Appendices

A. Unpolarised Non-singlet Splitting Function in Leading Order(LO)

The explicit form of the unpolarised non-singlet splitting function in LO is

$$P^{(0)}(x) = C_F\left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x)\right], \tag{10.1}$$

where the + sign is defined by

$$\int_0^1 dx \frac{f(x)}{(1-x)_+} = \int_0^1 \frac{dx}{1-x}[f(x)-f(1)] + f(1)\ln(1-x) \tag{10.2}$$

B. Unpolarised Non-singlet Splitting Function in Next-to-Leading Order(NLO)

The unpolarised non-singlet splitting function in NLO is given by

$$\begin{aligned} P^{(1)}(x) = C_F^2\left[P_F(x) - P_A(x) + \delta(1-x)\{\frac{3}{8} - \frac{1}{2}\pi^2 + \zeta(3) - 8\tilde{S}(\infty)\}\right] \\ +\frac{1}{2}C_F C_A\left[P_G(x) + P_A(x) + \delta(1-x)\left\{\frac{17}{12} + \frac{11}{9}\pi^2 - \zeta(3) + 8\tilde{S}(\infty)\right\}\right] \\ +C_F T_R N_F\left[P_{N_F}(x) - \delta(1-x)\left\{\frac{1}{6} + \frac{2}{9}\pi^2\right\}\right] \end{aligned} \tag{10.3}$$

where,

$$\begin{aligned} P_F(x) = -2\frac{1+x^2}{1-x}\ln x\ln(1-x) - \left(\frac{3}{1-x} + 2x\right)\ln x - \frac{1}{2}(1+x)\ln^2 x \\ -5(1-x), \end{aligned} \tag{10.4}$$

$$\begin{aligned} P_G(x) = \frac{1+x^2}{(1-x)_+}\left[\ln^2 x + \frac{11}{3}\ln x + \frac{67}{9} - \frac{1}{3}\pi^2\right] + 2(1+x)\ln x \\ +\frac{40}{3}(1-x), \end{aligned} \tag{10.5}$$

$$P_{N_F}(x) = \frac{2}{3}\left[\frac{1+x^2}{(1-x)_+}(-\ln x - \frac{5}{3}) - 2(1-x)\right], \qquad (10.6)$$

$$P_A(x) = 2\frac{1+x^2}{1+x}\int_{x/(1+x)}^{1/(1+x)} \frac{dz}{z}\ln\frac{1-z}{z} + 2(1+x)\ln x + 4(1-x) \qquad (10.7)$$

C. Unpolarised Non-singlet Splitting Function in Next-to-Next-to-Leading Order(NNLO)

Unpolarised non-singlet splitting function in NNLO has the form,

$$\begin{aligned} P^{(2)}(x) = N_F\Big[& -183.187D_0 - 173.927\delta(1-x) - \frac{5120}{81}L_1 - 197.0 \\ & +381.1x + 72.94x^2 + 44.79x^3 - 1.497xL_0^3 - 56.66L_0L_1 \\ & -152.6L_0 - \frac{2608}{81}L_0^2 - \frac{64}{27}L_0^3\Big] \\ +N_F^2\frac{64}{81}\Big[& -D_0 - \left(\frac{51}{16} + 3\zeta_3 - 5\zeta_2\right)\delta(1-x) + \frac{x}{1-x}L_0\left(\frac{3}{2}+5\right) \\ & +1 + (1-x)\left(6 + \frac{11}{2}L_0 + \frac{3}{4}L_0^2\right)\Big] \end{aligned} \qquad (10.8)$$

D. Polarised Non-singlet Splitting Function in Leading Order(LO)

In leading order the polarised non-singlet splitting function is given by

$$\Delta P_{ns}^{(0)}(x) = 2C_F\left(\Delta p_{qq}(x) + \frac{3}{2}\delta(1-x)\right) \qquad (10.9)$$

Here

$$\Delta p_{qq}(x) = \frac{2}{1-x} - 1 - x. \qquad (10.10)$$

E. Polarised Non-singlet Splitting Function in Next-to-Leading Order(NLO)

$$\begin{aligned}\Delta P_{NS}^{+(1)}(x) = 4C_F^2\Big[&2\Delta p_{qq}(-x)(\zeta_3 + 2H_{-1,0} - H_{0,0}) + 2\Delta p_{qq}(H_{1,0} \\ &+H_2 - 3/4H_0) - 9(1-x) - (1+x)H_{0,0} - 1/2(7+11x)H_0 \\ &+\delta(1-x)(3/8 + 6\zeta_3 - 3\zeta_2)\Big] \\ +4C_AC_F\Big[&-\Delta p_{qq}(-x)(\zeta_2 + 2H_{-1,0} - H_{0,0}) + \Delta p_{qq}(x)(H_{0,0} \\ &+11/3H_0 - \zeta_3 + 67/18) + 26/3(1-x) + 2(1+x)H_0 \\ &+\delta(1-x)(17/24 - 3\zeta_3 + 11/3\zeta_2)\Big] + 4/3C_FN_F \\ \Big[&-\Delta p_{qq}(x)(5/3 + H_0) - 2(1-x) - \delta(1-x)(1/4 + 2\zeta_2)\Big], \end{aligned} \tag{10.11}$$

with

$$\Delta p_{qq}(x) = \frac{2}{1-x} - 1 - x. \tag{10.12}$$

F. Polarised Non-singlet Splitting Function in Next-to-Next-to-Leading Order(NNLO)

$$\begin{aligned}P_{NS}^{+(2)}(x) \cong\ & 1174898D_0 + 1295.470\delta(1-x) + 714.1L_1 + 1860.2 \\ &-3505x + 297.0x^2 - 433.2x^3 + L_0L_1(684 + 251.2L_0) \\ &+1465.2L_0 + 399.2L_0^2 + 320/9L_0^3 + 116/81L_0^4 \\ +N_F\Big[&-183.187D_0 - 173.927\delta(1-x) - \frac{5120}{81}L_1 - 197.0 \\ &+381.1x + 72.94x^2 + 44.79x^3 - 1.497xL_0^3 - 56.66L_0L_1 \\ &-152.6L_0 - \frac{2608}{81}L_0^2 - \frac{64}{27}L_0^3\Big] + N_F^2\frac{64}{81}\Big[-D_0 - \Big(\frac{51}{16} + \\ &3\zeta_3 - 5\zeta_2\Big)\delta(1-x) + \frac{x}{1-x}L_0\Big(\frac{3}{2} \\ &+5\Big) + 1 + (1-x)\Big(6 + \frac{11}{2}L_0 + \frac{3}{4}L_0^2\Big)\Big] \end{aligned} \tag{10.13}$$

Here the following abbreviations are used,

$$D_0 = \frac{1}{(1-x)_+}, \quad L_1 = \ln(1-x), \quad L_0 = \ln x. \tag{10.14}$$