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# Abstract

Particle physics is a quest for the fundamental building blocks of the matter. As more than 99.9 % of the mass of any object in our solar system is carried by its nucleons, exploring the structure of the nucleon is a vital part of our effort to understand the structure of matter. A complete understanding of nucleon structure is expected to achieve through coherent interplay of Deep Inelastic Scattering (DIS) experiments at all energy scale, together with a proper theoretical approach. Our most reliable knowledge concerning the internal structure of the nucleon has been achieved through DIS off high energy beams of electrons, muons and neutrinos. DIS processes are described in terms of the structure functions which are expressed as a functions of the momentum fraction  $x$  of the nucleon carried by the parton and the four momentum transfer squared  $Q^2$  and these functions are directly related to the distribution of quarks and gluons inside a nucleon. These DIS structure functions are the objects of intensive investigation both theoretically and experimentally in order to understand the underlying theory of strong interaction. With the recent developments of dedicated experimental facilities significant progresses have been observed in the field of experimental investigation of structure functions. Simultaneously, in this regard, tremendous progress is observed in the field of theoretical investigation with a variety of theoretical approaches.

Quantum Chromodynamics (QCD) and Regge theory are two important approaches in order to account for the strong interaction processes observed at high energy particle colliders. However, the predictive power of both approaches is limited. Specifically, Regge theory has its predictions on the  $x$  dependence of the structure functions within the Regge limit ( $x \rightarrow 0$ ,  $Q^2$  fixed and of the order of a typical hadronic scale) and QCD is successful in describing  $Q^2$  dependency of the structure functions in accord with DGLAP (Dokshitzer-Gribov-Lipatov-Altarelli-Parisi) evolution equations in the perturbative regime i.e., within the Bjorken limit ( $Q^2 \gg 1$ ,  $x$  fixed and not too small). However the most important region in DIS, which has attracted much interest recently is the small- $x$  region, lies between the interface of Bjorken limit and the Regge limit.

Despite limitations in themselves, the combination of QCD and Regge theory is expected to provide proper understanding of the structure functions, particularly in the small- $x$  region. In QCD the structure functions are governed by a set of

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integro-differential equations, known as the DGLAP equations. Due to its complicated mathematical structure, an exact analytic determination of the structure functions is currently out of reach and one needs to apply approximated methods to arrive on predictions from the theory. Therefore, in current analysis this set of equations are usually solved numerically by using an initial input distribution of the structure function at a fixed  $Q^2$ , in terms of some free parameters, the parameters are so adjusted that the parameterization best fit the existing data. In order to perform a fit one must start with a particular ansatz for the structure functions at some reference  $Q_0^2$ . In most of the existing fitting analysis, including those in the experimental papers it has been performed by assuming a simple power behavior based on Regge theory. Although many parameterizations are available in literature in order to predict the initial distribution of structure functions to DGLAP equation, but most of them are full of different constraints and suffer of several drawbacks. Therefore explorations of the possibility of obtaining accurate solutions of DGLAP evolution equations with less number of parameters are always interesting. Under this motivation, this thesis is devoted to the exploration of a semi-analytic approach of solving DGLAP equation for non-singlet structure functions using two Regge inspired model with less number of parameters. Here particular emphasis is given to the non-singlet structure functions because they are considered as the starting ground for theoretical description of DIS structure functions. Besides being interesting in themselves, another significant advantage is that QCD analysis by means of non-singlet structure functions is comparatively technically simpler. This thesis concerns with the usefulness of the combination of Regge theory and QCD in order to have reliable understanding of both the spin independent and spin dependent non-singlet structure functions and determination of various sum rules associated with them. Here we explicitly specify how the usefulness of two  $Q^2$  dependent Regge ansatz, utilized as the required initial input to the DGLAP evolution helps in obtaining the small- $x$  behaviour of the non-singlet structure functions,  $F_2^{NS}(x, Q^2)$ ,  $xF_3(x, Q^2)$  and  $xg_1^{NS}(x, Q^2)$ . Obtained small- $x$  behaviour of these non-singlet structure functions are then utilized to calculate the sum rules, Gottfried sum rule(GSR), Gross-Llewellyn Smith(GLS) sum rule and Bjorken Sum Rule (BSR), which are associated to  $F_2^{NS}(x, Q^2)$ ,  $xF_3(x, Q^2)$  and  $xg_1^{NS}(x, Q^2)$  respectively. In addition to the prediction of structure functions and sum rules we have paid attention to their precision. Precise prediction of structure functions demand

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to incorporate the standard higher order approximation of pQCD and several non-perturbative effects. In this regard particular emphasis is given to the determination of structure functions and sum rules with pQCD corrections up to next-next-to-leading order (NNLO) and to the inclusion of the special non-perturbative effects, shadowing effect and higher twist effect.

