CHAPTER-7

GRAVITATIONAL FLUCTUATIONS IN SELF-GRAVITATIONALLY CONFINED SOLAR PLASMA

Abstract: *The Sun is a mysterious stellar structure formed due to self-gravitational instability of interstellar dust molecular cloud. Even after formation of its equilibrium structure, it is under constant self-gravitational effects, as reflected via helioseismic modes and surface oscillations. In this chapter, we develop a theoretical evolutionary model for investigating new nonlinear self-gravitational fluctuations associated with the bounded solar plasma system with the lowest-order inertial correction of the plasma thermal electrons considered. Application of multiscale analysis over the coupled gravito-electrostatic equilibrium solar structure equations results in a unique type of driven Korteweg-de Vries Burger (d-KdVB) equation, revealing mainly monotonous shock-like eigenmodes. The self-consistent new and unique nonlinear driving source here appears due to the inclusion of weak but finite electron inertia. The observed self-gravitational fluctuations are in good agreement with multispace satellite and imaging detections made by others in past. The results are significant for probing intrinsic structural properties of the solar, stellar, or other astrophysical media through which the fluctuations propagate.*

7.1 INTRODUCTION

Waves and oscillations are ubiquitous in the self-gravitating plasmas like the Sun, stars, and their atmospheres [1-5]. The Sun, which is the huge ionized gas ball, is known to support rich spectrum of nonlinear collective waves, oscillations, and fluctuations evolving in the form of shocks, solitons, vortices, hybrid structures, and so forth [1-5]. They are naturally excited by different internal energetic mechanisms [1-5]. Various experimental observations, multispace satellite data, and imaging findings equally support the existence of such eigenmodes [4-5] in the solar atmosphere. The launch of the *Extreme-ultraviolet Imaging Telescope (EIT)* onboard the *Solar and Heliospheric Observatory (SOHO)*, and *Transition Region and Coronal Explorer (TRACE)* spacecrafts, make possible the direct observations of wave activities in the solar atmosphere [1-5].

Many authors have reported the excitation, existence, and propagation of a wide variety of nonlinear wave dynamics in a star like the Sun and its atmosphere by applying different model approaches centered on multiscale analyses [1-3]. In addition, as far as seen, all the earlier nonlinear stability analyses of the solar plasma are based on Magneto-Hydro-Dynamic (MHD) equilibrium configuration [1-2]. The effects of space charge, plasma-boundary wall interaction, and sheath formation mechanism have hardly been taken into account. Besides, the inertial responses of the thermal electrons are absolutely neglected for physical and mathematical simplicity. Thus, the nonlinear eigenmode features in the light of all the factors mentioned above are yet to be understood both analytically as well as numerically.

In this chapter, we propose a simplified theoretical model to investigate the nonlinear self-gravitational eigenmodes supported in an idealized self-gravitating stellar plasma like the Sun and its atmosphere with the inertia-corrected Boltzmann distributed electrons. We consider the lowest-order inertial correction of the electrons [6] in the collective excitation processes of the self-gravitational solar eigenmodes in quasi-neutral hydrodynamic equilibrium. The principal stimulus of the investigation is to examine whether the nonlinear and dispersive nature of self-gravitational waves [1-2] are sustained in the Sun and its atmosphere in presence of electron inertia and plasma-boundary interaction. The inertia–induced acoustic excitation theory [6] is applied in the plasma-based Gravito-Electrostatic Sheath (GES) model [7] for the investigation. Basing on the equilibrium GES model [7], the entire solar plasma system divides into two basic parts: the Sun which is the subsonic Solar Interior Plasma (SIP) on the bounded scale, and the supersonic or hypersonic Solar Wind Plasma (SWP) on the unbounded scale. The Solar Surface Boundary (SSB) on the lowest-order couples the SIP (Sun) with the radially outflowing SWP through plasma-boundary wall interaction processes and act as an interfacial transitional surface.

A distinct set of non-autonomous self-consistently coupled nonlinear dynamical eigenvalue equations in the defined astrophysical scales of space and time is developed. In view of that, a unique form of driven Korteweg-de Vries Burger (*d*-KdVB) equation containing a self-consistent nonlinear driving source is derived on the SIP scale in terms of the lowest-order self-gravitational fluctuation by the application of multiple scaling technique [8]. It is interesting to note that, the source originates from the lowest-order inertial correction of the plasma thermal electrons. It is then studied both analytically by Tangent Hyperbolic method (approximate) [9] and numerically by the Runge-Kutta IV method (exact) [10] as an initial value problem in detail. It is found that the nonlinear self-gravitational potential fluctuations on the lowest-order evolve as a unique class of shock-like structures with some new features, which may have extensive applications in nonlinear self-gravitational plasma wave dynamics of helioseismic importance [4-5, 11]. The internal structure of the Sun can also be diagnosed with the help of such analyses. The fluctuations can carry information of the plasma medium through which they propagate, and therefore, they can provide a unique tool for solar plasma diagnostics.

