"Never theorize before you have data. Invariably, you end up twisting facts to suit theories instead of theories to suit facts"

Sherlock Holmes in "A Scandal In Bohemia"

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Introduction

In this introductory chapter we first aim at presenting a literature survey of the present updates on neutrino oscillation parameters, what we have with how much accuracy. Then we briefly discuss the Standard Model of particle physics and its inadequacy in realizing some observed phenomena. Here we also discuss the neutrino oscillation phenomena and the class of seesaw scenarios in short with the motivation of going beyond the Standard Model for explaining light neutrino mass via the inclusion of heavy right handed heavy neutrinos. The seesaw models considered for this task correspond to high energy scale and some other, relatively low energy scale. We keep a section for detailed discussion on matter-antimatter asymmetry of the universe. We also have dedicated one section for dark matter history. Finally we end up with a section discussing the non-Abelian discrete flavour symmetries like S_4 and A_4 which have extensively been used in model building purpose in this thesis.

Study of Neutrinos and its associate observables continue to intrigue. An Austrian physicist Wolfgang Pauli [1–4] in 1930, proposed the existence of a neutral particle called *neutrino* (as a mathematical trick) in order to preserve energy-momentum conservation in nuclear β decay. This proposal of Pauli opened up a new avenue in the particle physics ball park. Thus to start with, neutrinos are electrically neutral fermions. Their mass was long thought to be zero, although neutrino oscillation experiments have confirmed the tiny mass that they possess. However Pauli had also supposed that nobody would ever be able to detect this new particle due to the fact that they interact feebly with matter. Then in the year 1956, Clyde Cowan and Fred Reins [5] had gone through an observation of anti-neutrinos emitted by a nuclear reactor at Savannah River at South Carolina, USA. It was later found that the observed neutrino was an electron neutrino which is a partner of an electron. The SM is unable to accommodate neutrino mass as there is no right handed counter part of neutrino in the SM. And this fact calls for some BSM frameworks, by the inclusion of right handed (RH) Majorana neutrinos to the SM fermion sector. Neutrinos being electrically neutral are allowed to possess Majorana masses. For, a Majorana neutrino mass can not arise from the neutrino analogue of the SM coupling that gives quarks and charged lepton their masses. That analogue would be a Yukawa coupling of the form $H_{SM}\bar{\nu}_R\nu_L$, where H_{SM} is the SM Higgs field. Rather, Majorana masses must come from couplings such as $H_{SM}H_{SM}\bar{\nu_L^c}\nu_L$ or $H_{I_W=1}\bar{\nu_L^c}\nu_L$, the first of which implies non-renormalizability and therefore outside the scope of the SM but the second involves a Higgs Boson with weak isospin $I_W = 1$, which the SM does not accommodate. In this way within the SM, neutrinos remain mass less. Although this theoretical prediction was consistent with the experiments till 1960 due to lack of evidence of neutrino mass but this fact had gained much interest due to the fact that results from solar neutrino experiment and atmospheric neutrinos indicated towards a massive neutrino.

1.1 Present status of neutrino parameters

In the year 1968, an American physicist Raymond Davids Jr. while detecting solar neutrinos [6, 7] for the first time from a deep underground experiment, observed that the number of electron neutrino measured was one third of the actual number that was expected to come from Sun, the phenomenon later on named as Solar neutrino problem. In the same way, was found a discrepancy [8, 9] while measuring muon neutrino flux coming from earth atmosphere and this is familiar as atmospheric neutrino problem. In this context Mikheyev, Smirnov along with Wolfenstein told that only electron neutrinos are emitted by the Sun and they could be converting into muon and tau neutrino which were not being detected on earth. Such a scenario of inter-conversion from one kind to another is termed as neutrino oscillation [10]. Theoretical justification of neutrino oscillation beautifully fix the puzzle created from solar and atmospheric neutrino fluxes. Then in the year 1998, Super Kamiokande [11, 12] experiment, piloted by Takaaki Kajita from Japan, evinced that there was a deficit in the number of muon neutrinos reaching from earth when cosmic rays strike with earth's atmosphere. This experiment was able to detect only half of the muon neutrinos actually expected. Then in the year 2001/2002, Arthur B. McDonald in Canada guided the Sudbury Neutrino observatory (SNO) collaboration, and did a detailed measurement of the fluxes of both the neutrinos along with total flux of all the three types of neutrinos. Interestingly, the result found in SNO collaboration was consistent with the theoretically predicted result for electron neutrinos coming from the Sun. This experiment confirmed the conversion of electron neutrino to the other two kinds i.e., muon and tau neutrino. This phenomenon of oscillation from one particular kind of neutrino to the other two is termed as neutrino flavor oscillation, where the term flavor is used to mean the three kinds of neutrinos namely electron (ν_e), muon (ν_μ) and tau neutrino (ν_τ). For the above extensive and nontrivial study led by Takaaki Kajita and Arthur B. McDonald, they shared the 2015 Nobel Prize in Physics. Later on many experiments such as KamLAND [13, 14] nuclear reactor in Japan, K2K [15] long base line experiments also in Japan, Fermilab-MINOS [16] in U.S. put concrete evidence of the phenomenon called neutrino oscillation.

