Chapter-5

A THEORETICAL MODEL FORMALISM OF THE NONEXTENSIVE *GES* STRUCTURE

Abstract: *The well-established gravito-electrostatic sheath (GES) model*† *is reexamined in the presence of nonextensive electrons (Tsallis thermostatistics) and turbulence pressure (nonlinear logatropic pressure law). It is shown that the GES structure gets drastically modified due to the considered factors. The electronic nonextensivity parameter is seen to play an important role in the decrement of the GES field strength, GES potential, and so on.*

5.1 INTRODUCTION

The gravito-electrostatic sheath (GES) model is the first plasma wall interaction-based theoretical model which has been successful in explaining the fundamental issues associated with the solar plasma flow dynamics [1-5]. The key plasma sheath-centric idea underlying this is that a gravito-electrostatic force, originated from the electrostatic surface-polarization effects between the light unbounded electrons and the heavy bounded ions, is responsible for energization and acceleration of the solar wind particles. Consequently, the entire solar plasma system consists of two concentric spherical plasma layers: the bounded solar interior plasma (SIP, quasi-hydrostatic), the Sun and the unbounded solar wind plasma (SWP, hydrodynamic), the solar wind. The SIP and SWP are dynamically intercoupled by the diffused solar surface boundary (SSB) formed by an exact gravito-electrostatic force equilibration. It has successfully explored the source mechanism of the supersonic SWP flow as a quasi-linear transformation of the subsonic SIP via the GES force field. It is recognized that such plasma-based analyses have also been reported in the past to explore the dynamics of solar wind, polar wind, magnetized planets, and so forth [6-8]. It can, however, be seen that the main difference of our approach is that the GES formalism exploits the basic physics

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of plasma sheath formation, which has so long been remaining confined in ground laboratory plasmas alone, but on the astrophysical scales of space and time for the first time.

It is admitted that, in all the subsequent GES-based analyses [1-5], the electrons have been treated as isothermal inertialess fluid described by the Boltzmann distribution law. The realistic picture of the Sun and its atmosphere however predicts that the solar electrons are nonextensive in nature because of diversified long-range nonlocality and inhomogeneity effects [9, 10]. In addition, the self-gravitationally confined solar plasma volume is highly turbulent in character giving rise to diversified waves, instabilities and oscillations [11-13]. The effects of turbulence arise in such astrophysical environments because of nonlinear energy cascading processes by virtue of which large-scale flow energy is arrested and converted into small-scale energy. In this chapter, we propose a theoretical model of the GES structure accounting for the solar nonthermal electron dynamics and nonlinear logatropic pressure effects arising from turbulence. It is mainly seen that the new GES field for the bounded solar solution to exist decreases by 21.67 %, and the electrostatic potential by 30 %.

5.2 PHYSICAL MODEL

A two-fluid solar plasma model in the earlier GES model framework [1-5] in a spherically symmetric geometrical configuration (1-D) is considered. It consists of electrons as the nonthermal species and ions as the inertial (fluid) species amid a presumed global quasi-neutrality. The GES in such a situation is formed because of the space charge polarization effects across the SSB owing to the surface leakage of the lighter hot electrons leaving the heavier cold ions behind. The GES force field, thus developed, gives rise to a new source mechanism for the SWP acceleration and its subsequent supersonic flow dynamics [1-3].

5.3 MATHEMATICAL FORMULATION

The normalized separate sets of the basic governing equations in a time-stationary (steady) form describing the equilibrium dynamics of the solar plasma on the bounded SIP and unbounded SWP scales are formulated afresh with the help of the basic inputs available in the literature [1, 2]. In addition to the normal fluid dynamical equations, the sets include the coupling equations in the Poisson form owing to gravito-electrostatic interplay. Before presenting the formulated equations, it is pertinent to introduce various relevant physical parameters required for the solar plasma description alongside the standard astrophysical normalization scheme adopted herein. Here, Table 5.1 serves the purpose.