7.2 THE MODEL

A simplified solar plasma fluid model is adopted to study the nonlinear self-gravitating solar plasma fluctuations under a global hydrodynamic type of homogeneous equilibrium configuration. The self-gravitationally bounded quasi-neutral field-free plasma by a spherically symmetric surface boundary of non-rigid and non-physical nature is considered. An estimated value $\sim 10^{-20}$ of the ratio of the plasma Debye and Jeans length scales of the total solar mass justifies the global quasi-neutrality behavior on both the bounded SIP and unbounded SWP scales for $T_e = 10^6$ K. The exact hydrostatic condition of the gravito-electrostatic force balancing defines the zeroth-order boundary surface, which encloses the solar plasma mass at some arbitrary radial position $(\xi \approx 3.50\lambda_J)$ from the center of the SIP mass distribution [1, 9]. A bulk non-isothermal uniform flow is assumed to pre-exist. For minimalism, we consider spherical symmetry because this helps to reduce the three-dimensional problem of describing the GES into a simplified one-dimensional problem in the radial direction. The curvature effects are ignorable for small scale size of the fluctuations. The entire solar plasma is assumed to consist of a single component of hydrogen ions as the inertial species, and electrons as the thermal species. The electrons are assumed to show weak inertial characteristics on the lowest-order. Again, the ions are assumed to exhibit their full inertial response dynamics governed by normal fluid equations. This includes the force density conserving ion fluid momentum equation as well as the flux conserving ion continuity equation. The entire dynamical response is finally closed by the electrostatic Poisson equation. The self-gravitational Poisson equation may not justifiably close up the full system. The weak inertia of the electrons produce insignificant contribution to the net self-gravitational potential distribution as compared with that developed by the ions. This, in addition, may be pertinent to add that all the model equations being considered here are in the nonrelativistic classical limit of the Newtonian gravity without involving any contribution from the special theory of relativity in the Minkowski space. This model setup sustains nonlinearity due to fluidity, dispersion due to self-gravity and dissipation due to collective collisional dynamics of intrinsic solar origin. The average strength of the electric forces due to space charge polarization effects (local charge imbalance) is assumed to be too weak to excite higher order contributions on the Jeans scale.

This is to elucidate that our plasma-based theory of the nonlinear self-gravitational stability is quite idealized in the sense that it does not include any complication like the magnetic forces, nonlinear thermal forces, spatio-temporal inhomogeneities, rotation and the role of interplanetary medium or any other difficulties like collisions, viscous processes, and so forth.

7. 3 BASIC NORMALIZED SET OF EQUATIONS

When the lowest-order inertial correction of the electrons is taken into account [6], their Maxwellian population density distribution is modified [6] and expressed in normalized form as,

$$N_e = exp\left[\left(\frac{m_e}{m_i}\right)M_{eo}^2\left(1-exp\,2\theta\right)-\theta\right], \tag{7.1}$$

where, m_e is the electronic mass, m_i the ionic mass of hydrogen, M_{eo} the equilibrium electron Mach number, and θ the normalized plasma potential associated with the GES.

The plasma ions are described by their full inertial response in their dynamical evolution in hydrodynamic equilibrium. The normalized (with all the standard astrophysical parameters) set of the basic structure equations are enlisted as follows,

$$\frac{\partial M}{\partial \tau} + M\frac{\partial M}{\partial \xi} = -\frac{\partial \theta}{\partial \xi} - \in_T \frac{1}{N_i}\frac{\partial N_i}{\partial \xi} - \frac{\partial \eta}{\partial \xi}, \tag{7.2}$$

$$\frac{1}{M}\frac{\partial N_i}{\partial \tau} + N_i\frac{2}{\xi} + \frac{N_i}{M}\frac{\partial M}{\partial \xi} + \frac{\partial N_i}{\partial \xi} = 0, \tag{7.3}$$

$$\left(\frac{\lambda_{De}}{\lambda_J}\right)^2\left(\frac{\partial^2 \theta}{\partial \xi^2} + \frac{2}{\xi}\frac{\partial \theta}{\partial \xi}\right) = exp\left[\left(\frac{m_e}{m_i}\right)M_{eo}^2\left(1-exp\,2\theta\right)-\theta\right] - N_i, \text{ and} \tag{7.4}$$

$$\frac{2}{\xi}\frac{\partial \eta}{\partial \xi} + \frac{\partial^2 \eta}{\partial \xi^2} = N_i. \tag{7.5}$$

