

APPENDIX-A

A GENERALIZED BI-FLUIDIC MODEL FORMALISM OF PLASMA SHEATH EVOLUTION

A generalized two-fluid model[†] to study the equilibrium structure of plasma sheath in normal two-component plasma in the presence of both viscoelastic effects [1] and weak but finite electron-inertia [2] is built up. The Bohm ionic-flow condition for the sheath formation in viscoelastically modified form is strategically derived. It is reliably bolstered by an exact reproduction of the pre-reported results on normal plasma sheaths [3]. The role of both plasma viscoelasticity and electron-inertia on the sheath evolutionary dynamics is specifically discussed. The findings may be useful to explore realistic plasma boundary-wall interaction processes relevant in diversified laboratory, star-space and astro-environs.

The local Bohm ionic-flow criterion, which is found to be considerably modified with the plasma viscoelasticity and electron-inertia in our model, is systematically obtained as

$$M_{i0} \geq \left[\left[1 + \mu_{ei} M_{e0}^2 - \left\{ \frac{e}{T_e \sqrt{\epsilon_0} n_0 m_i} \right\} \left\{ \chi \left\{ \left(\frac{\partial M_e}{\partial \xi} \right) + \frac{\partial}{\partial \Phi} \left(\frac{\partial M_e}{\partial \xi} \right) \right\} \right] \right] \times \exp \left[\left(\frac{e}{T_e \sqrt{\epsilon_0} n_0 m_i} \right) \chi \left(\frac{\partial M_e}{\partial \xi} \right) \right] \right]^{-1/2}, \quad (\text{A1})$$

where, ξ is the normalized (by electron plasma Debye length) spatial coordinate, $\mu_{ei} = m_e/m_i$ is the electron-to-ion mass ratio, M_e is the normalized (by ion sound phase speed) electron flow velocity, Φ is the normalized (by electron thermal potential) electrostatic potential, T_e is the electron temperature (in eV). Furthermore, $\chi_{e(i)} = (\zeta_{e(i)} + 4\eta_{e(i)})/3$ is the effective generalized electron (ion) fluid viscosity, with $\zeta_{e(i)}$ and $\eta_{e(i)}$ the bulk and shear viscosity of the electron (ion) fluid. Here, χ depicts the average effective generalized dynamic viscosity in a mean fluid approach.

A numerical analysis over the sheath structure equations is strategically performed to see the reality on the sheath evolution characteristics by employing the fourth-order Runge-Kutta method. The resultant sheath evolution is graphically displayed in figure A1.

[†]Gohain, M. and Karmakar, P. K. A generalized two-fluid model of plasma sheath equilibrium structure. *Europhysics Letters*, 112:45002(1-6), 2015.

