CHAPTER-4

NEW NONLINEAR EIGENMODE PATTERNS IN SPHERICAL CHARGED DUST MOLECULAR CLOUD

Abstract: *A hydrodynamical model to study nonlinear self-gravitational eigenmode excitations in spherical charged dust molecular cloud with full convective flow-dynamics is developed. The spherically symmetric cloud mass is assumed to be greater than the critical mass limit required to exhibit collective fluctuation dynamics. The eigenmode patterns evolve as new damped oscillatory shock-like structures governed by a unique form of modified Korteweg-de Vries Burger (m-KdVB) equation with self-consistent derivative source. Their formation mechanisms, distinctive features, and tentative astrophysical applicability leading to prestellar cores are summarily highlighted.*

4.1 INTRODUCTION

The study of various waves, oscillations, and fluctuations supported in self-gravitating dusty plasmas, such as in interstellar clouds, have been an important emerging area of research for years [1-7]. The growing interest of investigations of such challenging areas is mainly due to the roles played by them in the dynamics of self-gravitational collapse leading to the formation of various bounded equilibrium structures such as stars, stellar rings, planets, planetesimals, planetary rings, cometary tails, clusters, etc., in astrophysical situations [1-7]. Usually, the major constituents of interstellar clouds are the thermal electrons, ions, and the inertial dust grains such as silicates, carbon, aromatic hydrocarbons, and so on [1, 4-7]. The clouds have irregular shapes. Most of the larger clouds such as giant molecular clouds (GMCs) appear elongated, part of larger filamentary structures as discussed detail in chapter 1. It has even been suggested that the geometry of the interstellar clouds may be better represented by fractals [4-5]. However, generally, the shape of the clouds are considered as spherical for mathematical simplicity. The self-gravitational collapse of the massive-dust grains in such clouds by satisfying the Jeans criterion plays a crucial role in the formation processes of prestellar cores that eventually spawn stars [1-7]. In interstellar clouds, the grains acquire a non-negligible electric charge due to interstellar radiation fields ionizing the background gas, resulting in plasma collision effects and some other energetic mechanisms [1, 4].

Avinash and Shukla have recently reported about the existence of a new class of astrophysical objects, in which the self-gravity of the dust is balanced by the intensity originating from the shielded electric fields on the charged dust [8]. They show the stable equilibrium mass limit ($M_{AS} \sim 10^{19}$ kg) for the maximum dust cloud mass, which could be supported against self-gravitational collapse by these fields in the stationary dust configuration. The concerned mass limit has been shown to conform to Chandrasekhar's mass limit for compact objects like white dwarfs and neutron stars [9]. Although stable and neutral on the Jeans scale, a wide spectrum of fluctuation eigenmodes exists in the entire cloud owing to composite-type gravito-electrostatic interaction. Therefore, it would be interesting and important to investigate the basic physics and characteristics of various nonlinear eigenmodes of the associated gravito-electrostatic waves in such a situation including all possible realistic agencies.

In the present chapter, we consider Avinash-Shukla model [8] of the hydrodynamic dust molecular cloud (DMC), but in spherical geometry with dust flow convective dynamics included in full form to study the nonlinear stability of the charged DMC in the astrophysical scales of space and time. A distinct set of non-autonomous self-consistently coupled nonlinear dynamical eigenvalue equation in the defined Jeans scales of space and time configuration is accordingly derived in normalized form. The standard methodology of multiple scaling technique [10] around the defined cloud equilibrium [8] is systematically applied. We, in addition, assume that the dust gravitational to electrostatic force ratio $F_{dg}/F_{de} = Gm_d^2/q_d^2 \approx 0(1)$, so that the self-gravitational field and electric field of the grains are comparable. Besides, this ratio is too small for the electrons and ions; hence, the self-gravitational field effects on them will be neglected in our model. Thus, the self-gravitational potential fluctuation dynamics, governed by a new type of modified Korteweg-de Vries Burger (*m*-KdVB) equation, is contributed collectively by relatively massive charged-grains in the form of damped oscillatory shock-like structures. This gives some new dynamical aspects of star formation mechanism in the form of nonlinear eigenmodes as initial conditions.