It is further assumed that the weak electron inertia is too skinny to contribute to self-gravitational potential given by equation (7.5). Here, $\in_T = T_i/T_e$ is the ratio of ion to electron temperatures (in eV). The parameters $M(\xi)$, $\theta(\xi)$, and $\eta(\xi)$ represent the normalized equilibrium ion flow Mach number, electrostatic potential, and solar self-gravitational potential, respectively. They are, correspondingly, normalized by plasma sound phase speed (c_s), electron thermal potential (T_e/e)

, and plasma sound phase speed squared $\left(c_s^2\right)$. The independent variables like position (ξ) and time (τ) are normalized with the Jeans length (λ_J) and the Jeans time $\left(\omega_J^{-1}\right)$ scales, respectively. Moreover, N_i and N_e are, in that order, the ion and electron population densities normalized by the equilibrium bulk plasma density (n_o). Equations (7.2)-(7.5) constitute the basic normalized set of the coupled dynamical structure equations for the equilibrium GES picture with the weak electron inertia. It is well known that, the self-gravitating plasmas are inhomogeneous in nature. So, all the normalization parameters should keep on changing globally in radial direction. But, to see the fluctuation dynamics in an idealized situation, as in the present situation, we presumed them as constants. Our approach is justifiable on the basis of the Jeans assumption of homogeneous medium in mathematical simplification for nonlinear local node analyses.

7.4 DERIVATION OF *d*-KdVB EQUATION

We apply standard methodology of multiple scaling technique [8-9] to describe the self-gravitational fluctuations of the GES model in presence of the lowest-order inertial correction of the thermal electrons on the Jeans scales of space and time. The dependent variables are expanded nonlinearly (in various $\in$-powers) around the respective characteristic equilibrium values as,

$$\begin{pmatrix} N_e \\ N_i \\ M \\ \theta \\ \eta \end{pmatrix} = \begin{pmatrix} N_{eo} \\ N_{io} \\ M_o \\ \theta_o \\ \eta_o \end{pmatrix} + \in \begin{pmatrix} N_{e1} \\ N_{i1} \\ M_1 \\ \theta_1 \\ \eta_1 \end{pmatrix} + \in^2 \begin{pmatrix} N_{e2} \\ N_{i2} \\ M_2 \\ \theta_2 \\ \eta_2 \end{pmatrix} + \ldots\ldots\ldots\ldots\ldots, \tag{7.6}$$

where, the equilibrium values of densities are usually unity, and those of others are zero in equation (7.6) for idealized plasma situation. But, we adopt the numerical values consistent with the GES model [7] discussed later. The independent variables are stretched into a new space coordinatized by the transformations $X = \in^{1/2} (\xi - \mu\tau)$, and $T = \in^{3/2} \tau$, where μ is the phase speed of the self-gravitational fluctuations, and $\in$ is a minor scaling parameter characterizing the dimensionless amplitude of the lowest-order fluctuations. Now, substituting equation (7.6) over the equations (7.2)-(7.5) for order-by-order analyses of various powers of $\in$ and applying the systematic methodology of elimination and simplification, we get,

$$\frac{\partial \eta_1}{\partial T} + \chi_1 \eta_1 \frac{\partial \eta_1}{\partial X} + \chi_2 \frac{\partial^3 \eta_1}{\partial X^3} + \chi_3 \frac{\partial^2 \eta_1}{\partial X^2} = S_o, \tag{7.7}$$

where, $S_o = \left[\left\{-\chi_4 - \chi_6\left(\frac{\partial \eta_1}{\partial X}\right) - \chi_7\left(\frac{\partial^2 \eta_1}{\partial X^2}\right)\right\}\left(\frac{\partial \eta_1}{\partial X}\right) - \chi_5 \eta_1\right]$.

Equation (7.7) represents a unique form of driven Korteweg-de Vries Burger (*d*-KdVB) equation containing a self-consistent nonlinear driving source S_0 originating due to the lowest-order inertial correction of the electrons in our model. The coefficients associated with equation (7.7) are

$$\chi_1 = \left[-\frac{4\mu T\{M_o(M_o-\mu) - \in_T\}}{X^3 M_o Z}\right], \tag{7.8}$$

$$\chi_2 = \left[-\frac{\{M_o(M_o-\mu) - \in_T\}^2}{N_{io} M_o}\right], \tag{7.9}$$

$$\chi_3 = \left[\left\{\frac{4\mu^2 T^2 \in_T - X^4 N_{io} + 4\mu T X\{M_o(\mu - M_o) + \in_T\} - 4XM_o^2 \mu T}{2X^2 N_{io} \mu T M_o}\right\}\{M_o(M_o-\mu) - \in_T\}\right], \tag{7.10}$$

