Chapter 8

Conclusion and Outlook

In this thesis we have examined the behaviour of DIS structure functions in the framework of both linear DGLAP and non-linear GLR-MQ evolution equations at small-x. The small-x behavior of quark and gluon densities, where x is the Bjorken scaling variable, is one of the challenging issues of QCD. A key discovery of the past years is the prevalent role of gluons with very small fractional momentum x in nucleons when observed by a high energy probe. On that account, the study of lepton-nucleon DIS or in particular the determination of the gluon density in the region of small-x is of great significance. The increase of energy generates a rapid growth of the gluon density in the limit $x \to 0$ which is eventually expected to saturate in order to preserve unitarity. Accordingly, the corrections of the higher order QCD effects, which suppress or shadow the growth of the parton densities, have been rigorously studied in the last few years.

The linear DGLAP evolution equations are the standard and the basic theoretical tools to explore the scale dependence of the PDFs and ultimately the DIS structure functions are. In part I of this thesis we have solved the DGLAP equations for the singlet and non-singlet structure functions, as well as the gluon distribution function at LO, NLO and NNLO respectively in an analytical approach by using the Taylor series expansion method. The Taylor series expansion transforms the integro-differential DGLAP equations into first order partial differential equations which are much easier to solve. The resulting equations are then solved by the Lagrange's auxiliary method to obtain Q^2 and x evolutions of the singlet and non-singlet structure functions and the gluon distribution functions. We have also calculated the Q^2 and x evolutions of

deuteron structure function as well as the Q^2 evolution of proton structure function from the solutions of the singlet and non-singlet structure functions. We compare our predictions of deuteron and proton structure function with the NMC data, E665 data, H1 data as well as with the results of NNPDF parametrization. Our results show that at fixed x the structure functions increase with increasing Q^2 whereas at fixed Q^2 the structure functions decrease as x decreases which is in agreement with perturbative QCD fits at small-x. We further observe that our computed results can explain the general trend of data in a decent manner in the kinematic region $10^{-3} < x < 10^{-1}$ and $0.5 \leq Q^2 \leq 40$ GeV^2. On the other hand, our results of gluon distribution function obtained by solving DGLAP equation are compared with the GRV1998NLO, MRST2004NNLO, MSTW2008NNLO and JR09NNLO global QCD analysis as well as with the BDM model. The obtained results can be described within the framework of perturbative QCD. We perform our analysis in the x and Q^2 range, viz. $10^{-4} \leq x \leq 0.1$ and $5 \leq Q^2 \leq 110$ GeV^2 and find that in this domain our predictions are comparable with different global analysis of parton distributions. It is observed from our phenomenological analysis that the inclusion of the NNLO contributions provides better agreement of our results with the experimental data and parametrizations. The Taylor series expansion is a very feasible and convenient method for analytical solution of DGLAP equations. We have considered some numerical parameters to obtain the solution of DGLAP equations, however the number of parameters are less compared to the numerical. Moreover, this approach also enables us to calculate the x-evolution of deuteron structure function in addition to the Q^2-evolution. Even though various numerical methods are available in order to obtain the solution of DGLAP evolution equations, but it is always interesting to obtain an analytical solution and in this regard the Taylor series expansion method is a good alternative.

In the very small-x region the growth of the gluon distribution is incredibly enunciated. Accordingly at small-x the likelihood of interaction between two gluons can no longer be overlooked and therefore, gluon recombination will be as important as gluon splitting. So the standard linear DGLAP evolution equation will have to be modified in order to include the the modifications due to the correlations among initial gluons to the evolutionary amplitude. A traditional tool in this research is the

