# Studies on Longitudinal Structure Function $F_L$ of Proton at Small-$x$

## Abstract

The knowledge of structure of the matter, its properties and interactions are important for better understanding of the origin of the universe. High Energy Physics has been dealing with the understanding of the fundamental constituents of the matter, i.e., the leptons, quarks, the intermediate gauge bosons and the interaction between them. Scattering experiments starting from Rutherford's famous experiment have played an important role in the investigation of the inner structure of matter. Deep Inelastic lepton-nucleon Scattering (DIS) provides quantitative test of Quantum Chromodynamics (QCD) i.e., the measurement of quarks and gluon densities inside the nucleon. The interaction among these constituents of nucleon can be described by different QCD evolution equations. The structure functions of the nucleon which provide the information about the partons (quarks and gluons) can be obtained as a solution of these evolution equations.

Proton is one of the familiar particles around us and it is being used in the present colliders where investigations are going on in search of new physics. So, the knowledge of its structure is essential for the detailed perturbative QCD (pQCD) calculations of any process involving proton. Among the proton structure functions, longitudinal structure function $F_L$ is important one to study as it is directly sensitive to the gluon distribution in the proton. Theoretically and phenomenologically the measurement of $F_L$ structure

function helps one to distinguish different models describing the QCD evolutions at small-$x$. Moreover, the structure function measurement remains incomplete without the inclusion of this particular structure function measurement. Experimentally it was measured at electron proton collider HERA over a wide range of Bjorken variable $x$ and the four momentum transfer in DIS process $Q^2$.

In pQCD among the QCD evolution equations, Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equation is the most fundamental one to study the $Q^2$ and $x$- evolution of structure function. Once a quark or gluon distribution function at some reference point is given, one can compute it for any value of $Q^2$ using this equation. The structure function of the nucleon reflects the momentum distributions of the quarks and gluons in it. It is also important to study the gluon distribution inside a hadron at small-$x$ because gluons are expected to be dominant in this region. In the framework of the DGLAP equation the parton distributions grows at small-$x$ as a result of their $Q^2$-evolution. The steep rise of $F_2(x, Q^2)$ structure function towards small-$x$ observed at HERA also indicates a similar increase in the gluon distribution towards small values of $x$ in pQCD. That is, the perturbative QCD predicts a strong power law rise of the gluon distribution in the small-$x$ region. At small values of $x$, the behaviour of $F_L$ is driven mainly by gluons through the transition $g \to q\bar{q}$. Therefore, once the distribution of gluon inside the proton is known, $F_L$ structure function can be calculated from it. The behaviour of $F_L$ structure function also shows power law rise as that of the gluon distribution function.

Along with the light flavours, the inclusion of heavy flavours (charm and beauty quarks) in the study of evolution of $F_L$ structure function is also important. It is already well known that the scaling violations are different in case of the massless and massive pQCD calculation. Thus, in all precision measurement of structure functions, a detailed treatment of heavy flavour contribution is required. At small-$x$, all the heavy quark structure functions are dominated by the gluon content of the proton. Therefore, the behaviour of these structure functions can be studied using the gluon distribution function.

In this thesis, we have studied the behaviour of proton longitudinal structure function $F_L$ in the region of small-$x$ up to next-to-next-to-leading orders. Here the evolution of structure function in higher orders are studied using the higher order kernels in the QCD evolution equation. The inclusion of higher order kernel in the study of the hard processes in QCD becomes significant as compared to leading one due to consistency and accuracy of the results. This consideration is particularly important for studying the physical processes at the present colliders LHC and TEVATRON. Also, in these colliders major emphasis has been given on the small-$x$ region.

In Chapter 1, we present a brief introduction to the structure of matter, standard model of elementary particle physics, deep inelastic scattering, DIS cross section and structure function, Quantum Chromodynamics, Quark Parton Model, QCD evolution equation, Longitudinal structure function $F_L$, heavy quarks in the proton, small-$x$ physics, experimental measurement of $F_L$ structure function and the related experiments.

In Chapter 2, we have solved the QCD evolution equation for $F_L$ structure function up to next-next-to-leading order at small-$x$. Here we use Taylor expansion method to obtain the analytical expression for $t$- and $x$-evolution of $F_L$ structure function. The computed results are compared with recent H1, ZEUS data, Donnachie-Landshoff (DL) model results and the theoretical prediction of MSTW08, CT10, ABM11 and NNPDF2.3 parameterizations.

In Chapter 3, we have presented the $t$- and $x$-evolutions of $F_L$ structure function obtained as a solution of QCD evolution equation for $F_L$ structure function up to next-to-leading order at small-$x$ using Regge like behaviour of structure function. The results obtained are compared with H1, ZEUS data, DL model results and the theoretical prediction of MSTW08, CT10, ABM11 and NNPDF2.3 parameterizations. We have also compared these results with our results from Chapter 2.

In Chapter 4, we have presented the approximate relation between the $F_L$ structure function and gluon distribution function at small-$x$ using Taylor expansion method. From this relation we have calculated the $x$-evolution of $F_L$ structure function and the results obtained are compared with H1, ZEUS data, DL model results and the theoretical prediction of MSTW08, CT10, ABM11 and NNPDF2.3 parameterizations.

In Chapter 5, the $t$- and $x$-evolutions of $F_L$ structure function up to next-to-next-to-leading order at small-$x$ using the Regge like behaviour of gluon distribution function obtained as a solution of DGLAP evolution equation is presented. The results obtained are compared with H1, ZEUS data, DL model results and the theoretical prediction of MSTW08, CT10, ABM11 and NNPDF2.3 parameterizations. The behaviour of DIS cross section ratio $R$ with respect to $x$ is also presented. We have also presented a comparative study of our results with those from Chapter 4.

In Chapter 6, we have presented the behaviour of heavy flavour structure function $F_L^c$, $F_L^b$, $F_2^c$ and $F_2^b$ with respect to $x$ using Taylor expansion method and Regge like behaviour of gluon distribution function. Our results are compared with H1, ZEUS data and results of DL, Colour-Dipole (CD) model. We use these results to analyse the behaviour of heavy quark DIS cross section ratio $R^h$ and reduced cross section $\sigma_r^h$. Finally, the behaviour of heavy quark content to the $F_L$ structure function with respect to $x$ is also presented here. We have also presented a comparative analysis of our results obtained by both the methods.

In Chapter 7, we have summarised the overall conclusions drawn from our work. □