$$\chi_4 = \left[\left\{\frac{-2M_o \mu^2 TX - 3X^4 N_{io} - 2\mu TX\{M_o(\mu - M_o) + \in_T\} - 4\mu^2 T^2 \in_T + 4X\mu T M_o^2 + 6X\mu T M_o(\mu - M_o)}{X^3 N_{io} \mu T M_o}\right\}\{M_o(M_o-\mu) - \in_T\}\right], \tag{7.11}$$

$$\chi_5 = \left[-\frac{2\{M_o(M_o-\mu) - \in_T\}}{M_o \mu T}\right], \tag{7.12}$$

$$\chi_6 = \left[\left\{\frac{8\mu^3 T^2 M_o}{X^5 N_{io}} - \frac{8\mu^2 T^2}{ZX^5 N_{io}\left(1 + 2\frac{m_e}{m_i} e^{2\theta_o} M_{eo}^2\right)} - \frac{2\mu T}{X^2 Z} + \frac{2\mu T}{X^2}\right\}\frac{\{M_o(M_o-\mu) - \in_T\}}{M_o}\right], \text{ and} \tag{7.13}$$

$$\chi_7 = \left[\left\{\frac{4\mu^2 T^2\{M_o(M_o-\mu) + \in_T\}}{ZX^4 N_{io}^2} - \frac{4\mu^3 T^2 M_o}{X^4 N_{io}^2}\right\}\frac{\{M_o(M_o-\mu) - \in_T\}}{M_o}\right]. \tag{7.14}$$

We look for possible steady-state structures of the fluctuations. Equation (7.7) is transformed into an ordinary differential equation (ODE) with the Galilean type of transformation, $\xi = X - VT$, with normalized frame velocity $V = 1$ without any loss of generality of the fluctuation dynamics as,

$$\frac{\partial \eta_1}{\partial \xi} + A\eta_1 \frac{\partial \eta_1}{\partial \xi} + B\frac{\partial^3 \eta_1}{\partial \xi^3} + C\frac{\partial^2 \eta_1}{\partial \xi^2} = S_o, \tag{7.15}$$

where, the transformed nonlinear source $S_o = \left[a\eta_1 + \left\{b\left(\partial\eta_1/\partial\xi\right) + c\left(\partial^2\eta_1/\partial\xi^2\right)\right\}\partial\eta_1/\partial\xi\right]$. The various transformed coefficients are given by $A = \chi_1/(\chi_4 - 1)$, $B = \chi_2/(\chi_4 - 1)$, $C = \chi_3/(\chi_4 - 1)$, $a = -\chi_5/(\chi_4 - 1)$, $b = -\chi_6/(\chi_4 - 1)$, and $c = -\chi_7/(\chi_4 - 1)$. The strength of the coefficients appearing in equation (7.15) for the normal GES description [7] with $\in_T = 0.40$, and $T = 55.00$ can be estimated as $A = -0.1781$, $B = 0.2313$, $C = 0.0270$, $a = 0.0150$, $b = 0.2743$, and $c = -0.5486$. It shows the possibilities for the existence of shock-like structures (due to energy dissipation) and soliton-like structures (due to energy dispersion) jointly. The excitation of the *d*-KdVB eigenmodes here is due to the integrated interplay of nonlinearity, dispersion and dissipation of internal solar origin.

7.5 RESULTS AND DISCUSSIONS

The results obtained from our simplified and idealized calculation schemes on the solar plasma fluctuation dynamics are discussed in detail in the following subsections.

7.5.1 Analytical Results

The *d*-KdVB dynamics in our model is of highly nonlinear type. So, the exact analytical solution in an explicit form is difficult to achieve. We apply the systematic methodology of *Tangent Hyperbolic method*, abbreviated as *tanh*-method [9], to obtain it. Accordingly, the explicit solution of equation (7.15) is assumed as series expansion in functional form given by,

$$\eta_1(\xi) = \sum_{j=0}^{p} A_j \tanh^j(K\xi). \quad (7.16)$$

The upper limit p can be determined by the homogeneous balance of the highest-order nonlinear (convective) term with the highest-order linear (dispersive) term in the equation (7.15) so that the hydrodynamical equilibrium conditions of the expected eigenmode formation are sustained in it. In our present case $p = 2$, and equation (7.16) can thus be written as,

$$\eta_1(\xi) = A_o + A_1 \tanh(K\xi) + A_2 \tanh^2(K\xi). \quad (7.17)$$

Here, A_o, A_1, A_2, and K are determined by substituting equation (7.17) and its various derivative in equation (7.15) with the help of order-by-order analysis. Accordingly, the approximate solution of equation (7.15) governing the self-gravitational fluctuations of solar plasma is found as,

$$\eta_1(\xi) = \frac{3\chi_2\chi_1(2\chi_2\chi_1 + \chi_7 - \chi_4\chi_7)}{\chi_7(\chi_6\chi_7\chi_5 + 3\chi_2\chi_1^2)} + \frac{6\chi_2}{\chi_6}\sqrt{\frac{\chi_1}{2\chi_7}}\, tanh\left[\left\{\sqrt{\frac{\chi_1}{2\chi_7}}\right\}\xi\right]. \tag{7.18}$$