4.2 PHYSICAL MODEL

A simplified idealistic charged DMC is considered in spherical geometry approximation in hydrodynamic equilibrium configuration with radial symmetry. The solid matter of the identical spherical dust grains is embedded in the gaseous phase of plasma. A bulk uniform flow is assumed to pre-exist. Global electrical neutrality is supposed to exist over the spherical gravito-electrostatic enclosure containing the various plasma particles, as in Avinash-Shukla model [8].

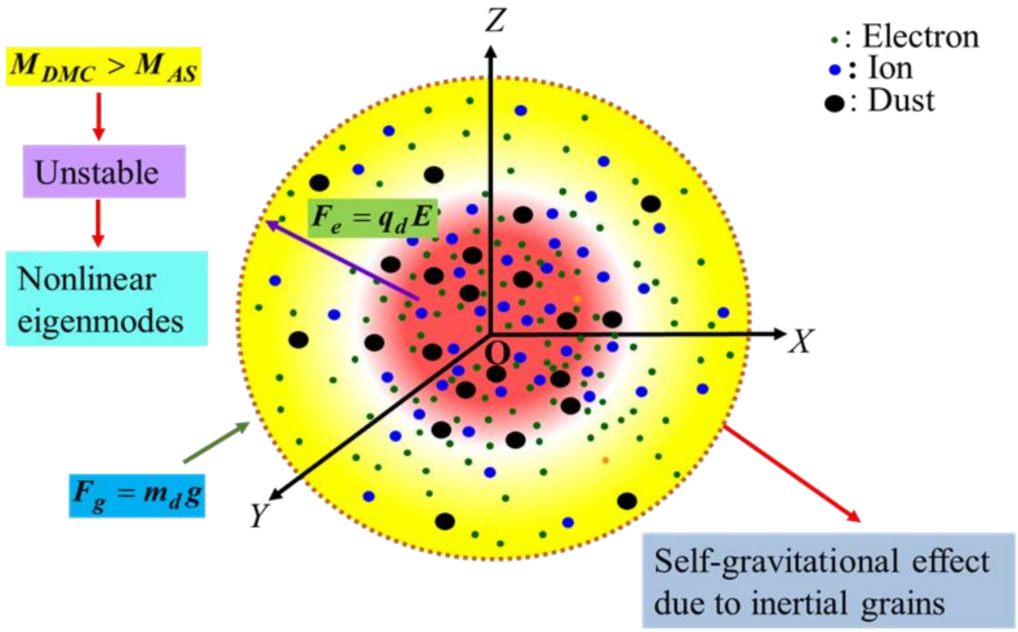

Figure 4.1 *Cartoon showing a typical spherical DMC model.*

For our observation on the Jeans time scale, the heavier dust grains are assumed to behave as a hydrodynamical fluid, whereas the lighter electrons and ions, as Boltzmannian thermal particles [8]. Since, we are interested in low frequency eigenmodes on the Jeans scale, the inertial terms in the dynamics of the thermal electrons and ions are ignored. The total cloud mass contributed collectively by the heavier grains is greater ($M_{DMC} > M_{AS}$) than the Avinash–Shukla mass limit [8]. The velocity convection dynamics in the dust fluid, however, is afresh included in our idealized model of current concern to see the real nature of the cloud eigenmodes. The electric forces generated due to the electrostatic polarization effects (local charge imbalance) are, judiciously, assumed to be weak, so that only the lowest-order contributions of various nonlinear terms in our model are being usefully considered, neglecting the higher-order ones. Figure 4.1 pictorially shows our spherical DMC, where the self-gravitational force ($F_g = m_d g$) originating due to the inertial dust grains, is balanced by the electrostatic force ($F_e = q_d E$) arising due to shielded electric fields on the charged-dust in the background plasma. When $M_{DMC} > M_{AS}$, due to the force imbalance, the cloud will no longer be in stable equilibrium configuration [8]. Thus, in such condition, cloud undergoes Jeans instability and generate different types of nonlinear eigenmode patterns.

