Abstract

Structure functions in lepton-nucleon deep inelastic scattering (DIS) are the well-established observables exploring Quantum Chromodynamics (QCD), the theory of the strong interaction, and they provide exclusive information about the deep structure of hadrons. Predominantly, the structure functions form the foundation of our knowledge of the parton densities, which are indispensable for analysis of hard scattering processes at proton-(anti-)proton colliders like the TEVATRON and the Large Hadron Collider (LHC). Parton distributions are the vital ingredients for most of the theoretical calculations at hadron colliders and they provide the number densities of the colliding partons (quarks and gluons) inside their parent hadrons at a given momentum fraction x, where x is the Bjorken scaling variable, and scale Q^2.

The small-x behavior of parton densities is one of the challenging issues of QCD. The chief and most salient phenomena in the region of small-x which determine the physical picture of the parton evolution are the increase of the parton density at $x \to 0$, the growth of the mean transverse momentum of a parton inside the parton cascade at small-x, and the saturation of the parton density. Therefore the determination of parton densities or to a great degree the gluon densities in the small-x region is particularly important because the gluon distribution function controls the physics at high energy or small-x in DIS. Moreover, precise knowledge of gluon distribution functions at small-x is useful to estimate backgrounds and explore new physics at the LHC. That being so, the dynamics of the the high density QCD, the regime of large gluon densities, is one of the present-day highly demanding undecided issues in the area of high energy or small-x physics.

The standard and the key tools for theoretical investigation of DIS structure functions are the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations. These equations can delineate the available experimental data in a decent manner considering a large domain of x and Q^2 with appropriate parameterizations. Consequently, the solutions of the DGLAP evolution equations have been reported in recent years with significant phenomenological success. The DGLAP evolution equation at the twist-2 level prognosticates a sharp growth of the gluon densities as x grows smaller which is clearly observed in the DIS experiments at HERA as well. In consequence, this generates cross sections which in the high-energy or small-x

limit fail to comply with the Froissart bound. Subsequently, the growing number of gluon densities, so as to approach small-x, demands a formulation of the QCD at high partonic density incorporating the unitarity corrections in a suitable manner.

Gluon recombination is usually assumed to be accountable for the unitarization of the cross section at high energies or a possible saturation of the gluon density at small-x. At small-x the likelihood of interaction between two gluons can no longer be neglected and sooner or later, the individual gluons necessarily start to overlap or shadow each other. Consequently, nonlinear phenomena are expected to arise which eventually bring about a taming of the maximum gluon density per unit of phase-space. Moreover, the pioneering finding of the geometrical scaling in HERA data as well as the existence of geometrical scaling in the production of inclusive jets in the LHC data provides strong experimental evidences of the saturation effects. The multiple gluon interactions towards small-x induce nonlinear corrections in the conventional linear DGLAP equation and accordingly the corrections of the higher order QCD effects have become the focus of in-depth studies in the last few years.

Gribov, Levin and Ryskin, at the onset, investigated the shadowing corrections of gluon recombination to the parton distributions. Following that Mueller and Qiu completed the equation numerically using a perturbative calculation of the recombination probabilities in the DLLA, and also formulated the equation for the change of gluons to sea-quarks. This is a great achievement as it authorises the GLR-MQ equation to be applied phenomenologically and thus provides the connection to experiments. This equation predicts a critical line separating the perturbative regime from the saturation regime, and it is legitimate just in the edge of this critical line. The study of the GLR-MQ equation is extremely important for the interpretation of the non-linear effects of gluon-gluon recombination due to high gluon density at sufficiently small-x as well as for the determination of the saturation momentum. Moreover, the Balitsky-Kovchegov (BK), Modified-DGLAP (MD-DGLAP), Modified-Balitsky-Fadin-Kuraev-Lipatov (MD-DGLAP) and Jalilian-Marian-Iancu-McLerran-Weigert-Leonidov-Kovner (JIMWLK) are some of other widely studied nonlinear evolution equations relevant at high gluon densities.

The work presented in this thesis is focused on the study of the small-x and Q^2 behaviours of the singlet and nonsinglet structure functions and gluon distribution functions in the context of linear DGLAP and nonlinear GLR-MQ evolution. The first part of this thesis is concerned with the study of the linear DGLAP equation upto next-to-next-to-leading order (NNLO); the second part is more specifically dedicated to the higher order QCD corrections in small-x physics where we address the issue of gluon-gluon recombination or shadowing corrections at very small values of x. In this work, we intend to check whether at very small-x the DGLAP equations can be ruled out in favour of the GLR-MQ equations which would mean evidence for gluon recombination. The structure of the thesis is organized as follows:

Chapter 1 is a general introduction of the elementary particles with a brief account of QCD, DIS, structure functions and parton distribution functions (PDFs). The importance of small-x physics and gluon shadowing are also concisely described here. Various high energy experiments as well as parametrization groups extracting PDFs from global data analyses are also briefly summarized.