Table 5.1: Adopted normalization scheme

S No.	Normalized parameters	Normalizing parameters
1	Position (ξ)	Jeans length (λ_J)
2	Population density (N_e, N_i)	Average SIP density (n_0)
3	Electric potential (θ)	Plasma thermal potential (T_e/e)
4	Mach number (M_i)	Sound phase speed (c_S)
5	Gravitational potential (η)	Square of sound speed (c_S^2)
6	Current density (J)	SIP Bohm current density (j_B)

5.3.1 SIP calculation scheme

The electron dynamics is thermostatistically described by the non-Maxwellian (nonextensive) distribution law [9, 10] in normalized form with entropic index q_e as

$$N_e = [1+(1-q_e)\theta]^{\frac{1}{1-q_e}} . \tag{5.1}$$

The ion continuity and momentum equations along with the closing gravito-electrostatic Poisson equations are respectively given in normalized form as

$$\frac{1}{N_i}\frac{\partial N_i}{\partial \xi} + \frac{1}{M_i}\frac{\partial M_i}{\partial \xi} + \frac{2}{\xi} = 0 , \tag{5.2}$$

$$M_i \frac{\partial M_i}{\partial \xi} = -\in_T \left[\frac{1}{N_i}\frac{\partial N_i}{\partial \xi} + \frac{1}{N_i^2}\frac{\partial N_i}{\partial \xi}\right] - \frac{\partial \theta}{\partial \xi} - g_s , \tag{5.3}$$

$$\frac{dg_s}{d\xi} + \frac{2}{\xi} g_s = N_i , \tag{5.4}$$

$$\left(\frac{\lambda_{De}}{\lambda_J}\right)^2 \left[\frac{\partial^2 \theta}{\partial \xi^2} + \frac{2}{\xi}\frac{\partial \theta}{\partial \xi}\right] = N_e - N_i \ . \tag{5.5}$$

The $2/\xi$-term, appearing in equations (5.2), (5.4) and (5.5), represents the contribution from the geometric curvature (spherical) effects. The contribution from the Larson logatropic pressure effects [12, 13] in the form of plasma fluid turbulence force density is represented by the nonlinear term, $1/N_i^2\,(\partial N_i/\partial\xi)$, as shown in equation (5.3).

Most of the relevant physical parameters required for the solar plasma description are already enlisted in Table 5.1. The additional parameters with all the generic significances involved in the analytic formalism here are described as follows. The term g_s $(=d\eta/d\xi)$ represents the normalized self-gravity, ρ_Θ the mean solar material density, $G = 6.67\times10^{-11}$ m^3 kg^{-1} s^{-2} the universal gravitational constant, m_e (m_i) the electron (ion) mass and T_e (T_i) the electron (ion) temperature (in eV). Finally, $\in_T = (T_i/T_e)$ signifies the ion-to-electron temperature ratio describing the inter-species thermodynamical coupling in the model.

The solar plasma system indeed is globally quasi-neutral in nature because of large spatial extension [1-5]. As a quantitative measure of the quasi-neutrality, the ratio between the Jeans scale length to the Debye scale length is estimated as $\lambda_{De}/\lambda_J \sim 10^{-20}$ [1, 2]. So, equation (5.1) for the solar plasma global quasi-neutrality condition yields

$$N_i \approx N_e = \left[1+(1-q_e)\theta\right]^{\frac{1}{1-q_e}}. \tag{5.6}$$

Thus, the SIP equations in the presence of nonextensivity and nonlinear pressure effects (turbulence) are now obtained from equations (5.2)-(5.4) with the help of equation (5.6) as

$$\frac{\partial\theta}{\partial\xi} = -\left[1+(1-q_e)\theta\right]\left[\frac{1}{M_i}\frac{\partial M_i}{\partial\xi}+\frac{2}{\xi}\right], \tag{5.7}$$

$$M_i\frac{\partial M_i}{\partial\xi} = -\left[1+(1-q_e)\theta\right]\left[\left\{1+\in_T\left\{1+\left[1+(1-q_e)\theta\right]^{-\frac{1}{1-q_e}}\right\}\right\}\frac{\partial\theta}{\partial\xi}\right]-g_s, \tag{5.8}$$

$$\frac{\partial g_s}{\partial\xi} = \left[1+(1-q_e)\theta\right]^{\frac{1}{1-q_e}}-\frac{2}{\xi}g_s. \tag{5.9}$$