After the discovery of neutrino oscillation there is no doubt that neutrino possess masses, however, tiny. That time the particle physics community did not remain silent only with this discovery and started to think about the other properties associated with this oscillation phenomenon. Some of them are what are their mixing angles, what is their absolute mass scale, then which flavored neutrino is the heaviest (and which is the lightest). Therefore the above mentioned queries are also extensively exercised both theoretically from some neutrino mass models and practically in some neutrino oscillation experiments. T2K [17], Double Chooz [18], Daya-bay [19] and RENO [20] are some of the experiments which provided us with information about the neutrino mass squared splittings and mixing angles with a very strong precision. These experiments gave bounds on the mass squared splittings of order $\Delta m^2_{\rm sol} \approx 10^{-5} eV^2$ and $\Delta m^2_{\rm atm} = 10^{-3} eV^2$. Such a small mass splitting not only hints towards the tiny magnitude of neutrino mass but also shows a 10^{12} order of mass difference between the neutrino and top quark mass. These experiments only could measure two mass squared splittings rather than the individual masses possessed by three flavor of them. Moreover, the leptonic mixing angles also are under huge discussion. There are three mixing angles in the neutrino sector: solar (θ_{12}), atmospheric (θ_{23}) and reactor (θ_{13}). Earlier it was believed that the value of reactor mixing angle is zero. But later on some dedicated neutrino oscillation experiments confirmed that the reactor mixing angle is non-zero although tiny as compared to the other two. In support of these neutrino data, there have been found several mixing schemes namely bimaximal (BM), Tri-bimaximal (TBM), hexagonal (HM) and Golden ratio mixing (GRM). Among them TBM has gained more popularity as, the mixing angles predicted by this mixing pattern is very much consistent with the angles observed in experiments. In TBM scenario we find $sin^2\theta_{12} = 0.33$, $sin^2\theta_{23} = 0.5$ with $sin^2\theta_{13} = 0$. However TBM has also lost the favor as the latest data ruled out a zero value for the reactor angle. In order to address a non-vanishing reactor angle, thus one needs to break the above mentioned mixing patterns, since all of them accommodate a zero value for reactor angle. Now if we look at the Pontecorvo Maki Nakagawa Sakata (PMNS) mixing matrix we find that a CP violating phase delta is associated with the reactor angle in the third column of it. The value of which is still unknown and hence is kept in the

"To find" list.

As already said that within the SM we do not have neutrino mass and at the same time the existence of neutrino mass is also a truth, thus one needs to go beyond the SM to validate the above two facts. And this task is carried out by an extension of the SM particle content by the inclusion of the missing RH neutrinos. Seesaw mechanisms are such methods to implement the consequences of adding two or more right handed heavy neutrinos to the SM fermion sector and generating the neutrino mass via some higher dimension terms. Below we discuss a brief overview on the SM and its drawbacks.

1.2 Standard model

It was in the year 1960, G. Glashow, S. Weinberg and P. Salam proposed the Standard Model [21–23] which is a quantum field theory particularly a spontaneously broken Yang-Mills theory, that takes all the three fundamental forces (strong, weak and electromagnetic) into account except the gravity. The SM gauge group is given by

$$G_{SM} \subset SU(3)_C \times SU(2)_L \times U(1)_Y \tag{1.2.1}$$

The symmetry group $SU(3)_C$ is the group of "color" that comes from quantum chromodynamics, having eight generators, particle representatives of which are gluons, the carrier of the strong force. The weak isospin group is named as $SU(2)_L$ which has three generators. The SM gauge group has four gauge bosons, three of which, namely $W^{\pm}$ and Z^0 are mediators of weak interaction and the particle representatives of $SU(2)$ group generators as, $SU(2)$ has three generators. Likewise $U(1)_Y$ is the group of hypercharge, the generator of the group corresponding to the massless boson: photon, which is the mediator of electro-magnetic interaction. The SM provides a concrete platform to describe the particles and their interactions that constitute the model itself. Now, on the basis of some physical properties of the particles, they are categorized as scalars, fermions and gauge bosons. Among them left handed fermions of the SM transform as $SU(2)$ doublets. The SM fermions are categorized into three generations. The scalar boson Higgs is also a doublet under $SU(2)$ and singlet of $SU(3)$ symmetry group. The entire particle content and their charges under each symmetry groups are listed in Table 1.1. In addition the newly discovered

		I	I_3	Y	Q
Lepton Doublet	$\binom{\nu_e}{e}_L$	1/2	1/2	-1	0
			-1/2	-1	-1
Lepton Singlet	e_R	0	0	-2	-1
Quark doublet	$\binom{u}{d}_L$	1/2	1/2	1/3	2/3
			-1/2	1/3	-1/3
Quark Singlets	U_R	0	0	4/3	2/3
	d_R		0	-2/3	-1/3
Higgs Doublets	$\binom{\phi^+}{\phi^0}$	$\frac{1}{2}$	1/2	1	1
			-1/2	1	0

Table 1.1: Charges of the SM particles and the Higgs boson under isospin(I), third component of isospin(I_3), Hypercharge(Y) and electric charge(Q)