GLR-MQ equation that takes into account the nonlinear corrections arising from the recombination of two gluon ladders into one gluon. In part II of this thesis we have made an deliberate attempt to explore the higher order QCD effects of the gluon recombination processes at very small-x in the framework of nonlinear GLR-MQ equation. We have solved the GLR-MQ equation in the leading twist approximation in a semi-analytical approach by employing the well-known Regge-like ansatz with considerable phenomenological success. We have investigated the behavior of the gluon distributions in the vicinity of saturation region. Our resulting gluon distributions are compared with different global QCD fits to the parton distribution functions, viz. GRV1998LO, GJR2008LO, MRST2001LO, MSTW2008LO, NNPDF, HERAPDF0.1, CT10 as well as with the H1 experimental data, and are found to be quite compatible. Furthermore, we present a comparative analysis of our computed results with the results of the EHKQS and BZ models. We have examined how the inclusion of nonlinear effects changes the behavior of gluon density and it is interesting to observe that although the gluon distribution increases with increasing Q^2 and decreasing x, but the rapid growth of gluon densities is tamed due to shadowing corrections as x grows smaller. This indicates that the gluon distributions unitarize leading to the restoration of Froissart bound in the small-x region. This tamed behaviour of gluon density is observed to be more the the hot-spots when the correlation radius between two interacting gluons is of the order of the transverse size of a valance quark, i.e. $R = 2 \text{ GeV}^{-1}$. We have further checked the effect of shadowing corrections in our results by comparing the gluon distributions obtained in the nonlinear GLR-MQ approach with those obtained in the linear DGLAP approach. Careful investigation of our results indicates that the nonlinear effects or shadowing corrections, emerged as a result of recombination of two gluon ladders, play a significant role on QCD evolution for gluon distribution in the kinematic region of small-x ($10^{-5} \leq x \leq 10^{-2}$) and moderate Q^2 ($1 \leq Q^2 \leq 30 \text{ GeV}^2$).

We have also obtained a semi analytical solution of the GLR-MQ equation for sea quark distribution in leading twist approximation using the Regge like ansatz. The solution of the GLR-MQ equation for singlet structure function with shadowing corrections is found to be legitimate in the kinematic domain $10^{-4} \leq x \leq 10^{-1}$ and $0.6 \leq Q^2 \leq 30 \text{ GeV}^2$. We have examined the effect of shadowing corrections on the

small-x and moderate-Q^2 behaviour of singlet structure function and compared our predictions with the NMC and E665 experimental data as well with the NNPDF collaboration. Our predictions are found to show the general trend of experimental data and parametrization, nevertheless with the inclusion of the nonlinear terms, the behaviour of singlet structure function is slowed down towards small-x leading to a restoration of the Froissart bound. Moreover we note that in the small-x region the logarithmic derivative of the singlet structure function has a tamed behavior related to shadowing corrections due to gluon recombination.

We have further made a comparative analysis of our predictions obtained in the framework of GLR-MQ equation in a semi-analytical approach with the results of the MD-DGLAP and BK equations. It is very fascinating to note that the predictions of nonlinear gluon density obtained from the GLR-MQ equation are in a very good agreement with the results of the BK equation. Our results are also found to almost comparable with those of the MD-DGLAP equation but with a completely different slope. The MD-DGLAP equation predicts a steeper gluon distribution caused by strong antishadowing effect, whereas a flatter gluon distribution is observed in our predictions due to significant shadowing corrections at small-x.

As a future prospect, this work encourages a more detailed study of the properties of the high density parton system. The GLR-MQ equation only includes the first non-linear term reporting the recombination of two gluon ladders into one. Therefore although it predicts saturation in the asymptotic regime, but its validity does not extend to very high density regime where significant contributions from the higher twist effects should be taken into account. Moreover, the suggested Regge type solution of the GLR-MQ equation has a limited range of validity. Nevertheless for more reliable predictions beyond this range, towards much smaller-Q^2 or smaller-x, further analysis is required incorporating the evolution dynamics at higher order. It will be interesting to study the other nonlinear equations relevant at high gluon density.

