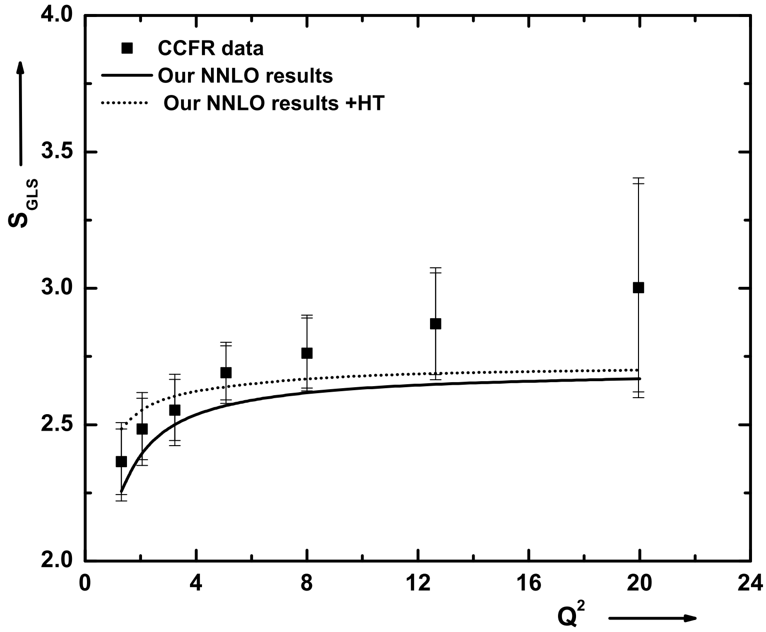

Figure 9.3: Hgher Twist corrections to GLS sum rule at NNLO. (Q^2's are taken in the unit of GeV^2).

9.3.1 Higher Twist Effect in Gross-Llewellyn Smith Sum Rule

The Gross-Llewellyn Smith sum rule(GLSSR), with the higher twist term, $\frac{\mu_4}{Q^2}$ is given by

$$S_{GLS}^{data}(Q^2) = S_{GLS}^{pQCD}(Q^2)\Big|_{LT} + \frac{\mu_4}{Q^2}. \tag{9.7}$$

Incorporating the results in accord with our NNLO prediction, (7.17) in (9.7), we have fitted the the expression with the available CCFR experimental data for GLSSR. The corresponding value of μ_4 for which best fitting is obtained in NNLO are summarised in Table 9.3 and depicted in Fig.9.3, along with the respective $\frac{\chi^2}{d.o.f}$ values.

	NNLO
μ_4	0.1840 ± 0.0842
$\frac{\chi^2}{d.o.f.}$	0.56

Table 9.3: Higher Twist corrections to GLS sum rule at NNLO.

In Fig. 9.3 we have presented the best fitting results of (9.7) for GLSSR in NNLO in comparison with CCFR experimental data. Our pQCD corrected results up to

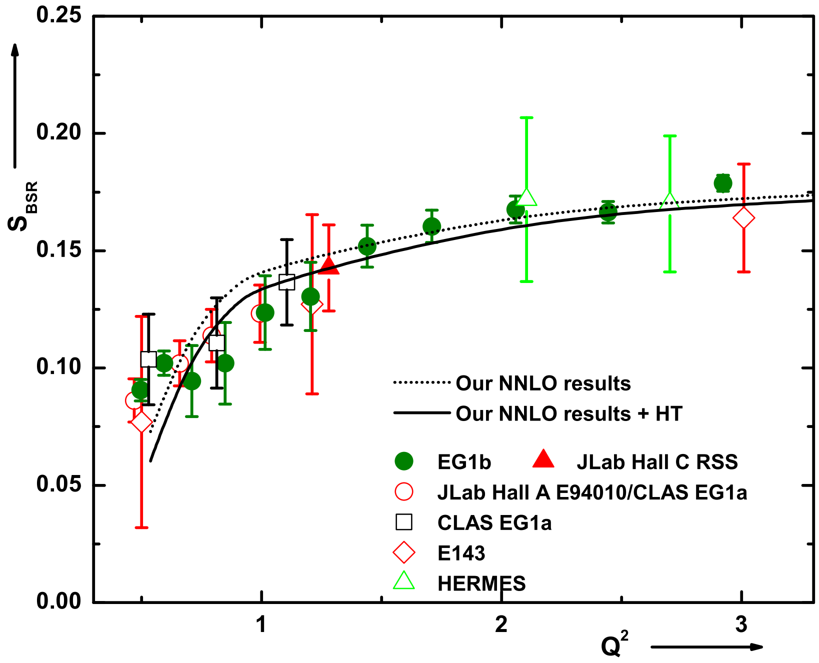

Figure 9.4: Hgher Twist corrections to BSR at NNLO. (Q^2's are taken in the unit of GeV^2).

NNLO are also included in this figure along with HT corrected results. We observe that our expressions along with the HT corrections provide better description of CCFR measurement of GLS sum rule.

9.3.2 Higher Twist Effect in Bjorken Sum Rule

The Bjorkan sum rule(BSR), with the higher twist term, $\frac{\mu_4}{Q^2}$ is given by

$$S_{BSR}^{data}(Q^2) = S_{BSR}^{pQCD}(Q^2)\Big|_{LT} + \frac{\mu_4}{Q^2}. \tag{9.8}$$

Incorporating our Q^2 dependent expressions (7.23) for BSR in NNLO as the LT term, we have fitted above parametrisation to the low Q^2 ($0.5 \leq Q^2 \leq 5GeV^2$) experimental data taken from COMPASS [71], HERMES[73], E143[75] and JLab experiments [76–78]. The corresponding value of μ_4 for which best fitting is obtained in NNLO is summarised in Table 9.4, along with the $\frac{\chi^2}{d.o.f.}$ value. In Fig. 9.4, we have presented the best fitting results in comparison with other experimental data.. Here both the results, with HT and without HT are shown. We observe that our expressions along with the HT corrections provide well description of BSR data.

	NNLO
μ_4	-0.007 ± 0.0024
$\frac{\chi^2}{d.o.f.}$	1.3

Table 9.4: Higher Twist corrections to BSR at NNLO.

9.4 Summary

In this chapter we have extracted the higher twist contribution to F_2^{NS} and xF_3 structure functions and to the GLSSR and BSR using a simple model. We then incorporated the higher twist contributions to our NNLO results for all of F_2^{NS}, xF_3, GLSSR and BSR. We observe that our NNLO expressions for these structure functions and sum rules along with the higher twist corrections provide well description of their respective experimental data. □□