Finally, the normalized form of the net electric current density [3] on the bounded SIP scale is derived and cast as

$$J_{SIP} = M_i - \left(\frac{m_i}{m_e}\right)^{\frac{1}{2}} \left(\frac{\xi_0}{\xi}\right)^2 \left[1 + (1 - q_e)\theta\right]^{\frac{1}{1-q_e}}, \tag{5.10}$$

where, ξ_0 is the initial normalized radial position (core).

5.3.2 SWP calculation scheme

It is well known that the entire mass of the bounded SIP acts as a source of external gravity to tailor and monitor the SWP dynamics [1, 2]. Thus, the solar self-gravity is switched off and the gravitational Poisson equation gets redundant. As a result, the modified normalized set of the SWP equations composed of nonextensive continuity equation, reduced momentum equation and net current density are respectively presented as

$$\frac{\partial \theta}{\partial \xi} = -\left[1 + (1 - q_e)\theta\right] \left(\frac{1}{M_i} \frac{\partial M_i}{\partial \xi} + \frac{2}{\xi}\right), \tag{5.11}$$

$$M_i \frac{\partial M_i}{\partial \xi} = -\left[1 + (1 - q_e)\theta\right] \left[\left\{1 + \in_T \left\{1 + \left[1 + (1 - q_e)\theta\right]^{-\frac{1}{1-q_e}}\right\}\right\} \frac{\partial \theta}{\partial \xi}\right] - \frac{a_0}{\xi^2}, \tag{5.12}$$

$$J_{SWP} = M_i - \left(\frac{m_i}{m_e}\right)^{\frac{1}{2}} \left(\frac{\xi_{SSB}}{\xi}\right)^2 \left[1 + (1 - q_e)\theta\right]^{\frac{1}{1-q_e}}. \tag{5.13}$$

Here, $a_0 = GM_\Theta / c_S^2 \lambda_J \sim 95$ is a constant [1], which gives an indirect measure of the SWP temperature, and M_Θ is the mean solar mass.

5.4 RESULTS AND DISCUSSIONS

A detailed numerical analysis is carried out over the basic set of equations (5.7)-(5.10) for the SIP [figure 5.1] and equations (5.11)-(5.13) for the SWP [figure 5.2] to see their equilibrium dynamics. The numerical technique employs the fourth-order Runge-Kutta (RK-IV) method as an initial value problem [14]. It may be noted that the initial values of the relevant solar parameters for the SIP are obtained by nonlinear stability analysis in analogy with the earlier works [1-5]. The numerically calculated SSB values for the same relevant solar plasma parameters are then utilized as a new combination of initial values to simulate the SWP.

