Appendices

Appendix A

The analytical expressions for the gluon splitting kernel $K_G^0(x)$ and $K_G^1(x)$ are given by:

$$K_G^0(x) = \sum_{i=1}^{N_f} e_i^2 8x^2(1-x) \tag{A.1}$$

$$\begin{aligned} K_G^1(x) = \Big[\sum_{i=1}^{N_f} e_i^2\Big] 16C_A x^2 & \left[\begin{array}{c} 4Li_2(x) - 2(1-x)lnxln(1-x) + 2(1+x)Li_2(-x) \\ +3ln^2x + 2(x-2)\zeta(2) + (1-x)ln^2(1-x) \\ +2(1+x)lnxln(1+x) + \dfrac{24+192x-317x^2}{24x}lnx \\ +\dfrac{1-3x-27x^2+29x^3}{3x^2}ln(1-x) \\ +\dfrac{-8+24x+501x^2-517x^3}{72x^2} \end{array}\right] \\ -16C_F x^2 & \left[\begin{array}{c} \dfrac{5+12x^2}{30}ln^2x - (1-x)ln(1-x) \\ +\dfrac{-2+10x^3-12x^5}{15x^3}[Li_2(-x) + lnxln(1+x)] \\ +2\dfrac{5-6x^2}{30}\zeta(2) + \dfrac{4-2x-27x^2-6x^3}{30x^2} \\ +\dfrac{(1-x)(-4-18x+105x^2)}{30x^2} \end{array}\right] \end{aligned} \tag{A.2}$$

Here the colour factors $C_A = 3$ and $C_F = \frac{4}{3}$ associated with the colour group SU(3). $Li_2(x)$ and $\zeta(2)$ are dilogarithmic function and Riemann Zeta function respectively.

The function $f(w)$ used in chapter 2 and 3 is defined as

$$f(w) = C_A w^2 \left[\begin{array}{c} 4Li_2(w) - 2(1-w)lnwln(1-w) + 2(1+w)Li_2(-w) \\ +3ln^2 w + 2(w-2)\zeta(2) + (1-w)ln^2(1-w) \\ +2(1+w)lnwln(1+w) + \frac{24+192w-317w^2}{24w}lnw \\ +\frac{1-3w-27w^2+29w^3}{3w^2}ln(1-w) + \frac{-8+24w+501w^2-517w^3}{72w^2} \end{array} \right]$$
$$-C_F w^2 \left[\begin{array}{c} \frac{5+12w^2}{30}ln^2 w - (1-w)ln(1-w) \\ +\frac{-2+10w^3-12w^5}{15w^3}[Li_2(-w) + lnwln(1+w)] \\ +2\frac{5-6w^2}{30}\zeta(2) + \frac{4-2w-27w^2-6w^3}{30w^2} \\ +\frac{(1-w)(-4-18w+105w^2)}{30w^2} \end{array} \right] \quad \text{(A.3)}$$

The analytical expressions of gluon co-efficient functions for F_L structure function are given by :

$$C^1_{L,g}(w) = 8N_f w(1-w) \quad \text{(A.4)}$$

$$\begin{aligned} C^2_{L,g}(w) \;\; &\cong \;\; N_f\{(94.74 - 49.20w)L_1^2 + 864.8w_1 L_1 + 1161wL_1L_0 \\ &+60.06wL_0^2 + 39.66w_1L_0 - 5.333(w^{-1} - 1)\} \end{aligned} \quad \text{(A.5)}$$

$$
\begin{aligned}
C^3_{L,g}(w) \;\cong\; & N_f\Big\{\Big(144L_1^4 - \frac{47024}{27}L_1^3 + 6319L_1^2 + 53160L_1\Big)w_1 + 72549L_0L_1 \\
& + 88238L_0^2L_1 + (3709 - 33514w - 9533w^2)w_1 + 66773wL_0^2 \\
& - 1117L_0 + 45.37L_0^2 - \frac{5360}{27}L_0^3 - (2044.70w_1 + 409.506L_0)w^1\Big\} \\
& + N_f^2\Big\{\Big(\frac{32}{3}L_1^3 - \frac{1216}{9}L_1^2 - 592.3L_1 + 1511wL_1\Big)w_1 + 311.3L_0L_1 \\
& + 14.24L_0^2L_1 + (577.3 - 729.0w)w_1 + 30.78wL_0^3 + 366.0L_0 \\
& + \frac{1000}{9}L_0^2 + \frac{160}{9}L_0^3 + 88.5037w^{-1}w_1\Big\} \\
& + fl_{11}^g N_f^2\Big\{(-0.0105L_1^3 + 1.550L_1^2 + 19.72wL_1 - 66.745w \\
& + 0.615w^2)w_1 + \frac{20}{27}wL_0^4 + \Big(\frac{280}{81} + 2.260w\Big)wL_0^3 - (15.40 \\
& - 2.201w)wL_0^2 - (71.66 - 0.121w)wL_0\Big\}. \qquad \text{(A.6)}
\end{aligned}
$$

In equation (A.5) and (A.6), $w_1 = 1 - w$, $L_0 = lnw$, $L_1 = lnw_1$ and fl_{11}^g denote the charge factor which is defined as $fl_{11}^g = \dfrac{<e>^2}{<e^2>}$.