In our model, the thermal screening species are, due to Boltzmann density distributions, in thermodynamic equilibrium on the slow Jeans time-scale. This assumption of thermalization of the thermal species is valid provided the phase velocity of fluctuations is much smaller than their thermal velocity, i. e., any fluctuation in the electron-ion temperature is instantly smoothened out. In addition, for further simplicity, complications like the effects of dispersed dust grain rotation, kinetic viscosity, non-thermal energy transport (wave dissipation process), and magnetic field due to convective circulation dynamics of plasma particles are all neglected.

4.3 BASIC GOVERNING EQUATIONS

The simplified spherical charged DMC under consideration consists of the lighter electrons and ions, and the heavier dust grains in presence of grain velocity convection dynamics. Avinash and Shukla have adopted such a model in the recent past for the investigation of stability behavior over a critical mass limit of the cloud as a whole [8]. We are going to apply a nonlinear perturbation analysis over the same model, but originally in spherically symmetric geometry with the dust fluid nonlinearity (velocity convection) taken into account. Accordingly, for low-frequency eigenmode investigation on the Jeans scale, the dynamics of the plasma thermal electrons and ions are governed by normalized equations with all usual notations respectively as follows,

$$N_e \frac{\partial \theta}{\partial \xi} - \frac{\partial N_e}{\partial \xi} = 0 \text{, and} \tag{4.1}$$

$$N_i \frac{\partial \theta}{\partial \xi} + \frac{\partial N_i}{\partial \xi} = 0 . \tag{4.2}$$

The dynamics of the inertial grains in field-free fluid model approach is collectively governed by the following set of normalized equations enlisted below,

$$\frac{\partial M_d}{\partial \tau} + M_d \frac{\partial M_d}{\partial \xi} = -\frac{q_d}{q} \frac{\partial \theta}{\partial \xi} - \frac{m_d}{q} \frac{\partial \eta}{\partial \xi} \ , \tag{4.3}$$

$$q\left(\frac{\lambda_{De}}{\lambda_J}\right)^2 \left[\frac{2}{\xi} \frac{\partial \eta}{\partial \xi} + \frac{\partial^2 \eta}{\partial \xi^2}\right] = G m_d N_d \text{, and} \tag{4.4}$$

$$q(N_i - N_e) + q_d N_d = 0 . \tag{4.5}$$

Equations (4.3)-(4.5) represent a neutral hydrodynamic equilibrium of the DMC on the Jeans scale. In such equilibrium, there is no electric field, and the free energy source is purely due to the self - gravitational field of the charged-grains. Moreover, invoking the Jeans swindle [2] in self-gravitational Poisson equation, we neglect the zeroth-order self-gravitational field, and regard the equilibrium initially as a 'homogeneous' one. In the above set of equations, space ξ, and time τ are normalized by the Jeans wavelength λ_J, and Jeans time ω_J^{-1}, respectively. The Jeans wavelength (λ_J) is defined as the critical length scale above which any astrophysical structure such as molecular cloud, interstellar dust molecular cloud, undergoes a gravitational collapse and becomes unstable. Similarly, the Jeans time (ω_J^{-1}) is the critical free-fall time scale below which the clouds undergo gravitational collapse. The Both the electrostatic potential θ, and self-gravitational potential η are normalized by the same plasma thermal potential T/q. The notations N_e, N_i, and N_d represent the number densities of the electrons, ions, and the dust grains normalized by the equilibrium plasma population density n_0 each, respectively. Dust flow velocity M_d is normalized by dust sound phase velocity $C_{ss}\left(=\sqrt{T/m_d}\right)$. Charge of the electron, ion, and the dust are respectively given by $q=-e$, $q=e$, and $q_d=Z_d q$. In addition, the thermal pressures of the lighter electrons and ions are governed by their isothermal equations of state $p_e=n_eT_e$, and $p_i=n_iT_i$ with $T_d<<T_e=T_i=T$ (due to $m_d>>m_i>m_e$ mass scaling), respectively. Moreover, charged-neutral dynamics does not arise here, so ignored in our model description.