$L_L(1,2,-\frac{1}{2})$	$Q_L(3,2,\frac{1}{6})$	$E_R(1,1,-1)$	$U_R(3,1,\frac{2}{3})$	$D_R(3,1,-\frac{1}{3})$
$\binom{\nu_e}{e}_L$	$\binom{u}{d}_L$	e_R	u_R	d_R
$\binom{\nu_\mu}{\mu}_L$	$\binom{c}{s}_L$	μ_R	c_R	s_R
$\binom{\nu_\tau}{\tau}_L$	$\binom{t}{b}_L$	τ_R	t_R	b_R

Table 1.2: Charge assignments of SM particle contents [24]

Higgs field gets the charges under SM gauge group as,

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \sim (1, 2, \frac{1}{2}) \tag{1.2.2}$$

The vacuum expectation value of the Higgs field breaks the gauge symmetry,

$$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \implies G_{SM} \to SU(3)_C \times U(1)_{em} \tag{1.2.3}$$

Therefore, the SM has only three active neutrinos with their charge conjugate partners. Charged lepton mass eigenstates are denoted as e, μ and τ with their $SU(2)_L$ partners ν_e, ν_μ and ν_τ respectively. The active neutrinos undergo weak charged current (CC) interaction in the following manner

$$-\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} \sum_{l} \bar{\nu}_L \gamma_\mu l_L^- W_\mu^+ + H.C. \tag{1.2.4}$$

Moreover the three active neutrinos undergo neutral current (NC) interactions,

$$-\mathcal{L}_{NC} = \frac{g}{2cos\theta_W} \sum_{l} \bar{\nu}_{Ll} \gamma_\mu \nu_{Ll} Z_\mu^0 + H.C. \tag{1.2.5}$$

where θ_W is termed as Weinberg or weak mixing angle. All the interaction by SM neutrinos are described by the above two equations. The SM also follows an accidental global symmetry

$$G_{\text{SM}}^{\text{global}} = U(1)_B \times U(1)_{L_e} \times U(1)_{L_\mu} \times U(1)_{L_\tau} \tag{1.2.6}$$

where $U(1)_B$ is the baryon number and $U(1)_{L_{(e,\mu,\tau)}}$ are the symmetries of the three lepton flavor with total lepton number $L = \sum_i L_i$ where i represents three flavors of lepton: e, μ and τ. The Lepton Number (LN) is said to be an accidental symmetry as it is not an imposed symmetry rather generated as a result of the gauge symmetry. The fermions and gauge bosons get their masses from Higgs mechanism via the spontaneous symmetry breaking, that we discuss in the following section. But only the neutrinos remain massless. Fermions in the SM gets masses from the Yukawa interactions of a left handed doublet with its right handed counter part and SM Higgs field. The complete Yukawa Lagrangian of the SM is given by

$$-\mathcal{L}_{\text{Yukawa}} = Y_{ij}^d \bar{Q}_{Li} H D_{Rj} + Y_{ij}^u \bar{Q}_{Li} \tilde{H} U_{Rj} + Y_{ij}^l \bar{L}_{Li} H E_{Rj} + \text{H.C.} \tag{1.2.7}$$

with $\tilde{H} = i\sigma_2 H^*$, the isospin conjugate of the Higgs doublet with σ_2 as the Pauli's spin matrix and also one of the generators of the weak isospin group $SU(2)$. The SM enlightened on the existence of three massive gauge bosons, later on the existence of which got verified in LEP experiment at CERN, Geneva. In addition it also predicts nine massless gauge bosons and existence of massive fermions.

1.3 Spontaneous symmetry breaking and Higgs mechanism

Spontaneous Symmetry breaking is a scenario where, the symmetry of the Lagrangian is not the symmetry of the vacuum state or the minimum energy state. If the vacuum state takes a nonzero value v of any field H ($<H>= v$), then any physical field can be written as $H^{\text{phys}} = H - v$. Where we call v as the vacuum expectation value (VEV) of the field ϕ. For a scalar particle the Lagrangian is written as,

$$\mathcal{L} \equiv T - V = \frac{1}{2}(\partial_\mu H)^2 - (\frac{1}{2}\mu^2 H^2 + \frac{1}{2}\lambda H^4), \tag{1.3.1}$$

with positive definite λ, provided that, the Lagrangian remains the same under the interchange of H by $-H$. Depending on the sign of the μ^2 term, the minimum of the potential implies the following conditions:

$$<H^2>= 0, \mu^2 > 0 \tag{1.3.2}$$

$$<H^2>= v^2 = -\frac{\mu^2}{\lambda} > 0, \mu^2 < 0 \tag{1.3.3}$$

Now the extremum $H = 0$ does not interpret the minimum energy state which we are looking for. Whereas $H = \pm v$ with $v = \sqrt{-\mu^2/\lambda}$ represents the spontaneous breaking of the symmetry as the ground state of the system corresponds to a nonvanishing value of H. When the vacuum takes a value $<H>= v$, this is called the vacuum expectation value of H.