In figure 5.1, we show the profiles of the normalized (a) solar self-gravity ($g_s = d\eta/d\xi$, upper curves) and electric field ($d\theta/d\xi$, lower curves), (b) electrostatic potential (θ), (c) Mach number (M_i) and (d) net electric current density (J) on the SIP scale with the variation in the normalized radial distance (ξ). The different lines correspond to q_e = 0.50 (blue lines), 0.55 (red lines), 0.60 (green lines) and 1.00 (black lines); respectively. The different input initial values used are $\xi_0 = 0.01$, with $\Delta\xi$ = 0.001, $g_{s0} = 0.005$, $\theta_0 = -0.001$ and $M_{i0} = 0.005$ [1-3]. The fixed input physical parameters are $m_e = 9.1\times10^{-31}$ kg, $m_i = 1.67\times10^{-27}$ kg, and $\in_T = (T_i/T_e)$= 0.4. The choice of the judicious initial values is based on the assumption that the solar plasma contained in the thermonuclear core is isothermal in nature. It is found that a bounded solution for the SIP with the maximum solar self-gravity, g_s= 0.47, exists at ξ=3.5, which is in turn, perfectly balanced by the electric force field, $d\theta/d\xi = g_s =$ 0.47 [figure 5.1(a)]. Thus, the critical values characterizing the SSB formation get reduced due to the nonextensive electrons. The effect of the nonlinear logatropic pressure on the GES evolution is evident in isolation [figure 5.1(a)] for q_e =1 (Boltzmannian electrons) in contrast with $q_e \neq 1$ (non-Boltzmannian electrons). It is seen that the lowest-order SSB is formed at ξ=3.5 on the Jeans scale in accordance with the principle of maximization of the solar self-gravity and exact counter-balancing of the GES force fields [1-5]. It is noticeable that the electrostatic potential associated with the SIP decreases monotonically in sense and attains $\theta = -1.3$ (instead of $\theta = -1$ in the idealized case [1]) at the SSB [figure 5.1(b)]. The electrostatic potential becomes more positive with the q_e-parameter value; and vice-versa. An interesting feature of the GES width, $\xi \approx 2$ demarcating the plasma layers acted upon by strong self-gravity and intense electric force field, remains invariant due to the nonextensivity [figure 5.1(c)]. The flow speed becomes slightly reduced with the q_e-parameter value; and vice-versa. It is further seen that the SSB draws finite non-zero (negative) electric current dominated only by the nonthermal electrons [figure 5.1(d)]. The high electric current in the present case with the nonthermal electrons is due to enhancement of the electric potential magnitude.