Equation (7.18) represents shock-like structures as the nonlinear eigenmodes of the GES in the presence of the lowest-order inertial electrons. The shock amplitude A_{sh}, shock front thickness Γ_{sh}, and dissipation strength S from equations (7.15) and (7.18) are given respectively as follows,

$$A_{sh} = \frac{6\chi_2}{\chi_6}\sqrt{\frac{\chi_1}{2\chi_7}},\ \Gamma_{sh} = \sqrt{\frac{2\chi_7}{\chi_1}}\ \text{, and } S = \chi_3/(\chi_4 - 1).$$

The different coefficients $\chi_1 \approx 0.1232$, $\chi_2 \approx -0.1600$, $\chi_3 \approx -0.0187$, $\chi_4 \approx 0.2083$, $\chi_5 \approx 0.0104$, $\chi_6 \approx 0.1897$, and $\chi_7 \approx -0.3795$ appearing in equation (7.18) for $\in_T = 0.40$ and $T = 55.00$ are now approximately known. One can, therefore, calculate the various shock parameters as $|A_{sh}| \sim 2.03$, $|\Gamma_{sh}| \sim 2.48$, and $S \sim 0.02$ in the normal GES condition [7].

The main features of the graphical nature based on the explicit solution represented by equation (7.18) may be discussed as follows. Figure 7.1 shows the graphical profile of the lowest-order perturbed self-gravitational potential obtained analytically on the bounded scale (SIP). Various lines correspond to Case (1): $M_o = 1.00 \times 10^{-10}$ (blue line), Case (2): $M_o = 3.00 \times 10^{-10}$ (red line), Case (3): $M_o = 4.00 \times 10^{-10}$ (green line), and Case (4): $M_o = 6.00 \times 10^{-10}$ (black line), respectively. The other parameters kept constant are $M_{eo} = 1.00 \times 10^{-4}$, $m_e/m_i = 1.00 \times 10^{-3}$, $\theta_o = -0.001$, $\mu = 5.00$, $T = 127.30$, $\in_T = 0.40$, and $(\lambda_{De}/\lambda_J) = 1.00 \times 10^{-20}$. It shows monotonic shock-like structures with amplitude $\sim 10^{-3}$. The average amplitude at the SSB is 0.88×10^{-3}, which is physically $\eta_{phy}(\xi_\Theta) \sim \in \eta_1(\xi) c_s^2 = 2.64 \times 10^5$ J kg^{-1} for $\in \sim 10^{-2}$ [1, 7, 9] and $T_e \sim 10^3$ eV [7]. This is observed that the lowest-order perturbed self-gravitational potential shows unique class of shock-like structures. The uniqueness is due to the fact that the shock-transition takes place only after the SSB, but never before that. Thus, our idealistic field-free solar plasma model analytically supports the existence of a new type of shock-like eigenmodes due to the homogeneous balancing between hydrodynamic nonlinearity, and the combined action of self-gravitational dispersion and collective dissipative processes of internal solar origin on the Jeans scale. The analytical profiles show no self-gravitational potential fluctuations in the core of the Sun. This reveals that the core is the most stable area compared to the rest of the entire Sun from self-gravitational stability point of view. Physically, it may be inferred that, the absence of self-gravitational fluctuations in the core is due to highly stabilized and compressed density distribution of the plasma particles present therein. This is an exciting outcome going as per the usual particle flux conservation rule in accordance with the hydrodynamical equation of continuity of the solar inertial species. The analytical evolutionary dynamics of the self-gravitational solar plasma fluctuations by varying other sensitive plasma parameters are discussed elaborately in the Ref. [13].