4.4 DERIVATION OF *m*-KdVB EQUATION

We now apply the standard methodology of multiple scaling technique [10] over equations (4.1)-(4.5) around the defined cloud equilibrium. The cloud mass is assumed to be greater than the critical mass limit required to exhibit collective fluctuation dynamics. The independent variables are stretched into a new space defined by the transformations $X=\in^{1/2}(\xi-\mu\tau)$, and $T=\in^{3/2}\tau$, where $\in$ is a minor parameter characterizing the balanced strength of nonlinearity and dispersion, and μ is the fluctuation phase velocity (normalized by C_{ss}). The dependent variables like densities, potentials, and velocities in equations (4.1)-(4.5) are now expanded nonlinearly (in $\in$-powers) around the respective equilibrium values of the defined equilibrium dust cloud as follows.

$$\begin{pmatrix} N_e(\xi,\tau) \\ N_i(\xi,\tau) \\ N_d(\xi,\tau) \\ M_d(\xi,\tau) \\ \theta(\xi,\tau) \\ \eta(\xi,\tau) \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \in \begin{pmatrix} N_{e1}(\xi,\tau) \\ N_{i1}(\xi,\tau) \\ N_{d1}(\xi,\tau) \\ M_{d1}(\xi,\tau) \\ \theta_1(\xi,\tau) \\ \eta_1(\xi,\tau) \end{pmatrix} + \in^2 \begin{pmatrix} N_{e2}(\xi,\tau) \\ N_{i2}(\xi,\tau) \\ N_{d2}(\xi,\tau) \\ M_{d2}(\xi,\tau) \\ \theta_2(\xi,\tau) \\ \eta_2(\xi,\tau) \end{pmatrix} + \cdots\cdots, \tag{4.6}$$

In equation (4.6), due to the gravity-induced space-charge electric polarization effects [11], both the electrostatic ($\theta_{eq} \neq 0$) and self-gravitational ($\eta_{eq} \neq 0$) potentials should have some finite non-zero equilibrium values in realistic astrophysical scenarios. But, these effects are neglected for mathematical simplicity of our physical problem of local analysis as appear in above equation. We now substitute equation (4.6) in equations (4.1)-(4.5). Equating the like terms in various powers of $\in$ from both sides of equations (4.1)-(4.5) and applying the systematic methodology of elimination and simplification, we obtain a nonlinear modified Korteweg-de Vries Burger (*m*-KdVB) equation with a self-consistent driving linear derivative source on the lowest-order perturbed self-gravitational potential fluctuation η_1 . The time stationary form of the *m*-KdVB equation by using the Galilean type of transformation $\rho = X - T$, in reduced form is as follows.

$$\frac{\partial \eta_1}{\partial \rho} - \gamma_1 \eta_1 \frac{\partial \eta_1}{\partial \rho} - \gamma_2 \frac{\partial^2 \eta_1}{\partial \rho^2} - \gamma_3 \frac{\partial^3 \eta_1}{\partial \rho^3} = -\gamma_4 \frac{\partial \eta_1}{\partial \rho}, \tag{4.7}$$

where,

$$\gamma_1 = \frac{m_d}{q(\mu - M_{do})} + \frac{2 m_d \mu T}{(\rho + T)^3 N_o (\mu - M_{do})} \frac{Z_d^2}{q} N_{do}, \tag{4.8}$$

$$\begin{aligned} \gamma_2 = {} & \frac{(\mu - M_{do})}{(\rho + T) N_o} Z_d^2 N_{do} + \left\{ -\frac{\mu T m_d}{(\rho + T)^2 N_o (\mu - M_{do})} \frac{Z_d^2}{q} N_{do} \right\} \eta_1 \\ & + \left\{ \frac{\mu^2 T^2}{(\rho + T)^4 N_o^2 G m_d (\mu - M_{do})} Z_d^3 q_d N_{do} \right\} \frac{\partial \eta_1}{\partial \rho}, \end{aligned} \tag{4.9}$$