Chapter 2 is an overview of the different QCD linear and nonlinear evolution equations with a more or less detailed description of the DGLAP and the GLR-MQ equations. The numerical as well as analytical solutions of the evolution equations, widely available in the literature are also outlined very briefly in this chapter.

It is always very alluring to explore the prospect of obtaining analytical solutions of DGLAP equations somewhat in the restricted domain of small-x. In **chapter 3**, we solve the DGLAP equations for the singlet and non-singlet structure functions analytically at LO, NLO and NNLO by using a Taylor series expansion valid at small-x and obtain the Q^2 and x-evolutions of deuteron structure function, $F_2^d(x,Q^2)$, along with the Q^2-evolution of proton structure function, $F_2^p(x,Q^2)$, upto NNLO. We compare our predictions with NMC, E665 and H1 experimental data as well as with the NNPDF parametrizations. The results obtained are in agreement with perturbative QCD fits at small-x and can explain the general trend of data in a decent manner. Moreover the inclusion of NNLO contributions provide excellent consistency with the experimental data and parametrizations.

In **chapter 4**, we find analytical expressions for gluon distribution function, $G(x,Q^2)$, at LO, NLO and NNLO by solving the corresponding DGLAP evolution equations using a Taylor series expansion as in chapter 3 and evaluate the Q^2 and x-evolutions of $G(x,Q^2)$ upto NNLO. We note that the NNLO approximation has appreciable contribution in the particular range of x and Q^2 under study. The obtained results can be described within the framework of perturbative QCD. We check the compatibility of our predicted gluon distributions and find satisfactory agreement with the GRV1998, MRST2004, MSTW2008 and JR09 global analysis as well as with the BDM model.

Although, the linear DGLAP equations can delineate the available experimental data, as far as HERA data are concerned, in a decent manner covering a large domain of x and Q^2 with appropriate parameterizations, however, in the very small-x region, due to the nonlinear corrections of gluon-gluon interactions, the conventional linear DGLAP evolution equation is expected to breakdown. We, therefore, turn our attention to the gluon recombination processes in **chapter 5** and estimate the importance of the corrections of these higher order QCD effects, which eventually saturate the growth of the gluon densities in the framework of nonlinear GLR-MQ equation. We investigate the effect of shadowing corrections on the small-x and moderate Q^2 behavior of gluon distribution by solving the nonlinear GLR-MQ equation in leading twist approximation incorporating the well known Regge ansatz in the kinematic region $10^{-5} \leq x \leq 10^{-2}$ and $1 \leq Q^2 \leq 30$ GeV^2. We also derive the condition of compatibility of the LO solution of linear DGLAP equation for gluon with the DLA solution in a finite range of x and Q^2. The predicted gluon distributions from GLR-MQ equation are compared with the GRV1998, GJR2008, MRST2001, MSTW2008, NNPDF, HERAPDF0.1 and CT10 parametrizations as well as with the H1 data. Our predictions are also compared with the EHKQS and BZ models respectively. We further analyse the ratio of the prediction of nonlinear GLR-MQ equation to that of linear DGLAP equation for $G(x, Q^2)$ and observe that the ratio decreases as x grows smaller signifying that the effect of nonlinearity increases towards small-x. It is enticing to note that, the rapid growth of gluon densities towards small-x is tamed by the gluon recombination processes. Results also indicate significant effect of shadowing corrections at $R = 2$ GeV^{-1} when the gluons are concentrated at the hot spots.

In **chapter 6**, we solve the nonlinear GLR-MQ equation for sea quark distribution in leading twist approximation incorporating the well known Regge like ansatz and investigate the effect of shadowing corrections to the small-x and Q^2 behaviour of singlet structure function $F_2^S(x,Q^2)$ in the kinematic region $10^{-4} \leq x \leq 10^{-1}$ and $0.6 \leq Q^2 \leq 30$ GeV^2. Our predictions are compared with NMC and E665 data as well as with the results of NNPDF collaboration. Results show that $F_2^S(x,Q^2)$ increases with increasing Q^2 and decreasing x, but this behaviour is slowed down towards small-x with the inclusion of the nonlinear terms. The logarithmic derivative, of singlet structure function with shadowing corrections is also calculated and compared with the H1 data. The behaviour of $\partial F_2^S(x,Q^2)/\partial \ln Q^2$ is seen to be tamed due to gluon recombination at small-x.

Chapter 7 concerns with the comparative analysis of the GLR-MQ equation with the more precise and more complicated BK equation as well as with the MD-DGLAP equation. It is interesting to note that the predictions of nonlinear gluon density obtained from the GLR-MQ equation are in very good agreement with the results of the BK equation. Our predictions are also observed to be almost comparable with those of the MD-DGLAP equation, however a flatter gluon distribution is observed in our predictions due to significant shadowing corrections at small-x.

Finally, the conclusions and the future outlooks of this work are drawn in **chapter 8**.