The outline of the thesis is as follows:

In **Chapter 1**, we have given a brief introduction to our current views of the basic building blocks of matter, deep inelastic scattering, structure functions, parton model, Regge theory and Quantum Chromodynamics and higher order corrections, various sum rules, non-perturbative QCD corrections such as nuclear effect, higher twist effect etc.

**Chapter 2** provides a general overview about the recent lepton deep inelastic scattering measurements which have enriched our phenomenological analysis performed in this thesis. Specifically the experimental results for non-singlet structure functions and associated sum rules for both polarized and unpolarized cases measured in electron, muon and neutrino DIS are reviewed. In addition, several parametrization associated with the determination of non-singlet structure functions are discussed.

In **Chapter 3**, along with a qualitative analysis of the available methods to solve DGLAP equation, I have allude the usefulness of two  $Q^2$  dependent Regge ansatz in solving DGLAP equation in order to have the small- $x$  behaviour of both the spin independent and spin dependent non-singlet structure functions. By means of fitting analysis, we have investigated the compatibility of the two ansatz with the available experimental data and then studied the possible role played by them in evolving the non-singlet structure functions in accord with DGLAP equation.

**Chapter 4** encompasses the evolution of the non-singlet structure function  $F_2^{NS}(x, Q^2)$  in charged lepton DIS by means of solving the DGLAP equations in LO, NLO and NNLO using the Regge ansatz as the initial input. Both the  $Q^2$  and  $x$  evolutions of  $F_2^{NS}(x, Q^2)$  structure functions, thus obtained are analysed phenomenologically in comparison with the experimental measurements taken from NMC and the results of NNPDF parametrization.

**Chapter 5** concerns with the determination of small- $x$  behaviour of the non-singlet structure function,  $x F_3(x, Q^2)$  originated in neutrino scattering. The DGLAP equation is solved up to NNLO for  $x F_3(x, Q^2)$  structure function and solutions are

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compared with the experimental data taken from CCFR, NuTeV, CDHSW and CHORUS experiments and also with the recent MSTW parametrization results.

**Chapter 6** deals with the understanding of the spin dependent non-singlet structure function  $xg_1^{NS}(x, Q^2)$  within small- $x$  region. The DGLAP equation is solved to have the  $Q^2$  as well as  $x$  evolution of  $xg_1^{NS}(x, Q^2)$  structure function with QCD corrections up to NNLO and perform a phenomenological analysis of our results in comparison with different experimental data taken from COMPASS, HERMES, E143 and JLab experiments, along with other available theoretical as well as phenomenological analysis.

**Chapter 7** utilises the small- $x$  behaviour of  $F_2^{NS}(x, Q^2)$ ,  $xF_3(x, Q^2)$  and  $xg_1^{NS}(x, Q^2)$  structure functions obtained in the previous chapters in prediction of sum rules associated with them, viz., Gottfried sum rule, Gross-Llewellyn Smith sum rule and Bjorken sum rule respectively. These sum rules are calculated incorporating higher order pQCD corrections up to NNLO and analysed phenomenologically by comparing with their respective experimental results and available theoretical predictions.

In **Chapter 8**, we present an analysis of the non-singlet structure functions and related sum rules taking into account the nuclear effects. In this regard, special attention is given to the nuclear shadowing effect as we are mostly concerning with the small- $x$  region. The corrections due to nuclear shadowing effect, predicted in several earlier analysis are incorporated to our results of structure function and sum rules for free nucleon and calculate the nuclear structure functions as well as sum rules for nuclei. The calculations are analysed phenomenologically in comparison with available experimental data and achieved at a very good phenomenological success in this regard.

In **Chapter 9**, the higher twist corrections to the non-singlet structure functions and sum rules associated with them are studied. Here, possible improvement in the accuracy of our results for the non-singlet structure functions and sum rules due to the inclusion of relevant higher twist terms is investigated. Based on a simple model we have extracted the higher twist contributions to the non-singlet structure functions and sum rules in NNLO perturbative orders and then incorporated them with our results. Our NNLO results along with higher twist corrections are observed to be compatible with experimental data..

Finally in **Chapter 10**, We have presented the overall conclusion drawn from our work. □□