Appendices

Appendix A

The explicit forms of the functions $A_i(x)$, $B_i(x)$ and $C_i(x)$ (where i=1,2,3,4) are

$$A_1(x) = 2x + x^2 + 4\ln(1-x), \tag{1}$$

$$A_2(x) = x - x^3 - 2x\ln(x), \tag{2}$$

$$A_3(x) = 2N_f\left(\frac{2}{3} - x + x^2 - \frac{2}{3}x^3\right), \tag{3}$$

$$A_4(x) = 2N_f\left(-\frac{5}{3}x + 3x^2 - 2x^3 + \frac{2}{3}x^4 - x\ln(x)\right), \tag{4}$$

$$B_1(x) = x\int_0^1 f(\omega)d\omega - \int_0^x f(\omega)d\omega + \frac{4}{3}N_f\int_x^1 F_{qq}(\omega)d\omega, \tag{5}$$

$$B_2(x) = x\int_x^1 \left[f(\omega) + \frac{4}{3}N_f F_{qg}^s(\omega)\right]\frac{1-\omega}{\omega}d\omega, \tag{6}$$

$$B_3(x) = \int_x^1 F_{qg}^S(\omega)d\omega, \tag{7}$$

$$B_4(x) = x\int_x^1 \frac{1-\omega}{\omega}F_{qg}^S(\omega)d\omega, \tag{8}$$

where the functions $f(\omega)$, $F_{qq}(\omega)$ and $F_{qg}^S(\omega)$ are defined in Appendix B. Again,

$$C_1(x) = N_f\int_0^{1-x} \frac{\omega d\omega}{1-\omega}R_1(\omega), \tag{9}$$

$$C_2(x) = N_f\int_0^{1-x} \frac{\omega x d\omega}{(1-\omega)^2}R_1(\omega), \tag{10}$$

$$C_3(x) = N_f\int_0^{1-x} \frac{\omega x d\omega}{(1-\omega)^2}R_1(\omega), \tag{11}$$

$$C_4(x) = N_f\int_0^{1-x} \frac{\omega x d\omega}{(1-\omega)^2}R_2(\omega), \tag{12}$$

with,

$$
\begin{aligned}
R_1(\omega) = & \{\ln(\omega)\ln(1-\omega)[-173.1 + 46.18\ln(1-\omega)] + 178.04\ln(1-\omega) \\
& + 6.892\ln^2(1-\omega) + \frac{40}{27}[\ln^4(1-\omega) - 2\ln^3(1-\omega)]\} + \omega\{\ln(\omega) \\
& (-163.9(1-\omega)^{-1} - 7.208(1-\omega)) + 151.49 + 44.51(1-\omega) \\
& -43.12(1-\omega)^2 + 4.82(1-\omega)^3\} + \omega^2\{-5.926\ln^3(\omega) \\
& -9.751\ln^2(\omega) - 72.11\ln(\omega) + 177.4 + 392.9(1-\omega) \\
& -101.4(1-\omega)^2 - 57.04\ln(1-\omega)\ln(\omega) - 661.6\ln(1-\omega) \\
& +131.4\ln^2(1-\omega) - \frac{400}{9}\ln^3(1-\omega) + \frac{160}{27}\ln^4(1-\omega) \\
& -506.0(1-\omega)^{-1} - \frac{3584}{27}(1-\omega)^{-1}\ln(1-\omega)\} + N_f\omega\{1.778\ln^2(\omega) \\
& +5.944\ln(\omega) + 100.1 - 125.2(1-\omega) + 49.26(1-\omega)^2 \\
& -12.59(1-\omega)^3 - 1.889\ln(1-\omega)\ln(\omega) + 61.75\ln(1-\omega) \\
& +17.89\ln^2(1-\omega) + \frac{32}{27}\ln^3(1-\omega) + \frac{256}{81}(1-\omega)^{-1}\} \quad (13)
\end{aligned}
$$