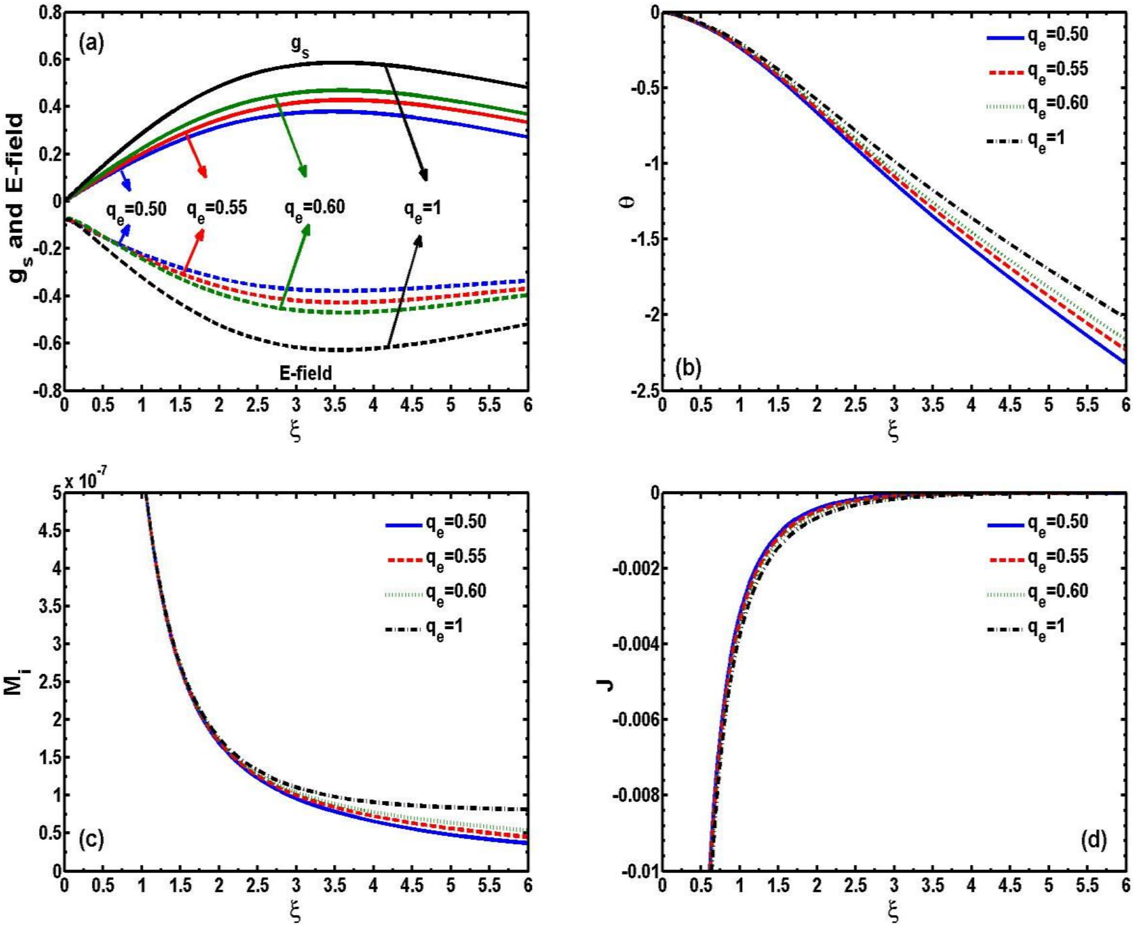

Figure 5.1: *Profile of the normalized (a) solar self-gravity ($g_s = d\eta/d\xi$, upper curves) and electric field ($d\theta/d\xi$, lower curves), (b) electrostatic potential (θ), (c) Mach number (M_i) and (d) net electric current density (J) on the SIP scale. The different lines link to q_e = 0.50 (blue lines), 0.55 (red lines), 0.60 (green lines) and 1.00 (black lines); respectively. The related fine details are given in the text.*

Figure 5.2 portrays the profile of the normalized (a) electrostatic potential (θ), (b) Mach number (M_i) and (c) net electric current density (J) on the SWP scale with the variation in the normalized radial distance (ξ). The different lines correspond to q_e = 0.90 (blue lines), 0.94 (red lines), 0.98 (green lines), and 1.00 (black lines); respectively. Different input values used are $m_e = 9.1\times10^{-31}$ kg, $m_i = 1.67\times10^{-27}$ kg, $\in_T = (T_i/T_e)$= 0.1, G = 6.67×10^{-11} m^3 kg^{-1} s^{-2} and $a_0 = 95$. The different initial values kept fixed are ξ_Θ=3.5 with

$\Delta\xi = 3.5$, and $\theta_{\Theta} = -1.3$ and $M_{\Theta} = 5\times10^{-7}$. These numerical results are completely based on the different SSB parametric values as initial inputs, obtained methodologically after having the SSB in the GES framework completely specified with the help of figure 5.2. This is in accordance with the SWP predictions reported in the previous works [5-9]. It is seen that the effect of the electrostatic potential becomes stronger beyond the SSB on the unbounded scale, then becomes almost uniform, and acts as a key source for the uniform SWP flow [figure 5.2(a)]. The GES force field, after undergoing quasi-linear transformation from the gravitational to electrostatic one, renders the supersonic SWP flow dynamics [figure 5.2(b)].