$$\gamma_3 = \frac{(\mu - M_{do})}{2 N_o} Z_d^2 N_{do}, \text{ and} \tag{4.10}$$

$$\begin{aligned} \gamma_4 = {} & \left\{ \frac{2 m_d \mu^2 T^2}{(\rho + T)^5 N_o^2 (\mu - M_{do})} \frac{Z_d^4}{q} N_{do}^2 + \frac{m_d \mu T}{(\rho + T)^2 N_o (\mu - M_{do})} \frac{Z_d^2}{q} N_{do} \right\} \frac{\partial \eta_1}{\partial \rho} + \\ & \frac{(\mu - M_{do})}{(\rho + T)^2 N_o} Z_d^2 N_{do}. \end{aligned} \tag{4.11}$$

Various coefficients of equation (4.7) are in terms of dust grain mass (m_d), flow (M_d), and charge number (Z_d). This is the mathematical construct contributed by the collective dynamics of the self-gravitating massive dust grains amidst the integrated interplay of diverse nonlinear (hydrodynamic in origin) and dispersive (self-gravitational in origin) effects in presence of internal dissipation (hydrodynamic in origin). This equation shows the possibility for the existence of both soliton-like and shock-like structures in presence of the joint action of all nonlinearity, dispersion, and dissipation effects of internal self-gravitationally evolving cloud origin. A soliton is a self-reinforcing solitary wave (a wave packet pulse) that maintains its shape while it progress at a constant velocity. Similarly, shock wave is defined as a type of propagating disturbance when a wave moves faster than the speed of sound in a liquid, gas, or plasma.

4.5 RESULTS AND DISCUSSIONS

A theoretical model is developed for the investigation of self-gravitational nonlinear eigenmodes of a spherically symmetric DMC in an external field-free hydrodynamic equilibrium configuration with dust grain velocity convective dynamics taken into account. The unique originality of the analysis lies in the perturbative treatment around cloud equilibrium extended over Avinash-Shukla model [8], applied to understand the self-gravitational collapse dynamics in terms of a critical cloud mass-limit. This is observed that the nonlinear self-gravitational fluctuation dynamics of the cloud is governed by a unique form of *m*-KdVB equation (4.7) with a linear driving derivative source, which analytically supports the possibility of existence of soliton-like and shock-like nonlinear eigenmodes. For actual details of the associated microphysics, the model is then numerically integrated as an initial value problem (by RK-IV method) with judiciously chosen plasma parameter values to yield the consequent numerical profiles as shown in figure 4.2.

Figure 4.2 gives the profile of the normalized lowest-order perturbed self-gravitational (a) potential, (b) potential gradient, (c) potential curvature (second derivative of potential), and (d) phase portrait. Various lines correspond to Case (1): $m_d = 1.02\times10^{-14}$ kg (blue line), Case (2): $m_d = 1.07\times10^{-14}$ kg (red line), Case (3): $m_d = 1.11\times10^{-14}$ kg (green line), and Case (4): $m_d = 1.14\times10^{-14}$ kg (black line), respectively. Different input initial values used are $\eta_i = -10^{-6}$, $(\eta_\rho)_i = 10^{-5}$, and $(\eta_{\rho\rho})_i = 10^{-7}$. The other parameters kept constant are $Z_d = 100$, $\mu = 1$,

$M_d = 11$, $N_{do} = 10^{-6}$, and $\in = 10^{-2}$ [12]. This is clear that the amplitude of the normalized lowest-order perturbed self-gravitational potential, potential gradient, and potential curvature increases with the mass of the dust grain and vice versa. The various new and distinct observations found in our stability analysis are summarized as follows.

(1) We observe that, the lowest-order nonlinear self-gravitational eigenmodes propagate in the form of damped oscillatory shock-like structures [13] as shown in figure 4.2(a). The oscillations show that the gravito-electrostatic interaction is not static, but dynamic in a periodic fashion. The cloud consists of plasma gas and solid dust. The oscillations may be due to the compression of one species and rarefaction of the other, and vice versa, due to the dynamically periodic gravito-electrostatic coupling processes. The shock amplitude comes out $\sim 10^{-4}$, which is physically $\sim 10^{-5}$ J kg^{-1}, for $T = 0.20$ eV, and $\in = 10^{-2}$ [8, 12]. At asymptotically large distance ($\rho \sim 10$, and beyond), the oscillatory fluctuation amplitudes gradually decrease because the self-gravity (variable) approaches external gravity (stable), and thereby decreasing the degree of compression and rarefaction of the cloud species. The oscillatory shock-like structures obtained on the Jeans length in the cloud are in qualitative agreement with various experimental [4, 14] and satellite observations under different conditions [1, 4, 15].