$$
\begin{aligned}
R_2(\omega) = & \{\frac{100}{27}\ln^4(\omega) - \frac{70}{9}\ln^3(\omega) - 120.5\ln^2(\omega) + 104.42\ln(\omega) + 2522 \\
& -3316(1-\omega) + 2126(1-\omega)^2 - 252.5(1-\omega)\ln^3(1-\omega) \\
& + \ln(\omega)\ln(1-\omega)\big(1823 - 25.22\ln(1-\omega)\big) + 424.9\ln(1-\omega) \\
& +881.5\ln^2(1-\omega) - \frac{44}{3}\ln^3(1-\omega) + \frac{536}{27}\ln^4(1-\omega) - 1268.3 \\
& (1-\omega)^{-1} - \frac{896}{3}(1-\omega)^{-1}\ln(1-\omega)\} + N_f\{\frac{20}{27}\ln^3(\omega) + \frac{200}{27}\ln^2(\omega) \\
& -5.496\ln(\omega) - 252.0 + 158.0(1-\omega) + 145.4(1-\omega)^2 \\
& -139.28(1-\omega)^3 - 98.07(1-\omega)\ln^2(1-\omega) + 11.70(1-\omega) \\
& \times \ln^3(1-\omega) - \ln(\omega)\ln(1-\omega)(53.09 + 80.616\ln(1-\omega)) \\
& -254.0\ln(1-\omega) - 90.80\ln^2(1-\omega) - \frac{376}{27}\ln^3(1-\omega) \\
& -\frac{16}{9}\ln^4(1-\omega) + \frac{1112}{243}(1-\omega)^{-1}\} \quad (14)
\end{aligned}
$$

Appendix B

The functions involved in the DGLAP equations for singlet and non-singlet structure functions at NLO are

$$f(\omega) = C_F^2[P_F(\omega) - P_A(\omega)] + \frac{1}{2}C_F C_A[P_G + P_A(\omega)] + C_F T_R N_f P_{N_f}(\omega), \qquad (15)$$

$$F_{qq}^S(\omega) = 2C_F T_R N_f F_{qq}(\omega), \qquad (16)$$

$$F_{qg}^S(\omega) = C_F T_R N_f F_{qg}^1(\omega) + C_G T_R N_f F_{qg}^2(\omega) \qquad (17)$$

where,

$$F_{qq}(\omega) = \frac{20}{9\omega} - 2 + 6\omega - \frac{56}{9}\omega^2 + \left(1 + 5\omega + \frac{8}{3}\omega^2\right)\ln(\omega) - (1+\omega)\ln^2(\omega), \qquad (18)$$

$$\begin{aligned}
F_{qg}^1(\omega) &= 4 - 9\omega - (1-4\omega)\ln(\omega) - (1-2\omega)\ln^2(\omega) + 4\ln(1-\omega) \\
&+ \left[2\ln^2(\frac{1-\omega}{\omega}) - 4\ln(\frac{1-\omega}{\omega}) - \frac{2}{3}\pi^2 + 10\right]P_{qg}^1(\omega), \qquad (19)
\end{aligned}$$

$$\begin{aligned}
F_{qg}^2(\omega) &= \frac{182}{9} + \frac{14}{9}\omega + \frac{40}{9\omega} + \left(\frac{136}{3}\omega - \frac{38}{3}\right)\ln(\omega) - 4\ln(1-\omega) \\
&- (2+8\omega)\ln^2(\omega) + \left[-\ln^2(\omega) + \frac{44}{3}\ln(\omega) - 2\ln^2(1-\omega) \right. \\
&\left. + 4\ln(1-\omega) + \frac{\pi^2}{3} - \frac{218}{3}\right]P_{qg}(\omega) \\
&+ 2P_{qg}(-\omega)\int_{\frac{\omega}{1+\omega}}^{\frac{1}{1+\omega}} \frac{dz}{z}\ln\frac{1-z}{z}, \qquad (20)
\end{aligned}$$

Here, the Casimir operators of the color group SU(3) are defined as $C_G \equiv N_C = 3$, $C_F = \frac{N_c^2 - 1}{2N_c} = \frac{4}{3}$ and $T_R = \frac{1}{2}$.