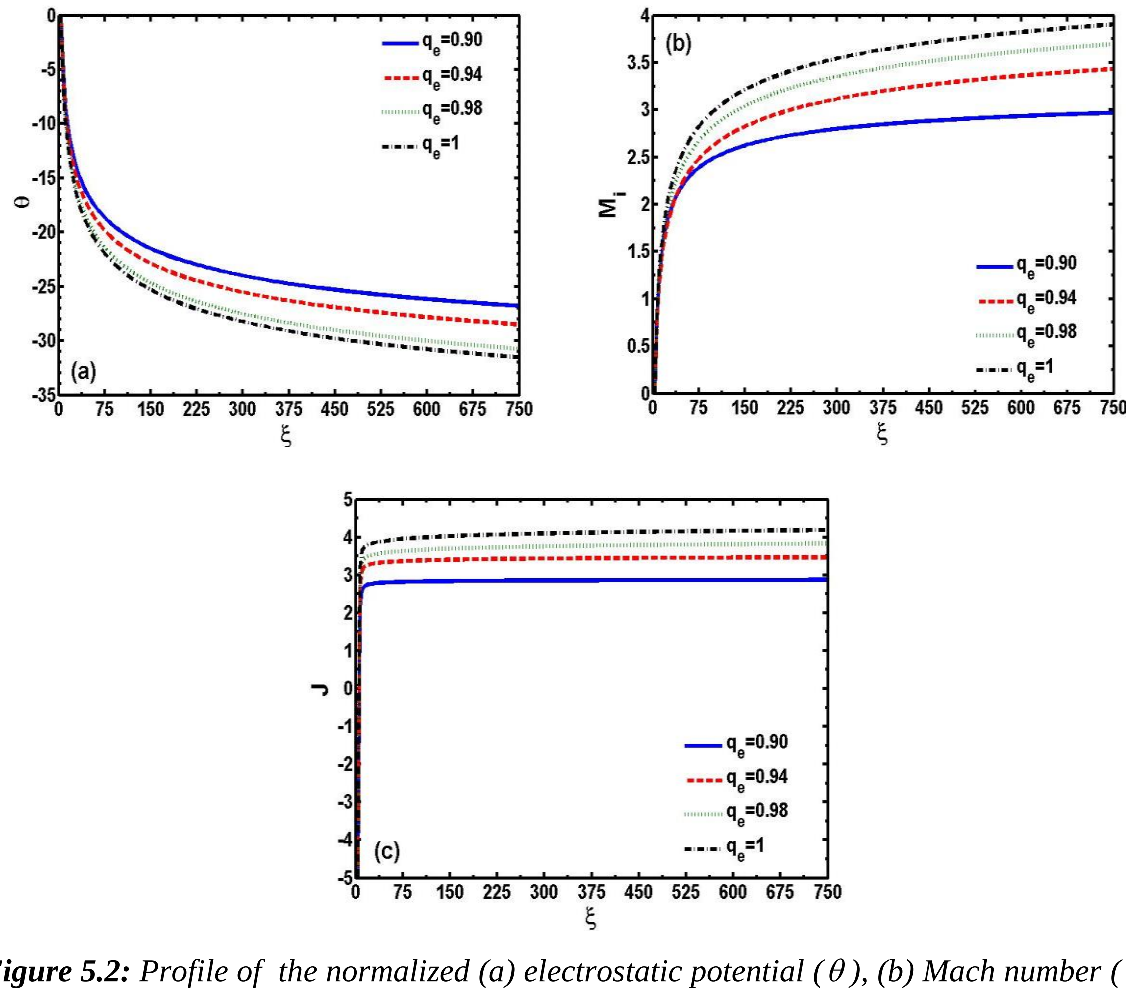

Figure 5.2: *Profile of the normalized (a) electrostatic potential (θ), (b) Mach number (M_i) and (c) net electric current density (J) on the SWP scale. The different lines link to q_e = 0.90 (blue solid), 0.94 (red dashed), 0.98 (green dotted), and 1.00 (black dashed dotted); respectively. The related fine details are presented in the text.*

An interesting speculation to be noted here is that the transonic zone, as found in the previous work [figure 5.2(c) in [1]), no more exists here due to the consideration of the nonthermal electron dynamics and nonlinear pressure (turbulence) effects. The high electric current in the SWP may be ascribable to the enhancement in the GES acceleration processes under the augmented electric force field [figure 2(a)]. A unique type of reversibility transformation from negative electric current (in the SIP) into positive one (in the SWP) is found to exist [figure 2(c)] in the considered model in the presence of the nonthermal electrons and nonlinear pressure effects (turbulence). The negative current is attributable to the strong gravitational attraction acting only over the massive ions, but not on the lighter electrons. In contrast, the strong electrostatic action over the ions, with negligible gravitational attraction, is responsible for the existence of positive electric current.

5.4.1 COMPARATIVE RESULTS

A comparison of the main outcomes investigated here over the original GES formalism on both the SIP and SWP scales can be highlighted respectively as in Table 5.2 and Table 5.3.