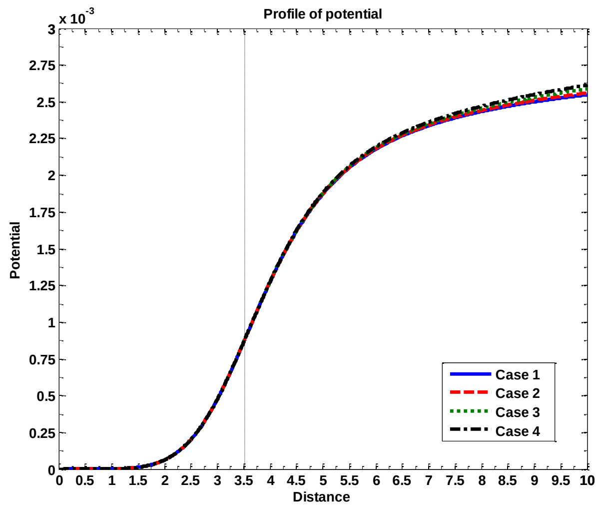
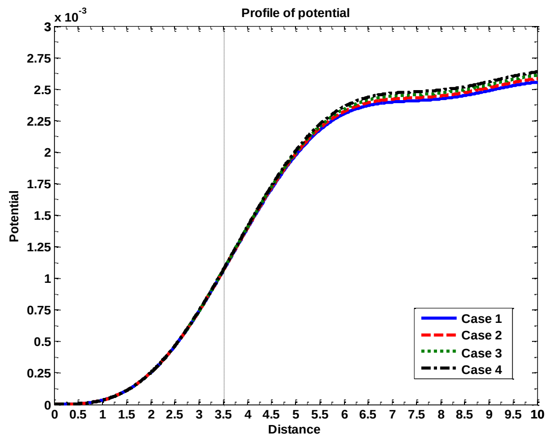


Figure 7.1 *Profile of the normalized lowest-order perturbed self-gravitational potential obtained analytically on a bounded scale (SIP). Various lines correspond to Case (1):* $M_o = 1.00\times10^{-10}$ *(blue line), Case (2):* $M_o = 3.00\times10^{-10}$ *(red line), Case (3):* $M_o = 4.00\times10^{-10}$ *(green line), Case (4):* $M_o = 6.00\times10^{-10}$ *(black line), respectively. Various inputs parameter are given in the text.*

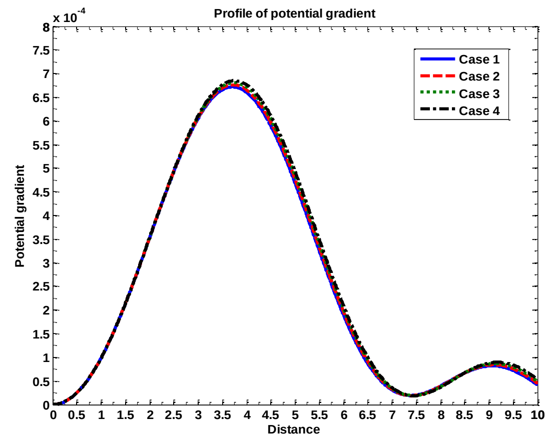
7.5.2 Numerical Results

The nonlinear self-gravitational solar fluctuation dynamics in presence of the thermal electron inertia under the GES model framework is governed by a unique type of *d*-KdVB equation. This equation due to highly nonlinear and complicated nature for exact integration, is numerically solved as an initial value problem with the help of the Runge-Kutta IV method [10] to get exact evolutionary profiles. Figure 7.2 shows the profile structures of the lowest-order perturbed self-gravitational (a) potential, (b) potential gradient, (c) potential curvature, and (d) phase portrait, respectively. Various lines correspond to Case (1): $M_o = 1.50\times10^{-3}$ (blue line), Case (2): $M_o = 4.50\times10^{-3}$ (red line), Case (3): $M_o = 7.50\times10^{-3}$ (green line), Case (4): $M_o = 10.50\times10^{-3}$

(black line), respectively. Different input initial values used are $\eta_i = 1.00 \times 10^{-6}$, $(\eta_\xi)_i = 9.00 \times 10^{-9}$, $(\eta_{\xi\xi})_i = 1.00 \times 10^{-11}$. The other parameters kept constant are $M_{eo} = 1.00 \times 10^{-4}$, $m_e/m_i = 1.00 \times 10^{-3}$, $\theta_o = -0.001$, $\mu = 1.00$, $T = 55.00$, $\in_T = 0.40$, and $(\lambda_{De}/\lambda_J) = 1.00 \times 10^{-20}$. The input Mach number value is chosen from the expression $M_{oi} = 1/2\xi_i \, e^{\theta_o/2}$ as in our earlier work [7].



(a)



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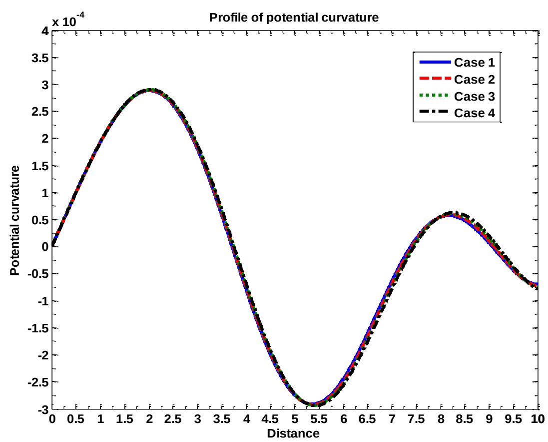