Higgs mechanism is the mass generation mechanism of all fermions and gauge bosons within the SM except neutrinos. Higgs is a complex scalar transforming as an $SU(2)$ doublet which has a hypercharge quantum number 1. H^0 is the neutral component of the scalar field which acquires a VEV and break the EW symmetry $SU(2)_L \times U(1)_Y$ to $U(1)_{em}$ at that scale making fermions and gauge bosons massive [25]. But, the gluon and photon remain massless as the $SU(3)_C$ and $U(1)_{em}$ symmetry are protected.

The relevant part of the Lagrangian particular in purpose of Higgs mechanism is give by

$$\mathcal{L}_{higgs} = (D_\mu H)^\dagger (D_\mu H) - V(H) \tag{1.3.4}$$

being D_μ as the covariant derivative has the following form,

$$D_\mu = (\partial_\mu - \frac{i}{2} g W^j_\mu \tau^j - \frac{iY_H}{2} g' B_\mu) \tag{1.3.5}$$

In the above expression for the covariant derivative we define τ^j as the Pauli spin matrices, Y_H is the hypercharge of the SM Higgs, g is the coupling constant for $SU(2)_L$ group and g' for $U(1)_Y$ gauge group. The scalar potential of the Higgs field has the following form

$$V(H) = \mu^2 H^\dagger H + \lambda (H^\dagger H)^2 \tag{1.3.6}$$

Minimization of this potential gives the solution for H, and is obtained as

$$< H >= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \text{ where, } v = \sqrt{\frac{-\mu^2}{\lambda}}. \tag{1.3.7}$$

The masses of the vector gauge bosons obtained from the Lagrangian (1.3.4) are written by,

$$\mathcal{L} = M_W^2 W^+_\mu W^-_\mu + \frac{1}{2} M_Z^2 Z_\mu Z^\mu \tag{1.3.8}$$

where, $W^+ = \frac{W^1_\mu - iW^2_\mu}{\sqrt{2}}$, $W^- = \frac{W^1_\mu + iW^2_\mu}{\sqrt{2}}$, $Z_\mu = cos\theta_W W^3_\mu - sin\theta_W B_W$. Masses of the vector boson thus can be written as; $M_W = \frac{gv}{2}$ and $M_Z = \frac{gv}{2cos\theta_W}$. Experiments gave a bound on mass of W boson equal to 80 GeV and Z boson of 90 GeV. However, the photon field remains massless as the $U(1)_{em}$ is preserved in the end. One can express the photon field in terms of the W^3_μ and B_μ field as

$$A_\mu = cos\theta_W W^3_\mu + sin\theta_W B_\mu. \tag{1.3.9}$$

The process of spontaneous symmetry breaking along with the Higgs mechanism together make this job of generating the masses of fermions (except neutrinos) and bosons easy in the SM. The fermion masses generated after the EWSB are given by

$$m_l = \frac{Y^l_{ij}}{\sqrt{2}} v, \; m_u = \frac{Y^u_{ij}}{\sqrt{2}} v, \; m_d = \frac{Y^d_{ij}}{\sqrt{2}} v. \tag{1.3.10}$$

where, Y^l_{ij},Y^u_{ij}, Y^d_{ij} are the Yukawa couplings of charged leptons, up-type quark and down-type quark respectively, v is the SM Higgs VEV.

1.4 Drawbacks of the standard model

Notwithstanding, there are some conclusive experimental evidences, such as neutrino masses, dark matter and the matter-antimatter asymmetry, along with theoretical issues, like the hierarchy problem, the strong-CP problem or the flavor puzzle, which are not addressed or explained within the SM, thereby inviting us to a journey towards new physics beyond the SM (BSM). It is in general believed that there exists new physics (NP) beyond the SM at a higher energy scale above the electroweak symmetry breaking scale ($\Lambda_{EW} \sim 100$GeV). Even the SM can not address physics at Planck scale (10^{19} GeV). Now within these two scales there lies some new physics and origin of whom are interrelated. It has become essential to list some relevant problems of the SM from experiment and observation point of view.

- Experimental evidences (the dedicated neutrino oscillation experiments and also the issue of solar and atmospheric neutrino problems) of massive neutrinos contradicts the facts that is apprised by the SM about the neutrino mass.

- Then some observational confirmations of NP consists of the Cosmic Microwave Background Radiation (CMBR) and the standard Big Bang Nucleosynthesis (BBN) scenario which push us to think seriously about the biggest mystery of the Universe– *the matter-antimatter asymmetry.* Now, in order to generate such asymmetry, adequate amount of CP violation is required which the SM interaction schemes are unable to produce. This again leads us to find a new source of CP violation which can account for the observed amount of baryon asymmetry set by CMBR and BBN data. The detail of this issue we address in Section 1.7.12.

- Another significant drawbacks of the SM is that, it does not enlighten us on the existence of dark matter, whose abundance is nearly the 26% of the total density of the Universe. The detail of dark matter observation we keep in Section 1.7.2.

- The unification of four gauge couplings of the strong, weak, electromagnetic and gravitational interactions is one of the chief concern in particle physics. The SM places only the electromagnetic and weak couplings in a single frame and unify them keeping gravity completely aside. Unification of electroweak and gravitational force is a difficult job to pursue within the SM, as the SM does not provide any quantum description of gravity. Strong and electroweak force unification too is not accommodated within the SM. String theory can unify all the four fundamental forces, but for that again one needs to go beyond the SM.