(2) The damped oscillatory self-gravitational gradient profile, as presented in figure 4.2(b), shows the inhomogeneous nature of the gravito-electrostatic force coupling processes. The self-gravitational intensity amplitudes decrease gradually at a distance of $\rho \sim 10$, and also beyond. Thus any external particle accreted to the solid dust matter cloud experiences a periodic force field of hydrodynamic periodic gravito-electrostatic interaction coupling origin.

(3) The damped periodic behavior of the self-gravitational Poisson term shows that the mass distribution of the heavier solid grains in the cloud is non-uniform on the Jeans space from the cloud center outwards. Due to strong self-gravity, the grain population density is very small asymptotically at large distance outward ($\rho \sim 10$, and also beyond) showing an almost zero-fluctuation value in amplitude (Figure 4.2(c)), which may be treated as a new investigation.

(4) The phase portrait (Figure 4.2(d)) shows open trajectories due to the damped oscillatory nature of the fluctuations in the self-gravitational cloud potential, thereby depicting the dissipative dynamics of the eigenmodes. Thus, the role of the dissipative dynamics of Avinash-Shukla

model [8] involving no dust flux conservation rule on the associated eigenmodes is further justified well.

(5) The evolutionary spectrum of self-gravitational fluctuations due to other sensitive plasma parameter variations are elaborately discussed in Ref. [16].