Table 5.2: Comparison of SIP results (past vs. present)

Properties of SSB	Earlier GES	Present GES	Deviation (in %)
Self-gravity (g_s)	0.6	0.47	21.67
E-potential (θ)	-1	-1.3	-30
Mach no. (M)	1.6×10^{-7}	8.7×10^{-8}	45.62

Table 5.3: Comparison of SWP results (past vs. present)

Properties of SWP (1 AU)	Earlier GES	Present GES	Deviation (in %)
Gravity (g_s)	NA	NA	NA
E-potential (θ)	-31.5	-26.79	14.95
Mach no. (M)	3.15	2.97	5.71

The main outcomes of the numerical analysis to see the role of the nonlinear pressure in the GES structure evolution from a quantitative perspective are in order. It is seen [figure 5.1(a)] that, the critical values characterizing the SSB formation is found to be 0.6, which is accordance with the earlier work [1]. But, the SIP electrostatic potential decreases by 20 % [figure 5.1(b)] and the SIP Mach number by 38.56 % [figure 5.1(c)]. On the other hand, in the case of the SWP, the electrostatic potential decreases by 0.06 % [figure 5.2(a)], while the Mach number decreases by 23.80 % [figure 5.2(b)]. Thus, we can say that the GES structural properties get noticeably modified due to the nonlinear pressure effects (turbulence).

5.5 CONCLUSIONS

A theoretical model study to understand the GES structure, accounting for the solar nonthermal electron dynamics and realistic logatropic fluid pressure effects is proposed. We demonstrate specifically that the GES structure gets drastically modified due to the inclusion of the new unavoidable factors. It is primarily shown that the GES field strength for the bounded solar solution on the self-gravity as a dynamical variable to exist, paving the way for the SSB formation, decreases by 21.67%, and the electrostatic potential by 30%.

Significant influences are found to exist also in the SWP flow dynamics on the unbounded scale. It is interestingly found herein that the supersonic SWP flow dynamics with the nonextensive electrons is a *two-step process*: subsonic-supersonic transition with no intervening transonic zone. This is against the earlier *three-step process* with the isothermal inertialess electrons: subsonic-supersonic transition via transonic zone of width ~ 37 λ_J [1]. It is, therefore, concluded that the joint action of the electronic thermostatistics and nonlinear fluid pressure effects enhances the quasi-linear gravito-electrostatic interplay causing the exterior solar plasma to accelerate into the supersonic or hypersonic regimes. Thus, the analysis presented here may find extensive application in the SWP flow dynamics from a new thermostatistical description of plasma wall interaction-based mechanism in the presence of both nonextensive electrons and nonlinear fluid pressure effects.

It is admitted finally that, our plasma-based theoretical approach (non-exospheric) is a simplistic one, based purely on the bi-fluidic treatment of the constituent species in the solar plasma system. The dynamical coupling mechanism between the electrons and ions lies solely on pure gravito-thermal interplay free from any kind of complication, such as electromagnetic force field, collisional effect, etc. It, for further analytic simplicity, ignores the relative abundances of the constituent neutrals, heavy non-hydrogenic ions, radicals, other chemical complex species, etc. Despite the facts and faults, as highlighted above, it describes both the interior and exterior of the solar plasmas methodically against the existing literature dealing with the exterior alone [15]. Moreover, it can qualitatively reproduce the exact profile structures of the electrostatic potential and supersonic flow velocity on the unbounded SWP scale as predicted by kinetic exospheric models (see figures 1, 3 in [15]). The relevant physical properties explored here on the SWP scale may be verifiable with the help of observable data under various spacecraft-based solar missions, such as the Solar Dynamics Observatory (SDO), in particular [11].

At the last, it indicates that the presented results, for better reproduction and more realistic outcomes, hereby opens a new ameliorative scope for further refinements of the proposed GES theory. In particular, the role of fluid turbulence could be investigated by employing a fully nonlinear power spectrum analysis [16]. Besides, it should be in the new framework of multi-fluidic approach with all the types of constituent heterogeneous species in diversified long-range force fields and large-scale inhomogeneities in futuristic direction.

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