Moreover, there are some other limitations also. *Naturalness* is one of the most serious ones: which says there are small parameters in the SM and it demands supernatural fine-tuning to explain them.

- Loop corrections to the Higgs mass are commonly quadratic in the mass of the heaviest particle present in the loop, which is also a property of the hierarchy problem. It is thought that the heavy particle which plays a role in making the neutrino mass, can also couple to the Higgs boson, which can result in making an impermissible contribution to the Higgs mass. And for this reason there is always a concern for the upper limit on the mass of the heavy particle responsible for the generation of neutrino mass. To be precise, the Higgs mass is unstable against quantum corrections [26, 27] and is not protected by any symmetry. If we impose a one loop correction to Higgs mass, it is proportional to $\Lambda_{\rm UV}^2$, where Λ_{UV} is the cutoff scale where NP is awaited. Difficulty arises if Λ becomes of the order of Planck Mass ($M_{\rm Pl}$), then value of the quantum correction turns out to exceed the required value of the Higgs Boson mass. Adjusting the Higgs mass to be around 100 GeV, one needs a tremendous fine-tuning. Supersymmetry (SUSY) can solve this fine-tuning problem, stabilizing the ratio $\Lambda_{\rm EW}/M_{\rm Pl}$ [28–31].

- The Yukawa couplings are quite small as compared to the top Yukawa coupling and thus hierarchical. The same fact holds good for masses of the fermions as well. For example, the electron mass is 0.5 MeV whereas the top quark mass is nearly 175 GeV which shows a 10^6 order of magnitude difference. There is no explanation of such vast hierarchy within the SM.

Keeping the above mentioned agendas in mind we look for a theory beyond the Standard Model which possibly will be able to shade light on these phenomena. Since the SM does not accommodate neutrino mass, a chief job will be to build a model which can easily make the neutrino mass non-zero however tiny. For that Stephen Weinberg introduced dimension 5 operator through the implementation of seesaw mechanisms. To implement seesaw mechanism one needs to incorporate the missing RH neutrinos to the SM fermion sector.

1.5 Neutrino mass beyond the SM

Although the SM does not offer the explanation for neutrino mass, but neutrino oscillation phenomena established the fact that neutrinos have tiny but nonzero mass. Now this fact needs a theoretical justification too. The justification for the solar and atmospheric neutrino anomaly reveals that neutrinos from one flavor oscillate to another flavor after traveling through a considerably large distance; the phenomenon known as neutrino flavor oscillation which is the observational evidence of neutrinos being massive. On the other hand, KamLAND and some recent experiments involving solar, atmospheric, reactor and accelerator neutrinos have confirmed the neutrino oscillation (please see [32] for a review).

Now, when a neutrino is produced, it is in a specific flavor state, which is expressed as a superposition of the mass eigenstates. Had the neutrinos been massless or degenerate in mass, all the mass eigenstates would have the same time evolution and, thereafter, the initial flavor state would remain unchanged. Here, in this section we briefly summarize the mathematical expressions showing the oscillation probability and mixing of flavor and mass eigenstates. The flavor and mass eigenstate have different bases. We denote the flavor state as ν_α for $\alpha = e, \mu, \tau$ and mass state as ν_i for $i = 1, 2, 3$. The flavor and mass eigenstate are related with each other by a unitary matrix of order three, popularly known as the PMNS mixing matrix. The name arises after Pontecorvo, who proposed neutrino oscillations, and from Maki-Nakagawa-Sakata, who introduced the mixing matrix. This is analogous to the CKM mixing matrix in the quark sector. The order three implies the number of three generations of neutrinos. One can write the following equation showing the relation between the neutrino mass and flavor eigenstate.

$$|\nu_\alpha\rangle = \sum_{i=1}^{3} U_{\alpha i}|\nu_i\rangle \tag{1.5.1}$$

The PMNS mixing matrix is parameterized in terms of three mixing angles and six phases, which are popularly known as CP-phases. Since all the phases are not physical and hence three of them gets removed by phase redefinition and rest three remains. Now the Dirac nature of neutrino leads to the removal of more two phases, thus we are left with only one physical phase δ popular as Dirac CP-phase. In the same context if we consider a Majorana type neutrino then we have two more phases α and β. All the above mentioned parameters are called neutrino mixing parameter which altogether construct the unitary matrix as the following

$$U_{\text{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} U_{\text{Maj}} \tag{1.5.2}$$

where $c_{ij} = \cos\theta_{ij},\ s_{ij} = \sin\theta_{ij}$. The diagonal matrix $U_{\text{Maj}} = \text{diag}(1, e^{i\alpha}, e^{i(\zeta+\delta)})$ contains the Majorana CP phases α, ζ. The oscillation probability for a neutrino going from a flavor α to β is given by

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i>j} \text{Re}[U_{\beta i}U^*_{\alpha i}U^*_{\beta j}U_{\alpha j}]\sin^2\Big(\frac{\Delta m^2_{ij}L}{4E}\Big) \tag{1.5.3}$$

$$- 2\sum_{i>j} \text{Im}[U_{\beta i}U^*_{\alpha i}U^*_{\beta j}U_{\alpha j}]\sin^2\Big(\frac{\Delta m^2_{ij}L}{2E}\Big) \tag{1.5.4}$$