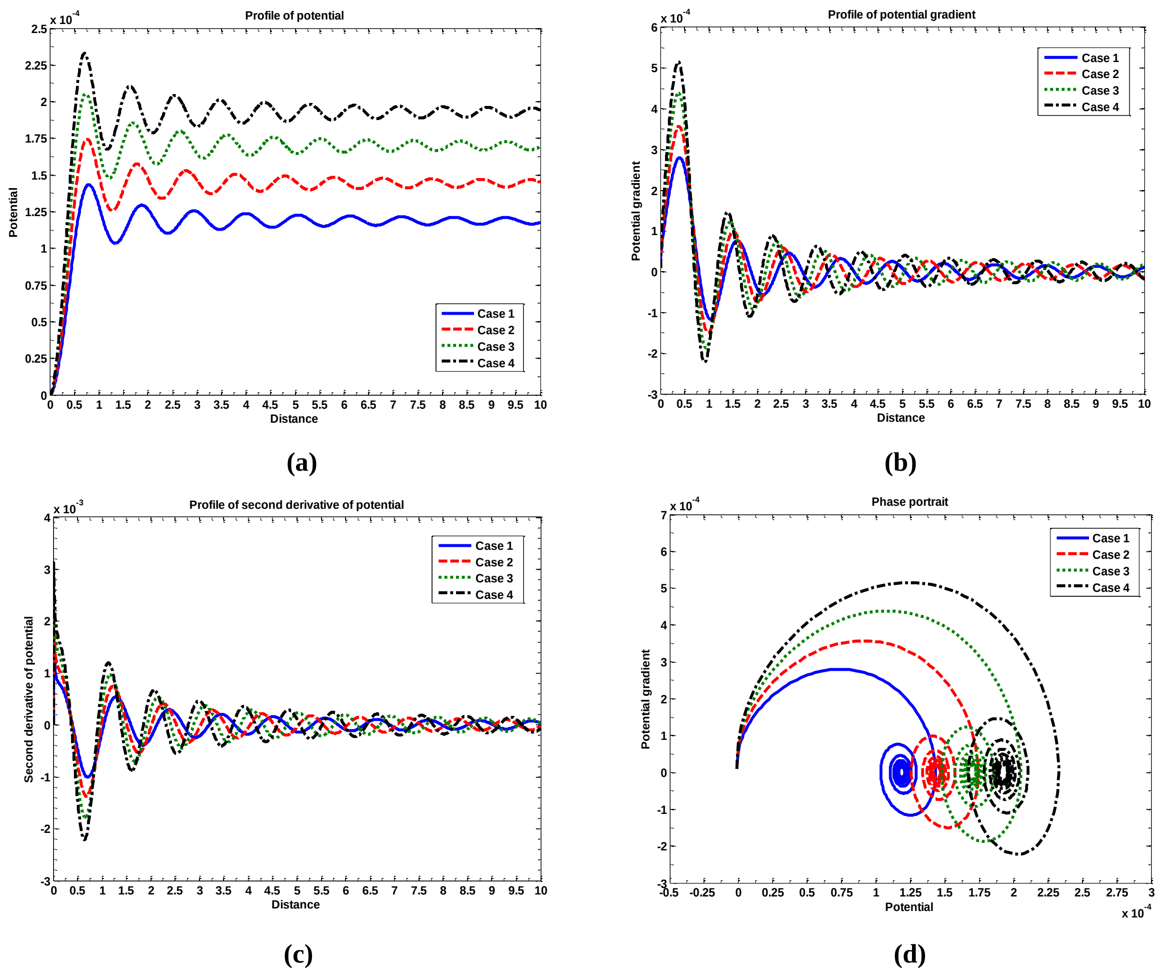

Figure 4.2 *Profile of the normalized lowest-order perturbed self-gravitational (a) potential, (b) potential gradient, (c) potential curvature (second derivative of potential), and (d) phase portrait. Various lines correspond to Case (1):* $m_d = 1.02 \times 10^{-14}$ *kg (blue line), Case (2):* $m_d = 1.07 \times 10^{-14}$ *kg (red line), Case (3):* $m_d = 1.11 \times 10^{-14}$ *kg (green line), and Case (4):* $m_d = 1.14 \times 10^{-14}$ *kg (black line), respectively. Various input and initial parameter values are given in the text.*

(6) Lastly, the following basic physical mechanism is supposed to contrive in producing the obtained numerical fluctuation plots (Figure 4.2) from the pure wave packet model point of

view. The damped oscillatory character in the form of *gravito-electrostatic wind* or *precursor* of the investigated oscillatory shock-like patterns is conjectured as due to the resonant (with phase-amplitude coherence) and non-resonant (without phase-amplitude coherence) coupling of the internal spectral components of the usual Burger shocks (wave packet model) with no source, and their background gravito-electrostatic fluctuations in presence of the source. Here, *precursor* refers to the emission of dispersing gravito-acoustic waves trailing behind the moving shock (observed from the shock reference frame). These oscillatory signatures are on an average qualitatively in agreement with others predictions reported elsewhere [1, 3, 13, 17].

4.5.1 Comparative Results

Stability analyses in dusty plasma have been carried out by many inquisitive authors in past to understand different eigenmodes they support in different equilibrium configurations [1-8]. However, only a few authors have considered self-gravitating DMC [5-7, 17-19] of stellar birth sites for nonlinear stability analysis. It can be seen that none of them has considered the Avinash-Shukla model [8] for investigating the nature of the nonlinear eigenmodes supported in the self-gravitational collapse dynamics in terms of a critical cloud mass limit with dust flow convection dynamics, and without dust flux conservation in spherically symmetric hydrodynamic neutral equilibrium configuration on the Jeans scale. We carried out our investigation to understand the model [8] from the nonlinear stability point of view in newer perspectives on the unique nonlinear eigenmode characteristics. Thus, the stability analysis of current concern differs from the existing analyses reported in the literature [5-7, 17-19] in fundamentality as well as observation. Table 4.1 lists in brief the main differences between our analysis and other existing analyses.