$$P_{N_f}(\omega) = \frac{2}{3}\left[\frac{1+\omega^2}{1-\omega}(-\ln\omega - \frac{5}{3}) - 2(1-\omega)\right], \qquad (21)$$

$$\begin{aligned}
P_F(\omega) &= -\frac{2(1+\omega^2)}{(1-\omega)}\ln(\omega)\ln(1-\omega) - \left(\frac{3}{1-\omega} + 2\omega\right)\ln\omega - \frac{1}{2}(1+\omega)\ln\omega \\
&+ \frac{40}{3}(1-\omega), \qquad (22)
\end{aligned}$$

$$\begin{aligned}
P_G(\omega) &= \frac{(1+\omega^2)}{(1-\omega)}\left(\ln^2(\omega) + \frac{11}{3}\ln(\omega) + \frac{67}{9} - \frac{\pi^2}{3}\right) - \frac{1}{2}(1+\omega)\ln\omega \\
&+ \frac{40}{3}(1-\omega), \qquad (23)
\end{aligned}$$

$$P_A(\omega) = \frac{2(1+\omega^2)}{(1+\omega)} \int_{\left(\frac{\omega}{1+\omega}\right)}^{\left(\frac{1}{1+\omega}\right)} \frac{dk}{k} \ln\left(\frac{1-k}{k}\right) + 2(1+\omega)\ln(\omega) + 4(1-\omega). \quad (24)$$

Appendix:C

The functions involved in the DGLAP equations for singlet and non-singlet structure functions at NNLO are given below.

The three-loop quark-quark splitting function is

$$P_{qq}^2 = P_{NS}^2 + P_{PS}^2, \quad (25)$$

The third-order pure-singlet contribution to the quark-quark splitting function is

$$
\begin{aligned}
P_{PS}^{(2)}(x) \simeq & \left[N_f(-5.92L_1^3 - 9.751L_1^2 - 72.11L_1 + 177.4 + 392.9x - 101.4x^2 \right. \\
& -57.04L_0L_1 - 661.61L_0 + 131.4L_0^2 - \frac{400}{9}L_0^3 + \frac{160}{27}L_0^4 - 506.0x^{-1} \\
& -\frac{3584}{27}x^{-1}L_0) + N_f^2(1.778L_1^2 + 5.944L_1 + 100.1 - 125.2x + 49.26x^2 \\
& \left. -12.59x^3 - 1.889L_0L_1 + 61.75L_0 + 17.89L_0^2 + \frac{32}{27}L_0^3 + \frac{256}{81}x^{-1}) \right](1-x),
\end{aligned}
$$
$$(26)$$

with $L_0 = \ln(x)$, $L_1 = \ln(1-x)$.

The non-singlet splitting function calculated upto third order is given by

$$
\begin{aligned}
P_{NS}^{(2)}(x) = & N_f \left[\{L_1(-163.9x^{-1} - 7.208x) + 151.49 + 44.51x - 43.12x^2 \right. \\
& +4.82x^3\}(1-x) + L_0L_1(-173.1 + 46.18L_0) + 178.04L_0 \\
& \left. +6.892L_0^2 + \frac{40}{27}(L_0^4 - 2L_0^3) \right]. \quad (27)
\end{aligned}
$$

The three-loop quark-gluon splitting function is

$$
\begin{aligned}
P_{qg}^{(2)}(x) \simeq & N_f\left(\frac{100}{27}L_1^4 - \frac{70}{9}L_1^3 - 120.5L_1^2 + 104.42L_1 + 2522 - 3316x + 2126x^2 \right. \\
& +L_0L_1(1823 - 25.22L_0) - 252.5xL_0^3 + 424.9L_0 + 881.5L_0^2 - \frac{44}{3}L_0^3 \\
& +\frac{536}{27}L_0^4 - 1268.3x^{-1} - \frac{896}{3}x^{-1}L_0) + N_f^2(\frac{20}{27}L_1^3 + \frac{200}{27}L_1^2 - 5.496L_1 \\
& -252.0 + 158.0x + 145.4x^2 - 98.07xL_0^2 + 11.70xL_0^3 - L_0L_1(53.09 \\
& \left. +80.616L_0) - 254.0L_0 - 90.80L_0^2 - \frac{376}{27}L_0^3 - \frac{16}{9}L_0^4 + \frac{1112}{243}x^{-1}\right).
\end{aligned}
$$
$$(28)$$