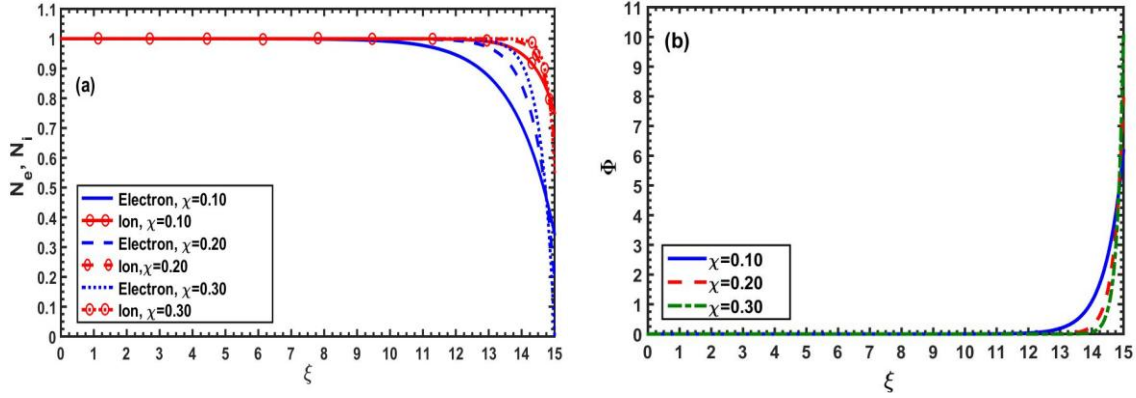


Figure A1: Spatial profiles of the normalized (a) electron and ion population densities and (b) electrostatic potential for different χ -values. Different lines in two distinct sets (blue and red) in figure A1(a) depict the densities for $\chi = 0.10, 0.20, 0.30 \text{ kg m}^{-1} \text{ sec}^{-1}$; respectively. The blue, red and green lines in figure A1(b) link the same χ -values; correspondingly.

It is seen that the joint action of both viscoelasticity and electron-inertia has significant influences on the local Bohm flow criterion and sheath characteristics. The deviation from quasi-neutrality increases with viscosity; and vice-versa [figure A1(a)]. Thus, the plasma sheath, so formed, is a non-neutral space charge layer with non-neutrality enhanced by viscosity. It is further seen that, with increase in viscosity, the sheath potential increases; and vice-versa. As a result, the sheath thickness decreases [figure A1(b)].

We admit that it is a simplified approach excluding external electromagnetic force fields, temperature variations, etc. The results are validated *a posteriori* only on an empiric basis against experimental verification. A future scope for further refinements is hereby opened. However, the presented analysis may be extensively applied for basic understanding of realistic plasma-boundary wall interaction processes in diversified plasma environs.

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APPENDIX-B

PULSATONAL MODE BEHAVIOR IN COMPLEX NONEXTENSIVE VISCOUS ASTROCLOUD

The pulsational mode dynamics of self-gravitational collapse [1] in a hydrostatically bounded complex non-thermal dusty astrocloud is theoretically investigated. The adopted multi-fluidic model[†] consists of nonextensive electrons and ions, and massive dust grains along with partial ionization in a flat space-time. A linear normal mode analysis reduces the basic cloud equations into a quartic (biquadratic) dispersion relation with unique set of multi-parametric coefficients. It is interestingly found with a numerical illustrative calculation that non-thermal associations in the cloud pave the way for faster normal mode propagation. The neutral dust viscosity plays a decisive role towards a transition of the pulsational mode from a non-dispersive to dispersive nature.

The $q_{e(i)}$ -nonextensive non-thermal electron (ion) population distributions ($N_{e(i)}$) in normalized form with all the conventional notations [2] is given as

$$N_{e(i)} = \left[1 \pm (1 - q_{e(i)}) \Phi \right]^{1/q_{e(i)}}, \quad (\text{B1})$$

where, $q_{e(i)}$ represents the non-thermal entropic index of electrons (ions), $N_{e(i)}$ is the normalized (by equilibrium concentration) electron (ion) population density, Φ is the normalized (by electron thermal potential) electrostatic potential developed by conjoint bipolar charge density fields. A plane-wave analysis over the linearly perturbed astrocloud relative to its equilibrium finally results in a linear generalized dispersion relation (quartic) as

$$D(\Omega, K) \equiv \Omega^4 + A_1 \Omega^3 + A_2 \Omega^2 + A_3 \Omega + A_4 = 0, \quad (\text{B2})$$

where, A_1 , A_2 , A_3 and A_4 are the various involved multi-parametric coefficients.[†]

It reveals that the pulsational mode propagatory properties are likely to be appreciably affected by the various dispersion coefficients. It is then numerically analyzed to explore the basic mode characteristic features in an exact form as shown in figure B1.

[†]Bhakta, B., Gohain, M., and Karmakar, P. K. Pulsational mode behaviors in complex nonextensive non-thermal viscous astrophysical fluids. *Europhysics Letters*, 119:25001(1-6), 2017.