Under the interchange of $U \to U^*$, the first two terms in the Lagrangian remain same, which reveals the conservation of CP, whereas the last term alters the sign implying the difference between neutrino and antineutrino oscillation probability, which we can express analytically as,

$$P(\bar{\nu_\alpha} \to \bar{\nu_\beta}) - P(\nu_\alpha \to \nu_\beta) = 4\sum_{i>j} \mathrm{Im}[U_{\beta i}U^*_{\alpha i}U^*_{\beta j}U_{\alpha j}]\sin^2\Big(\frac{\Delta m^2_{ij}L}{2E}\Big) \quad (1.5.5)$$

where, $U = U_{PMNS}$ in short, $E \sim |p|$ is the neutrino energy, L is the distance between the source and the detector, and $\Delta m^2_{ij} = m^2_i - m^2_j$ is the mass squared difference. Where, the condition $\alpha = \beta$ makes the RHS of the equation vanish, resulting into a zero CP asymmetry. Thus we need at least two generations or flavors of light neutrinos to have an estimation of CP asymmetry. For having a nonzero probability for flavor oscillation the mass squared difference is needed to be non-zero, which fact thereafter proves the existence of neutrino mass.

Various neutrino mixing parameters are under observation and study, e.g., KamLAND experiment has evinced a considerably large solar mixing angle and confirmed the solar neutrino oscillation. Very recently the value of reactor angle has been found to be tiny but non-zero as declared by Double CHOOZ [33–35], Daya Bay [19] and RENO [20]. Now there are series of questions after proven the existence of neutrino mass and nonzero reactor angle, which are worrying the neutrino physics community to a grater extent. Some of them are, (i) which hierarchy of mass pattern, does the neutrino mass follow? (ii) what is the absolute neutrino mass scale?, as we only have two mass squared splitting (solar and atmosphere) and the sum over absolute masses (iii) why there is a deviation of the atmospheric mixing angle from the maximal value and in which octane does it belong? Very recently some dedicated experiment groups planned to study all the above mentioned issues: Daya Bay, T2K, RENO, NOνA and Double Chooz are such examples. The information on sum over absolute neutrino masses come from cosmological observation: WMAP analysis set an upper bound on $\sum m_i$ and is found to be $\sum m_i \leq 0.17$ eV. The figure 1.1 evinces the possible hierarchy pattern, which the neutrino mass may follow. There are two possible, as such, namely inverted and normal. Even if we are sure about the neutrino mass, however it is still under observation and analysis, asking, what mass hierarchy the neutrino mass exactly does follow. Well, till date we do not have any answer for

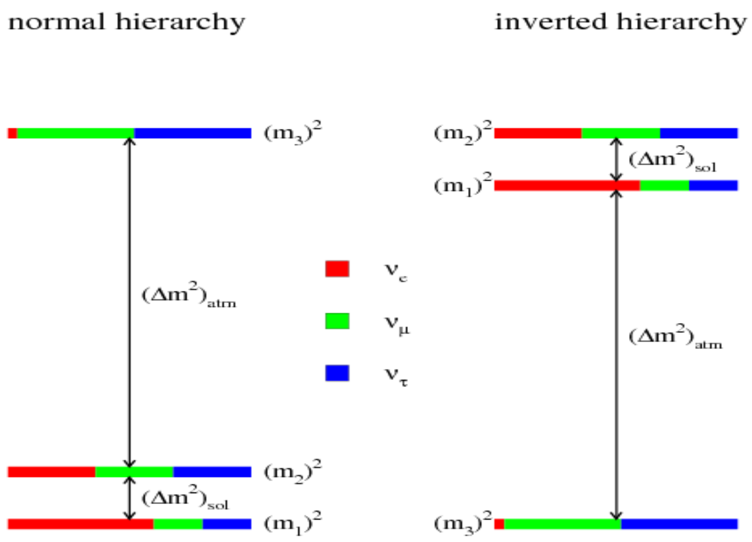

Figure 1.1: Possible hierarchy pattern of neutrino mass, we call them normal (left) and inverted (right) hierarchies [36]. The colors represent the flavor composition of each of the physical neutrinos: red for ν_e, green for ν_μ and blue for ν_τ

that although there have been several theoretical models which rule out the either mass ordering. The oscillation formula evince that the probability depends on the mass squared splitting, with no information on the absolute neutrino mass. The oscillation experiments always gave a positive value for the solar mass splitting which clearly implies $m_2 > m_1$, however they do not say about the sign of the atmospheric mass splitting Δm^2_{31}. Thus we can have two possible mass orderings [37–39] depending on the sign of Δm^2_{3l}.