The expression for gluon splitting function at small-x are written as :

$$
\begin{aligned}
P^1_{gg}(w) \;=\; & 2C_A\Big\{\frac{w}{(1-w)_+} + \frac{1-w}{w} + w(1-w)\Big\} \\
& + \delta(1-w)\frac{(11C_A - 4N_fT_R)}{6}, \qquad \text{(A.7)}
\end{aligned}
$$

$$
\begin{aligned}
P^2_{gg}(w) \;=\; & \frac{12C_FN_fT_R - 46C_AN_fT_R}{9w} + N_fT_R\Big\{\frac{-61}{9}C_F + \frac{172}{27}C_A\Big\} \\
& + C_A^2\Big\{\frac{1643}{54} - \frac{22}{3}\xi(2) - 8\xi(3)\Big\}, \qquad \text{(A.8)}
\end{aligned}
$$

$$
P^3_{gg}(w) = \frac{224C_AT_RN_f}{27w^2}(-C_A), \qquad \text{(A.9)}
$$

where the Casimir operators of color group $SU(3)$ are defined as $C_A = 3$, $C_F = \dfrac{4}{3}$ and $T_R = \dfrac{1}{2}$.

The expressions for gluon co-efficient function for F_2 structure function in LO, NLO and NNLO are given by :

$$C^1_{2,g}(w) = N_f\{(2-4ww_1)(L_1-L_0)-2+16ww_1\}, \tag{A.10}$$

$$\begin{aligned} C^2_{2,g}(w) &= \{(6.445+209.4(1-w))L_1^3-24L_1^2+(149w^{-1}-1483)L_1 \\ &\quad +L_1L_0(-871.8L_1-724.1L_0)+5.319L_0^3-59.48L_0^2-284.8L_0 \\ &\quad +11.90w^{-1}+392.4-0.28\delta(1-w) \end{aligned} \tag{A.11}$$

and

$$\begin{aligned} C^3_{2,g}(w) &= N_f\Big\{\frac{966}{81}L_1^5-\frac{1871}{18}L_1^4+89.31L_1^3+979.2L_1^2-2405L_1+1372x_1L_1^4 \\ &\quad -15729-310510x+331570x^2-244150xL_0^2-253.3xL_0^5 \\ &\quad +L_0L_1(138230-237010L_0)-11860L_0-700.8L_0^2-1440L_0^3 \\ &\quad +\frac{4951}{162}L_0^4-\frac{134}{9}L_0^5-x^{-1}(6362.54-932.089L_0)+0.625\delta(x_1)\Big\} \\ &\quad +N_f^2\Big\{\frac{131}{81}L_1^4-14.72L_1^3+3.607L_1^2-226.1L_1+4.762-190x \\ &\quad -818.4x^2-4019xL_0^2-L_0L_1(791.5+4646L_0)+739L_0+418L_0^2 \\ &\quad +104.3L_0^3+\frac{809}{81}L_0^4+\frac{12}{9}L_0^5+84.423x^{-1}\Big\}+fl_{11}^gN_f^2\Big\{3.211L_1^2 \\ &\quad +19.04xL_1+0.623x_1L_1^3-64.47x+121.6x^2-45.82x^3 \\ &\quad -xL_0L_1(31.68+37.24L_0)+11.27x^2L_0^3-82.40xL_0-16.08xL_0^2 \\ &\quad +\frac{520}{81}xL_0^3+\frac{20}{27}xL_0^4\Big\} \end{aligned} \tag{A.12}$$

respectively.

Appendix B

In the LO analysis, the heavy quark co-efficient functions used in chapter 6 are written as

$$C_{2,g}^{(0)}(w,\zeta) = \frac{1}{2}\Big([w^2+(1-w)^2 4w\zeta(1-3w)-8\zeta^2w^2]ln\frac{1+\beta}{1-\beta} \\ +\beta[-1+8w(1-w)-4z\zeta(1-w)]\Big) \quad \text{(B.1)}$$

and

$$C_{L,g}^{(0)}(w,\zeta) = -4w^2\zeta ln\frac{1+\beta}{1-\beta}+2\beta w(1-w), \quad \text{(B.2)}$$

where $\beta^2 = 1-\frac{4z\zeta}{1-z}$. In NLO, we have used the compact form of the co-efficient functions in high energy regime ($\zeta << 1$). The NLO co-efficient functions $C_{k,g}^{(1)}$ and $\overline{C}_{k,g}^{(1)}$ are given by

$$C_{k,g}^{(1)} = \frac{8}{3}C_A e_h^2 ln^2(Q^2/m_h^2) \quad \text{(B.3)}$$

and

$$\overline{C}_{k,g}^{(1)} = \frac{16}{3}C_A e_h^2 ln^2(Q^2/m_h^2), \quad \text{(B.4)}$$

where $k = 2, L$ and $h = c, b$. Here the colour factor $C_A = 3$, e_h is the charge of the heavy quark and m_h is the mass of the heavy quark. □