Table 4.1: Our analysis versus existing analyses

No	Items	Our analysis	Existing analyses [5-7, 17-19]
1	Model	Single fluid	Multi fluids
2	Geometry	Spherical	Planar
3	Eigenmode equation	*m*-KdVB equation	*m*-KdV [5-7, 18] and extended KdV equation [19]
4	Eigenmode structures	New oscillatory shock-like	Soliton-like
5	Nature of solutions	Self-gravitational shocks	Density [5-7] and gravito-electrostatic [18] solitons

6	Flux conservation	Not considered	Considered
7	Profile status	Understood by gas-solid matter dynamic analog	Understood by fluid-fluid dynamic analog [5-7, 18]
8	Dust flow convection dynamics	Added anew (modified model from [8])	Intrinsic [5-7, 18]
9	Eigenmode stability	Unstable	Stable
10	Main application	Specified as initial input elements for spherical star formation process	Not sharply specified
11	Global neutrality	Neutral	Quasi-neutral
12	Jeans swindle	Considered	Considered [18], and also not considered [5-7]
13	Charged grain state	Fully charged	Partially charged
14	Appearance of linear Derivative source in evolution equation	Yes, linear derivative source (due to dust convection dynamics)	No [5-7, 18], but the integral source that appears [19] vanishes asymptotically
15	Dust grain flow	Relatively high	Relatively low
16	Eigenmode formation condition	Only one fluid	At least two fluids (one cold, other hot) necessary for soliton formation
17	Gradient strength of eigenmode	Shown	Not shown
18	Self-gravitational curvature	Studied	Not studied
19	Eigenmode phase trajectories	Studied	Not studied
20	Self-gravitation contribution	Fully charged massive dust grains	Fully and partially charged massive dust grains
21	Eigenmode amplitude variation with grain mass	Increases	Decreases (due to above)

22	Neutral-charged dust interaction	Absent	Present
23	Main outcome	Oscillatory shock-like structures are supported in self-gravitating dust cloud	Soliton-like structures are only supported in self-gravitating dust cloud
24	Supporting experimental observations	Partially supported by multispace satellite observations (qualitatively)	Well supported by various satellite-based observations
25	Scope for future applicability	Input elements to study spherical star and planetary rings with various geometrical curvature in realistic conditions	Input to study interstellar and planetary space with no curvature (plane geometry approximation)

2.6. CONCLUSIONS

We study the stability of a charged DMC in the presence of velocity convection dynamics and self-gravity on the Jeans scales of space and time. We apply the standard methodology of analytical multiscale scheme under defined equilibrium configuration in spherical geometry to investigate the nonlinear eigenmode structures supported in the cloud. The main motivation behind it is to examine the effect of dust convection dynamics with inhomogeneous flux on the nature of eigenmodes. The cloud is found to be rich in new classes of different oscillatory shock-like structures. They are self-gravitational in origin due to the presence of massive charged-dust grains. This class of eigenmodes are collectively governed by a new type of *m*-KdVB equation having a self-consistent driving linear derivative source on the lowest-order perturbed self-gravitational potential arising from the massive charged dust flow convection dynamics. In addition, this simplified contribution idealistically shows how point particle approximation (within the validity limit of non-relativistic Newtonian dynamics) can give rise to the propagation of different oscillatory shock-like structures with new and unique properties in the presence of gravito-electrostatic coupling on the Jeans length for the first time. Such eigenmodes may be applied as initiation input elements in space science and modern astrophysics because of their crucial role in understanding self-gravitational collapse, the formation and evolution of interstellar clouds, star formation, galactic structure and its evolution, and so on. The obtained oscillatory shock-like structures are in qualitative agreement with those of the earlier predictions *in situ* made by various spacecraft instruments, on-board multispace satellite reports, and experimental findings under different conditions [1, 4, 14-15]. We must admit that our investigation is quite idealized for steady state observations. The adopted mathematical and numerical strategies may, however, be extended for further exploration of the self-gravitational fluctuation dynamics with more complications such as collisions, different gradient forces, grain-size distribution, charge variation, and so on, taken into account in different astrophysical situations. Lastly, our investigation may afford different and wider scopes for elaborate improvement and refinements to understand the temporal eigenmode evolution with various equilibrium spatiotemporal inhomogenetiies.

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