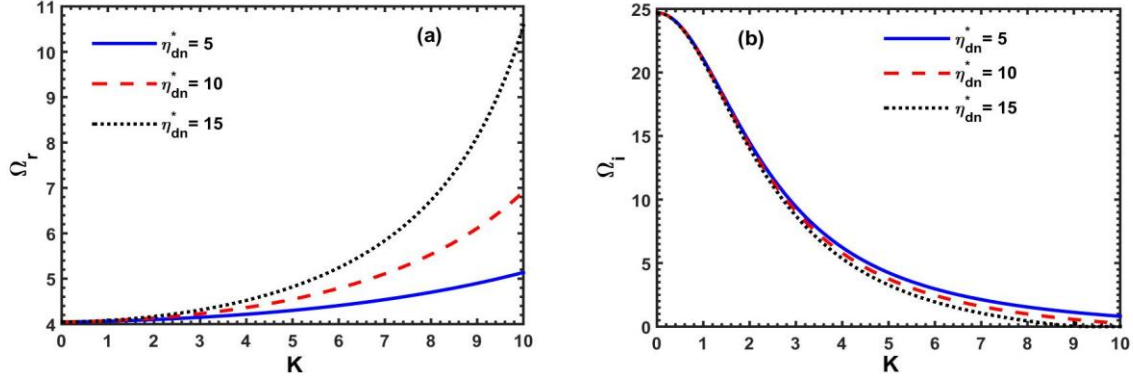


Figure B1: Profile of the normalized (a) real frequency part (Ω_r) and (b) imaginary frequency part (Ω_i) with the variation in the normalized wave number (K). The various lines indicate the diversified situations with different viscosities as $\eta_{dn}^* = 5$ (blue, solid line), $\eta_{dn}^* = 10$ (red, dashed line), and $\eta_{dn}^* = 15$ (black, dotted line); respectively.

It is seen that the real frequency of the pulsational mode increases with rise in the kinematic neutral dust viscosity; and vice-versa. It implies that the non-thermal associations in the cloud are responsible for faster normal mode propagation against a thermalized situation [1]. A unique transition from non-dispersive to dispersive nature is also speculated [figure B1(a)]. In contrast, the growth rate decreases with the neutral dust kinematic viscosity; and vice-versa [figure B1(b)]. Consequently, the neutral fluid viscosity has a stabilizing influence on the slightly perturbed non-thermal cloud, and, hence, in the pulsational dynamics. The results may be significant in understanding the self-gravitational cloud collapse dynamics leading to a hierarchical initiation of bounded structure formation in diverse astro-plasma-cosmic non-thermal equilibrium environments.

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APPENDIX-C

WAVE DAMPING BEHAVIORS IN SOLAR PROMINENCE PLASMAS

The evolutionary dynamics of non-adiabatic magneto-hydrodynamic (MHD) waves, in the prominence plasmas in the presence of kinematic viscosity and inertia-dependent stratification of the constitutive species is explored[†]. It judiciously implements a new energy equation devised to describe optically thin radiation losses, thermal conduction, viscosity, heating mechanisms, and so forth [1, 2]. A linear perturbative analysis over the basic governing equations for the prominence plasma dynamics results in a generalized dispersion relation (quintic) with a unique set of multi-parametric coefficients. It interestingly enables us to characterize two distinct classes of modes: the fast wave and slow ramified wave.

The dispersion relation in our visco-gravitational prominence model is derived as

$$a_5\omega^5 + a_4\omega^4 + a_3\omega^3 + a_2\omega^2 + a_1\omega^1 + a_0 = 0, \quad (\text{C1})$$

where, a_0 , a_1 , a_2 , a_3 , a_4 and a_5 are the various involved multi-parametric coefficients.[†]

Clearly, equation (C1) is a fifth-degree polynomial in ω with functional dependencies on k via the coefficients. We aim at the time-damping behaviors of the magnetoacoustic waves ($\omega = \omega_R + i\omega_I$, $k = k + i0$). We obtain five roots numerically, out of which, only one root is purely imaginary, indicating non-propagatory thermal or condensation mode [3]. The rest roots consist of two conjugative pairs, one representing the fast wave, and the other the slow one. The wave damping time is $\tau_D = 1/\omega_I$ and the period is $T = 2\pi/\omega_R$. A numerical analysis yields the results graphically displayed in figure C1.