Appendix:D

The explicit forms of the functions $A_i^g(x)$, $B_i^g(x)$ $(i=1,2,3,4)$ and $C_i^g(x)$ $(i=1,2)$ are

$$A_1^g(x) = -\frac{11}{6} + 2x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \ln(x), \tag{29}$$

$$A_2^g(x) = 1 + \frac{4}{3}x - 3x^2 + x^3 - \frac{1}{4}x^4 + 2x\ln(x). \tag{30}$$

$$A_3^g(x) = \frac{2}{9}(-\frac{3}{2} + 2x - \frac{1}{2}x^2 - 2\ln(x)), \tag{31}$$

$$A_4^g(x) = \frac{2}{9}(2 + \frac{1}{2}x - 3x^2 + \frac{1}{2}x^3 + 4x\ln(x), \tag{32}$$

$$B_1^g(x) = -\frac{52}{3}\ln(x), \tag{33}$$

$$B_2^g(x) = -\frac{52}{3}(1 - x + x\ln(x)), \tag{34}$$

$$B_3^g(x) = \int_x^1 A(\omega)d\omega, \tag{35}$$

$$B_4^g(x) = x\int_x^1 \frac{1-\omega}{\omega}A(\omega)d\omega, \tag{36}$$

$$C_1^g(x) = \int_x^1 P_{gg}^2(\omega)d\omega, \tag{37}$$

$$C_2^g(x) = x\int_x^1 \frac{1-\omega}{\omega}P_{gg}^2(\omega). \tag{38}$$

Here, the functions $A(\omega)$ and $P_{gg}^2(\omega)$ are defined in Appendices E and F respectively.

Appendix:E

The functions involved in the DGLAP equations for gluon distribution functions at NLO are

$$\begin{aligned}
P_{gg}^1(\omega) &= C_F T_F(-16 + 8z + \frac{20}{3}z^2 + \frac{4}{3}z - (6 + 10z)\ln(z) - (2 + 2z)lnz^2)\\
&\quad + C_A T_F(2 - 2z + \frac{26}{9}(z^2 - 1/z) - \frac{4}{3}(1 + z)\ln(z) - \frac{20}{9}P_{gg}(z))\\
&\quad + C_A^2(\frac{27}{2}(1 - z) + \frac{26}{9}(z^2 - 1/z) - (\frac{25}{3} - \frac{11}{3}z + \frac{44}{3}z^2)\ln(z)\\
&\quad + 4(1 + z)\ln(z^2) + 2P_{gg}(-z)S_2(z) + (\frac{67}{9} - 4\ln(z)\ln(1 - z)\\
&\quad + \ln(z^2) - \frac{\pi^2}{3})P_{gg}(z)).
\end{aligned} \tag{39}$$

$$A(\omega) = C_F^2 A_1(\omega) + C_F C_G A_2(\omega) + C_F T_R N_F A_3(\omega) \tag{40}$$

where

$$
\begin{aligned}
A_1(\omega) &= -\frac{5}{2} - \frac{7}{2}\omega + (2 + \frac{7}{2}\omega) + (-1 + \frac{\omega}{2})\ln^2\omega - 2\omega.\ln(1-\omega) \\
&\quad + (-3\ln(1-\omega) - \ln^2(1-\omega))\frac{1 + (1-\omega)^2}{\omega},
\end{aligned}
\tag{41}
$$

