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List of abbreviations

Sl. No	Abbreviation	Full form
1	DMC	Dust Molecular Cloud
2	ISM	Inter-Stellar Medium
3	C-Gs	Carbon-rich Giants
4	O-Gs	Oxygen-rich Giants
5	PN	Planetary Nebulae
6	Ν	Novae
7	SN	Supernovae
8	IMCs	Interstellar Molecular Clouds
9	GCs	Globular Clouds
10	DCs	Dark Clouds
11	GMCs	Giant Molecular Clouds
12	DDCs	Dense Dust Clouds
13	DDMCs	Diffuse Dust Molecular Clouds
14	CCs	Cirrus Clouds
15	SRCs	Supernova Remnant Clouds
16	eV	electron Volt
17	m-LEE	modified Lane-Emden equation
18	CSB	Cloud Surface Boundary
19	BIS	Bounded Interior Scale
20	UES	Unbounded Exterior Scale
21	ESMs	Electrical Stellar Models
22	TMC1	Taurus Molecular Cloud 1
23	L134N	Lynds 134N
24	KdV	Korteweg-de Vries
25	d-KdV	driven Korteweg-de Vries
26	m-KdVB	modified Korteweg-de Vries Burger
27	d-KdVB	driven Korteweg-de Vries Burger

28	EIT	Extreme-ultraviolet Imaging Telescope
29	SOHO	Solar and Heliospheric Observatory
30	TRACE	Transition Region and Coronal Explorer
31	MHD	Magneto-Hydro-Dynamic
32	GES	Gravito-Electrostatic Sheath
33	SIP	Solar Interior Plasma
34	SWP	Solar Wind Plasma
35	SSB	Solar Surface Boundary
36	ODE	Ordinary Differential Equation
37	tanh-method	Tangent Hyperbolic method
38	RK-IV	Fourth-order Runge-Kutta method
39	FD method	Finite-Difference method
40	GONG	Global Oscillation Network Group
41	SONG	Stellar Observations Network Group
42	HELAS	Helio-and Asteroseismology Network
43	BiSON	Birmingham Solar Oscillations Network
44	IRAS	Infrared Astronomical Satellite
45	ISO	Infrared Space Observatory

List of symbols

Sl. No	Symbol	Meaning
1	n_s , m_s , T_s , v_s ,	Number density, mass, temperature, flow velocity and thermal
	\mathcal{V}_{ts}	velocity of the s^{th} species ($s = e$ for the electrons, i for the ions,
		dc for the charged dust, and dn for the neutral dust)
2	N_s , M_s	Normalized population density and flow velocity of the s^{th}
		species ($s = e$ for the electrons, i for the ions, dc for the charged
		dust, and dn for the neutral dust)
3	n_0	Equilibrium plasma population density
4	n_{s0}	Equilibrium number density of the s^{th} species ($s = e$ for the
		electrons, i for the ions, dc for the charged dust, and dn for the
		neutral dust)
5	T (or, T_p)	Plasma temperature
6	I _e , I _i	Electron and ion currents, respectively
7	φ,ψ	Electrostatic and self-gravitational potentials, respectively
8	Φ (or, $ heta$)	Normalized electrostatic potential
9	Ψ (or, η)	Normalized self-gravitational potential
10	$ au_{ch}$, $ au_{dr}$	Dust charging and dust response time-scales, respectively
11	F_g , F_e	Gravitational and electrostatic forces, respectively
12	M_J , M_{AS} ,	Jeans mass, stable equilibrium mass limit of Avinash-Shukla
	M _{DMC}	model (Avinash-Shukla mass limit) and net mass of the dust molecular cloud, respectively
13	L_{AS}	Critical scale-size of Avinash-Shukla mass limit model
14	R_D	Dust molecular cloud scale-size
15	$ ho_{d}$, $ ho_{E}$	Mass and electric charge densities, respectively
16	Г	Gravito-electrostatic conversion factor
17	ξ, τ	Normalized space and time coordinates, respectively

18	$\lambda_{_J}$, $\omega_{_J}$, $\omega_{_J}^{-1}$	Jeans length, Jeans frequency and Jeans time, respectively
19	C_s , C_{ss}	Ion acoustic and dust acoustic phase speeds, respectively
20	μ	Normalized Phase velocity of the fluctuations
21	q_d , Z_d , L_d	Dust charge, dust charge number and dust scale-size length, respectively
22	Q_d	Normalized dust charge
23	$F_{\it edc}$, $F_{\it ed}$, $F_{\it idc}$,	Normalized collision frequencies of the electrons and charged
	F_{id} , F_{cn} , F_{nc}	grains, electrons and dust grains, ions and charged grains, ions and dust grains, neutral and charged grains, and finally, the charged and neutral grains, respectively
24	ξ_e, ξ_i	Electron and ion transit-scale-lengths, respectively
25	\in_T	Ion-to-electron temperature ratio
26	A_{sh} , Γ_{sh} , S	Shock amplitude, shock front thickness and dissipation strength, respectively
27	\in_0 , μ_0	Permittivity and permeability of free space, respectively
28	Γ_s	Polytropic index of the s^{th} species fluid (with $s = i$ for the ions and $s = d$ for the dust grains)
29	η_i , η_d	Average transit scale-lengths of the ions and dust grains, respectively
30	V _E , V _G	Electrostatic and self-gravitational Sagdeev pseudo-potentials, respectively