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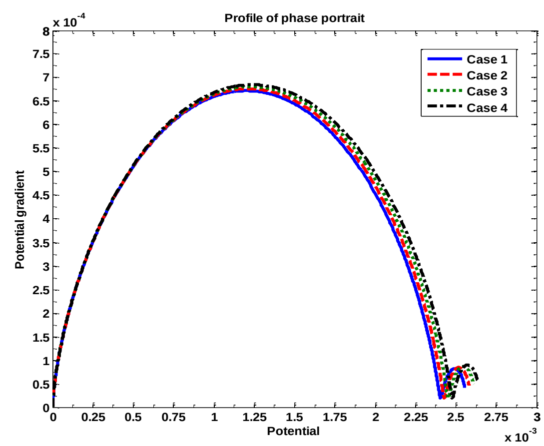

(d)

Figure 7.2 *Profile of the normalized lowest-order perturbed self-gravitational (a) potential, (b) potential gradient, (c) potential curvature, and (d) phase portrait on a bounded scale (SIP). Various lines correspond to Case (1):* $M_o = 1.50 \times 10^{-3}$ *(blue line), Case (2):* $M_o = 4.50 \times 10^{-3}$ *(red line), Case (3):* $M_o = 7.50 \times 10^{-3}$ *(green line), Case (4):* $M_o = 10.50 \times 10^{-3}$ *(black line), respectively. Various input and initial parameter values are given in the text.*

The numerical profile shows that the lowest-order perturbed self-gravitational potential in the Sun evolve as shock-like eigenmode structures (Figure 7.2(a)) with amplitude $\sim 10^{-3}$ which is physically $\eta_{phy}\left(\xi_{\Theta}\right) \sim \in \eta_1\left(\xi\right) c_s^2 \sim 10^5$ J kg^{-1} for $\in \sim 10^{-2}$ [1, 7, 9] and $T_e \sim 10^3$ eV [7]. Being of almost quasi-conservative nature (with no dissipative agencies in the governing equations), their gradients appear as soliton-like structures (Figure 7.2(b)) in accordance with the quasi-conservative rule for self-gravity, $g_s \sim \partial \eta_1 / \partial \xi$. Furthermore, the potential curvature profile (Figure 7.2(c)) give an empirical glimpse of the solar material density fluctuations in conformity with the self-gravitational Poisson equation on global mass-neutrality. Lastly, the fluctuation phase portrait (Figure 7.2(d)) shows the geometrical patterns (parabolic type) of the dynamical evolution of the self-gravitational solar eigenmodes. Thus, it is clear that although collisional effects are neglected in our model, collisionless shock-like shapes result due to the internal collective dynamics of the solar plasma particles, thereby providing the requisite thresholds for the shock-like eigenmode excitations. The evolutionary dynamics of the self-gravitational solar plasma fluctuations obtained by varying other plasma parameters under the same condition as figure 7.2 are shown and discussed in the Ref. [13].

7.5.3 Comparative Results

The primary motivation of our theoretical analyses is to provide qualitative supports to various theoretical, experimental, and multispace satellite observations on the self-gravitational solar shock features reported by a number of authors [1-5]. Besides, the second key stimulus is to examine whether the nonlinear nature of self-gravitational waves [12], and corresponding soliton-family eigenmode formation are possible in the self-gravitating solar plasma model based purely on the plasma-based GES theory [7]. Based on our findings on the classical Newtonian gravity, the following comparisons may possibly be worth mentioning.

7.5.3.1 Analytical and Numerical Results

Both analytical and numerical findings equally support the existence of shock-like eigenmode structures in the solar plasma. The amplitude of the lowest-order fluctuation at the SSB comes out $\sim 10^{-3}$ for both the calculation schemes, which is physically $\sim 10^5$ J kg^{-1}. Some of the main differences between approximate analytical results and exact numerical solutions are given below.

(1) The approximate analytical solutions are obtained by applying *tanh*-method, whereas the exact numerical solutions are obtained by applying the Runge-Kutta IV method.
(2) Analytical treatment shows heliocentre to radiation zone of the Sun as the higher stability regime of the GES equilibrium against external stimuli, whereas all numerical profiles depicts that the core is of the higher stability.
(3) Analytical profile gives approximate and qualitative nature of the eigenmodes, but numerical profiles give detailed characteristics with some microscale differences.
(4) Lastly, standard input values of solar plasma parameters are used in the simulation, whereas for analytical solution, both approximate and realistic values are used for some parameters.