- Normal mass hierarchy, which follow $m_3 > m_2 > m_1$ and
- Inverted mass hierarchy, which follow $m_2 > m_1 > m_3$

The list of queries does not end here! It is a long standing mystery asking whether neutrinos are Dirac or Majorana particle? Well for both the cases, the mass term can be generated via gauge invariant Yukawa like interaction followed by

$$\mathcal{L}_{\rm Dirac} = \bar{\nu_R} m_\nu \nu_L + {\rm H.C.} \tag{1.5.6}$$

$$\mathcal{L}_{\rm Majorana} = \nu_L^C m_\nu \nu_L + {\rm H.C.} \tag{1.5.7}$$

In essence, these two mass terms are different from each other in the point that for a Dirac type neutrino mass there is no violation of lepton number unlike the case for a Majorana neutrino mass term, which violates lepton number by two units. For both the scenario one has to look for a BSM scenario where there is an introduction of a gauge singlet RH neutrino. The RH neutrinos transform under the SM gauge group following a charge assignment like (1,1,0) under the respective symmetry groups. The Dirac mass term arises from a Yukawa coupling of the SM Higgs with the left handed neutral lepton of the lepton doublet in presence of the RH neutrino whose Yukawa Lagrangian reads

$$-\mathcal{L}_{\rm Yuk} = Y_\nu \bar{N_R}\tilde{H}L + \text{H.C.} \tag{1.5.8}$$

with $\tilde{H} = i\sigma_2 H^*$, ϕ being the SM Higgs field. After the EWSB, the neutral component of the Higgs acquires VEV of around 174 GeV and neutrino gets a Dirac mass as $\bar{N_R}m_\nu L$, where $m_\nu = Y_\nu v$.

As the neutrinos are only electrically neutral fermions in the SM, they could be Majorana particles, which by definition could be their own antiparticles. This would be in contrast to the rest of the SM Dirac fermions, for which their antiparticle is a different state. This hypothetical Majorana character of the neutrinos, although very common in theoretical models (as we will see later), does not have any impact on neutrino oscillations and, therefore, new observables to distinguish between Majorana and Dirac fermions need to be considered. The fact that a lepton can be its own antiparticle is directly related to the total lepton number (LN) violation, since a Majorana mass terms breaks LN symmetry by two units. Consequently, LN violating processes are usually considered as the smoking gun signatures for Majorana neutrinos, like neutrinoless double beta decay. Unfortunately, no experimental evidence has been found yet for any LN violating processes. Thus, knowing, if neutrinos are Majorana or Dirac fermions still an unprecedented job. Majorana mass comes from the seesaw mechanism where we introduce the RH heavy gauge singlet fermion. Seesaw models generate neutrino mass via the dimension-5 Weinberg operator, which we discuss in the following section. After the EWSB the Higgs doublet takes VEV and generates the following Majorana mass term

$$m_\nu = \frac{Y_\nu v^2}{4\Lambda_L} \tag{1.5.9}$$

For a coupling strength Y_ν of the order 1, in order to get a sub-eV light neutrino mass scale, Λ_L has to fall around $10^{14} - 10^{16}$ GeV where lepton number violation takes place.

1.6 Seesaw mechanism

Seesaw mechanisms play a non-trivial role in making the neutrinos massive. In essence this mechanism accounts for lepton number violation via the implementation of non-renormalizable dimension-5 Weinberg operator. Seesaw mechanism necessitates the extension of the SM by the incorporation of some extra fermions or scalars. Depending on the class of particle we add to the SM, different seesaws are named such as: type I seesaw, where right handed heavy neutrinos N_R (gauge singlets) are introduced; type II seesaw requires the inclusion of a scalar $SU(2)$ triplet Δ; and type III seesaw, which demands the introduction of fermion triplet (under $SU(2)$) field Σ. There is another kind of seesaw mechanism, termed as Inverse seesaw mechanism, which is a low scale seesaw scenario, we explain that in the end of this section.
Neutrino, being a member of the $SU(2)$ doublet representation, only one possible Weinberg operator is there to contribute to the Lagrangian, the expression for which one may write as

$$\delta\mathcal{L}_{d=5} = \frac{1}{2}\frac{c_{ij}}{\Lambda}(\bar{L_i^C}\tilde{H}^*)(\tilde{H}^\dagger L_j), \tag{1.6.1}$$

with c_{ij} as the dimensionless complex coefficient and $i, j = 1, 2, 3$. This operator yields a Majorana mass term for the neutrinos after the EWSB as,

$$\delta\mathcal{L}_{d=5} \to \frac{v^2 c_{ij}}{2\Lambda}(\bar{\nu_i^C}\nu_j + H.C.). \tag{1.6.2}$$

The higher the scale Λ, the smaller is the neutrino mass which naturally seems a seesaw scenario. And thus it is called so. The structure of the Weinberg operator decides the property of the new particles of a particular seesaw model. From the

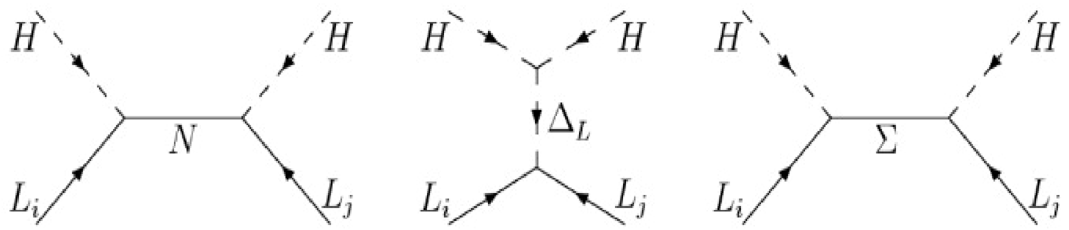

Figure 1.2: Schematic representation of type I, type II and type III seesaw mechanism

requirement of gauge invariance the new particle can be an $SU(2)$ singlet or triplet as it is supposed to couple to two $SU(2)$ doublets. despite this fact, the new particle can be either scalar or fermion depending on which we have a class of three chief seesaw models, pictorial representation of which we show in figure 1.2. In the next subsections all the above mentioned seesaw scenarios are shortly introduced.