$$
\begin{aligned}
A_2(\omega) &= \frac{28}{9} + \frac{65}{18}.\omega + \frac{44}{9}\omega^2 + (-12 - 5\omega - \frac{8}{3}\omega^2)\ln\omega + (4+\omega)\ln^2\omega \\
&\quad + 2\omega\ln(1-\omega) + (-2\ln\omega\ln(1-\omega) + \frac{1}{2}\ln^2\omega + \frac{11}{3}\ln(1-\omega) \\
&\quad + \ln^2(1-\omega) - \frac{1}{6}\pi^2 + \frac{1}{2})\frac{1 + (1-\omega)^2}{\omega} \\
&\quad - \frac{1 + (1+\omega)^2}{\omega}\int_{\omega/1+\omega}^{1/1+\omega}\frac{dz}{z}\ln(\frac{1-z}{z}),
\end{aligned}
\tag{42}
$$

$$A_3(\omega) = -\frac{4}{3}\omega - (\frac{20}{9} + \frac{4}{3}\ln(1-\omega))(\frac{1 + (1-\omega)^2}{\omega}). \tag{43}$$

Appendix:F

The functions involved in the DGLAP equations for gluon distribution functions at NNLO are

$$
\begin{aligned}
P_{gg}^2(\omega) &= 2643.524D_0 + 4425.894\delta(1-z) + 3589L_1 - 20852 + 3968z - 3363z^2 \\
&\quad + 4848z^3 + L_0 L_1(7305 + 8757L_0) + 274.4L_0 - 7471L_0^2 + 72L_0^3 - 144L_0^4 + \\
&\quad \frac{142141}{z} + \frac{2675.81}{z}L_0 + N_f(412.142D_0 - 528.723\delta(1-z) - 320l_1 \\
&\quad - 350.2 + 755.7z - 713.8z^2 + 559.3z^3 + L_0 L_1(26.85 - 808.7L_0) + 1541L_0 \\
&\quad + 491.3L_0^2 + \frac{832}{9}L_0^3 + \frac{512}{27}L_0^4 + \frac{182.961}{z} + \frac{157.271}{z}L_0) \\
&\quad + N_f^2(-\frac{16}{9}D_0 + 6.4630\delta(1-z) - 13.878 + 153.4z - 187.7z^2 + 52.75z^3 \\
&\quad L_0 L_1(115.6 - 85.25z + 63.23L_0) - 3.422L_0 + 9.680L_0^2 - \frac{32}{27}L_0^3 \\
&\quad - \frac{680}{2431z})
\end{aligned}
\tag{44}
$$

where, $L_0 = \ln(z)$, $L_1 = \ln(1-z)$ and $D_0 = \frac{1}{(1-z)}$.

Appendix G

To obtain the analytical solutions of DGLAP evolution equations for singlet structure function or gluon distribution function, we assume the following ansatz [a-c]

$$G(x, Q^2) = K(x)F_2^S(x, Q^2) \tag{45}$$

which gives the possibility to extract the gluon distribution function directly from the experimental data. Here $K(x)$ is a function of x or may be a suitable parameter which can be determined by phenomenological analysis.

In the DGLAP formalism the gluon distribution turns out to be very large at small-x and so it contributes crucially to the evolution of the parton distribution. Subsequently, the gluon distribution governs the structure function $F_2(x, Q^2)$ through the evolution $g \to q\bar{q}$ in the small-x region. For lower Q^2 ($Q^2 \approx \Lambda^2$), however, there is no such clear cut distinction between the two. Thus for small-x and high Q^2, the gluons are expected to more dominant than the sea quarks and therefore the determination of gluon density in the small-x region is particularly interesting. But the gluon distribution function $G(x, Q^2)$ cannot be measured directly through experiments. It is determined only via the quark distributions together with the evolution equations. The most precise determinations of the gluon momentum distribution in the proton can be obtained from a measurement of the deep inelastic scattering (DIS) proton structure function $F_2(x, Q^2)$ and its scaling violation. The Q^2-evolution of the proton structure function $F_2(x, Q^2)$ is related to the gluon distribution function $G(x, Q^2)$ in the proton and to the strong interaction coupling constant α_S. It is, therefore, important to measure the $G(x, Q^2)$ indirectly using $F_2(x, Q^2)$. Hence the direct relations between $F_2(x, Q^2)$ and $G(x, Q^2)$ are extremely important because using those relations the experimental values of $G(x, Q^2)$ can be extracted using the data on $F_2(x, Q^2)$. A plausible way of realizing this is through the above ansatz. The evolution equations of gluon distribution function and singlet structure function are in the same forms of derivative with respect to Q^2. Moreover the input singlet and gluon parameterizations, taken from global analysis of PDFs, in particular from the GRV1998, MRST2001, MSTW2008 parton sets [d-f], to incorporate different high precision data, are also functions of x at fixed Q^2. So the relation between singlet structure function and gluon parton densities can be expressed in terms of x at fixed-Q^2. Accordingly the above assumption is justifiable.