7.5.3.2 Experimental and Theoretical Results

A good numbers of astrophysicists [1-5] have reported the existence of wide-range spectrum of nonlinear wave dynamics in the solar atmosphere. Relying on various experimental reports and satellite data, it is confirmed that there are some shock-like nonlinear structures excited in the solar atmosphere. Our theoretical findings on collisionless shocks derived from the GES theory are qualitatively in agreement with others [1-5]. The main signatures of various nonlinear waves and associated eigenmodes have been analyzed by many curious minds theoretically [1-2] as well as experimentally [3-5] in different situations in recent past. The Global Oscillation Network Group (GONG), Stellar Observations Network Group (SONG), Helio-and Asteroseismology (HELAS) Network, and Birmingham Solar Oscillations Network (BiSON) are some examples of recent studies being undertaken to study various solar eigenmodes through space and ground based remote-sensing observations [4-5, 11]. The generation and detection of the self-gravitational waves and their nonlinear signatures proposed by others partially supports our theoretical results for self-gravitational helioseismic modes on an averaging process. Such helioseismic (asteroseismic) waves are extensively useful [11] as diagnostic tools of the structural interiors and thermodynamic behaviours of the stellar structures in different situations. Although idealized, ours' is a first effort on self-gravitational helioseismic modes in presence of the electron inertia in that new direction within the validity limit of the Newtonian gravity. Besides, it tries to provide theoretical support to the experimental findings as applications.

7.6 CONCLUSIONS

In this Chapter, an idealized plasma model is theoretically proposed to study the nonlinear self-gravitational fluctuation dynamics in the Sun and its atmosphere in presence of the electron inertia under the framework of the plasma-based GES theory in hydrodynamic equilibrium on the Jeans scale. The self-gravitational solar fluctuations undergo steep transition in the form of shock-like structures under the interplay of hydrodynamic nonlinearity, self-gravitational dispersion and collective dissipative processes of internal solar origin. The dynamical evolution of the structures is collectively governed by a new type of *d*-KdVB equation containing a self-consistent nonlinear source. This source naturally comes due to the contribution of the lowest-order inertial correction of the plasma electrons, but without affecting the Poisson gravity. Both analytical and numerical findings go hand in hand. To summarize in brief, the main points which may be drawn from our idealized theoretical analysis on the complex stratified solar plasmas are highlighted as follows.

(1) Nonlinear eigenmodes of self-gravitational fluctuations of the solar plasma in presence of the inertial electrons are collectively governed by a unique form of *d*-KdVB equation with a new nonlinear driving source self-gravitational in origin.

(2) Due to the active participation of the lowest-order inertial correction of the plasma thermal electrons, the nonlinear driving source is self-consistently originated in the model setup.

(3) The self-gravitational solar fluctuations evolve as shock-like structures due to the collective dynamics of the massive ions primarily, and electrons (due to weak inertial effect) secondarily under the considered hydrodynamic solar plasma conditions.

(4) The solar plasma system supports the self-gravitational shock-like eigenmode excitations under some judicious choice of the solar plasma conditions, even within the validity limit of the Newtonian gravity and point mass approximation, which are qualitatively in conformity with some multispace satellite observations as well [1-5].

(5) The self-gravitational fluctuation profiles reveal that, the entire solar plasma system is self-gravitationally unstable against nonlinear perturbations.

(6) Moreover, the various phase portraits show that the trajectories of the plasma particles overlap over one another for the potential value corresponding to that of the core of the Sun, which reveals that the core is the most stable area compared to the rest of the Sun. Physically, it can be explained that, the absence of self-gravitational potential fluctuations in the core is due to high, stabilized and compressed density distribution of the plasma particles.

(7) We admit that the theoretical formalism adopted is quite idealistic, since it ignores external field, rotation, convective circulation dynamics, spatio-temporal inhomogeneties, kinetic

viscosity, etc. Although simplified and idealized, our theoretical findings are in good agreement with various reported experimental observations on helioseismic fluctuations and eigenmodes [4-5, 11] qualitatively.

(8) The adopted simplified techniques, strategies and methodologies may extensively be useful in understanding nonlinear fluctuation dynamics of the Sun, other like stars and their atmospheres in more realistic model configurations amid various spatio-temporal inhomogeneities and force fields.

(9) Lastly, the self-gravitational nonlinear waves and eigenmodes stimulated by gravito-electrostatic coupling processes in the solar plasma system may be considered as explosive agencies generating various blast waves which are yet to be well understood. These, in turn, trigger plasma charge separation, and thereby excite a variety of solar-quake waves which are used in the helioseismic diagnosis of the Sun, its internal structure and its atmosphere [11]. These helioseismic and asteroseismic investigations and characterizations in detail in practical solar environment will extensively need similar mathematical and numerical strategies like ours' as an initial elementary input and application from plasma-based framework in presence of more realistic self-similar particle dynamics [1, 4-5]. These are, however, limited for presentation here due to limitations in lengthy mathematical calculations and modelings. We leave such complex interesting, important and challenging studies on the solar fluctuation dynamics induced by self-gravity for future works.

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