It is seen that, with the T_K -increment, τ_D of the fast wave [figure C1(A)] at first decreases followed by a rapid rise (at $k \sim 10^3 \text{ m}^{-1}$), and then, again decreases. On the other hand, τ_D of the slow wave [figure C1(C)] decreases in magnitude with increase in T_K . It implies that higher the temperature, the more is the time required to damp the waves; and

[†]Gohain, M. and Karmakar, P. K. Normal mode behaviors in solar prominence plasmas. *Journal of Physics: Conference Series*, 836:012006(1-5), 2017.

vice-versa. Likewise, T [figures C1(B), C1(D)] decreases similarly relative to τ_D of the slow wave [figure C1(C)]. Thus, the thermal conduction affects the damping mechanism. The results could be applied as prominence seismology test-bed to see the solar morphodynamics.

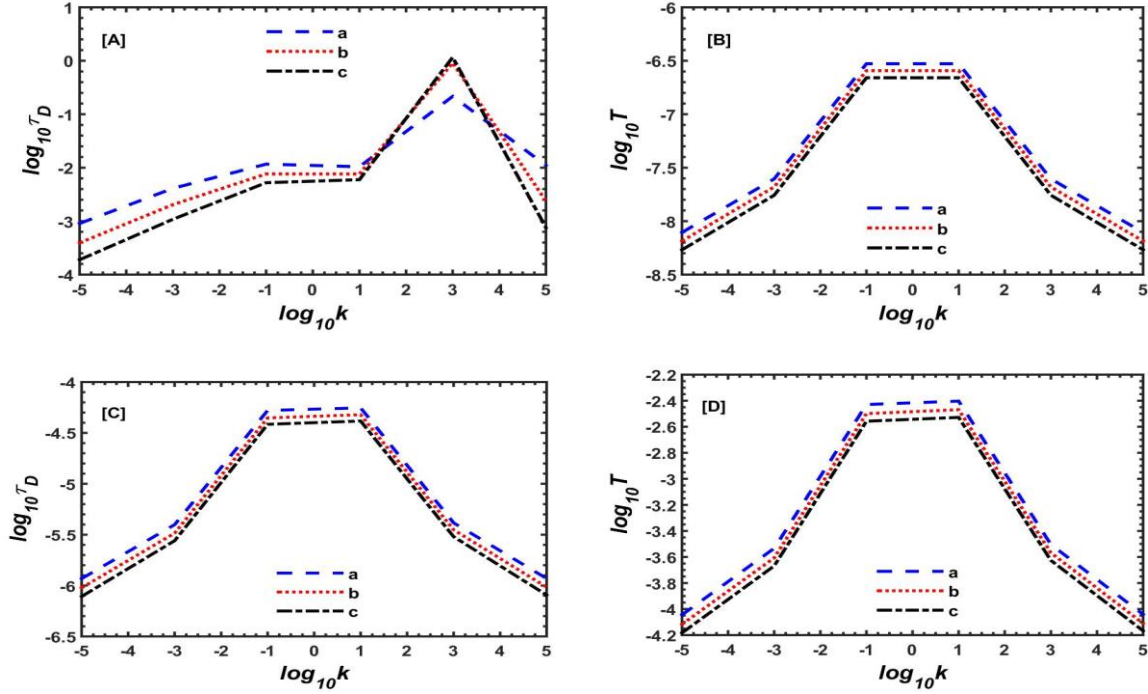


Figure C1: Profile of the logarithmic damping time ($\log_{10} \tau_D$) and logarithmic period ($\log_{10} T$) with the logarithmic wave number ($\log_{10} k$) for the fast ([A]-[B]), and slow ([C]-[D]) modes. The dashed, dotted, and dashed-dotted lines link to the three distinct prominence conditions: (a) $T_K = 6000$ K, (b) $T_K = 7000$ K and (c) $T_K = 8000$ K; respectively.

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