The function K(x) may be assumed to have some standard functional form such as $K(x) = K$, ax^b, ce^{dx} where K, a, b, c, d are suitable parameters which can be determined by phenomenological analysis, however we can not rule out the other possibilities [a-c, g, h]. The actual functional form of $K(x)$ can be determined by simultaneous solutions of coupled equations of singlet structure functions and gluon parton densities, nevertheless it is beyond the scope of thesis. In this thesis we perform our analysis considering the function $K(x)$ as an arbitrary constant parameter K for a particular range of x and Q^2 in defining the relation between gluon and singlet structure functions as the simplest assumption. But, we need to adjust its value for satisfactory description of different experiments. The best fit graphs are obtained by choosing an appropriate value of K for a proper description of each experiment.

Our phenomenological analysis reveals that the best fit results of singlet structure functions obtained from the solutions of linear DGLAP equations are in very good agreement with NMC data in the range $0.0045 \leq x \leq 0.19$ and $0.75 \leq Q^2 \leq 27$ GeV^2 for $0.92 < K < 1.2$, E665 data in the range $0.0052 \leq x \leq 0.18$ and $1.094 \leq Q^2 \leq 26$ GeV^2 for $0.45 < K < 0.87$ and NNPDF parametrizations in the range $0.0045 \leq x \leq 0.095$ and $1.25 \leq Q^2 \leq 26$ GeV^2 for $1.1 < K < 1.6$ respectively. Thus the parameter K lies in the range $0.45 < K < 1.6$ to obtain the best fit results of singlet structure functions compared with different experiments and parametrizations for the entire domain of x and Q^2 under study. Similarly we perform our analysis for gluon distribution functions obtained from the solutions of DGLAP equations in the x and Q^2 domain, viz. $10^{-4} \leq x \leq 0.1$ and $5 \leq Q^2 \leq 110$ GeV^2 and obtain our best fit results compared with different global analysis of parton distributions in the range $0.14 < K_1 < 0.85$, where $K_1 = 1/K$. We observe that our results show excellent consistency with the global parametrizations namely GRV1998, MRST2004, MSTW2008, JR09 and with the BDM model for $0.72 < K_1 < 0.85$, $0.5 < K_1 < 0.64$, $0.14 < K_1 < 0.48$, $0.56 < K < 0.68$ and $0.62 < K < 0.78$ respectively. On the other hand from the phenomenological analysis of singlet structure functions obtained from the solution of nonlinear GLR-MQ equation we note that the best fit results are obtained in the range $0.28 < K < 1.2$ for the entire domain of x and Q^2 under study. The computed values of singlet structure functions with shadowing corrections are found to be quite compatible with NMC data in the range $0.6 < Q^2 < 3.6$ GeV^2 and

$10^{-4} < x < 0.013$ for $0.52 < K < 0.9$, E665 data in the range $1 < Q^2 < 4$ GeV^2 and $10^{-4} < x < 0.01$ for $0.28 < K < 0.86$ and with the NNPDF parametrization in the range $1 < Q^2 < 27$ GeV^2 and $10^{-4} < x < 0.011$ for $0.72 < K < 1.2$ respectively.

To conclude, we examine the dependence of our predictions on the values of the arbitrary parameters K and K_1 for different experimental data or parametrizations and observe that the values of K or K_1 lie in a very small range. Therefore it is legitimate to take these parameters as constant parameters.

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