1.6.1 Type I seesaw

As already discussed that the implementation of type I seesaw needs the inclusion of RH neutrinos [40–43], this new field offers a possible Yukawa coupling between the SM neutrino and the Higgs field in addition having a Majorana mass for the new field itself. The relevant Lagrangian responsible for type I seesaw is given by

$$-\mathcal{L}_{\text{TypeI}} = Y_\nu \bar{N}_R \tilde{H}^\dagger L + \frac{1}{2} M_R \bar{N}_R N_R^C + \text{H.C.} \tag{1.6.3}$$

where, the second term in the above equation violates lepton number by two units. Y_ν is a complex 3×3 non-symmetric mass matrix and M_R is a symmetric matrix with order 3. The above Lagrangian can be written in terms of the column vector of the left handed field as

$$\mathcal{L}_{\text{TypeI}}^{\text{mass}} = \frac{1}{2}\left(\bar{\nu}_L^C \bar{N}_R\right) \begin{pmatrix} 0 & m_D^T \\ m_D & M_R \end{pmatrix} \begin{pmatrix} \nu_L \\ N_R \end{pmatrix} \tag{1.6.4}$$

from the above neutrino mass Lagrangian, we can display the Majorana neutrino mass matrix as

$$M_\nu^{\text{TypeI}} = \begin{pmatrix} 0 & m_D^T \\ m_D & M_R \end{pmatrix} \tag{1.6.5}$$

which is typically a 6×6 matrix for three RH neutrinos added to the SM particle spectrum. It is better to have one RH neutrino per generation. Block diagonalizing this matrix, results into following two eigenvalues,

$$m_\nu \approx -\frac{m_D^2}{M_R} = -\frac{v^2 Y_\nu^2}{M_R}, m_N \approx M_R \tag{1.6.6}$$

M_R is that mass scale for the RH neutrino, where Lepton number violation took place whereas m_ν is the mass scale for the SM neutrino. Thus one can conclude that SM neutrino mass is a ratio of the smaller Dirac mass scale with the large Majorana mass scale. Now, m_ν can be written as $m_\nu \sim m_D^T M_R^{-1} m_D = U_{PMNS}^* m_\nu^{\text{diag}} U_{PMNS}^\dagger$. We call this mechanism as the Type I seesaw mechanism.

1.6.2 Type II seesaw

Addition of the scalar triplet Higgs to the SM, allows us to generate the neutrino mass (for detail you may see references [44–46]) via the following Lagrangian,

$$\mathcal{L}_{\text{Type II}} = -\frac{1}{2} Y_\Delta^{ij} \bar{L_i^C} \tilde{\Delta} L_j - \mu H^T i\sigma_2 \Delta^\dagger H - \frac{1}{2} M_\Delta^2 Tr(\Delta^\dagger \Delta) + \text{H.C.} \tag{1.6.7}$$

where the scalar triplet is denoted by Δ, in terms of three complex scalars $\Delta^0, \Delta^+, \Delta^{++}$ and having the following form

$$\Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix} \tag{1.6.8}$$

The first term of Eq. (1.6.7) represents the Yukawa interaction between the scalar triplet with the SM lepton doublet, with a coupling Y_Δ with $\tilde{\Delta} = i\sigma_2 \Delta^*$. σ_2 is Pauli spin matrix. Under the SM gauge group the scalar triplet transforms as (1,3,+1). M_Δ is the mass of the Higgs triplet with μ as its coupling with two Higgs doublets. When the neutral component of the Higgs doublet generates a nonzero VEV it induces a tadpole term for the scalar triplet via the second term of Eq. (1.6.7) giving an induced VEV to the scalar triplet Δ, and thus neutrino mass is generated.

$$m_\nu = \frac{Y_\Delta v_\delta}{\sqrt{2}}, v_\Delta \approx \mu \frac{v^2}{M_\Delta^2}. \tag{1.6.9}$$

1.6.3 Type III seesaw

Type III seesaw is realized via the inclusion of a fermion triplet [47, 48], denoted by Σ, to the SM particle content. The fermion triplet field couples to the LH neutrinos and the SM Higgs doublet by the following Lagrangian,

$$\mathcal{L}_{\text{TypeIII}} = -Y_{\Sigma}^{ij}\bar{L}_i\tilde{H}\Sigma - \frac{1}{2}M_{\Sigma}^{ij}Tr(\bar{\Sigma_i^C}\Sigma_j) + \text{H.C.} \tag{1.6.10}$$

Here Y_ν is a 3×3 dimensionless Yukawa coupling matrix. The third panel of the figure 1.2 represents the type III seesaw model. The $SU(2)_L$ triplet fermion has a definition in terms of three components (η_1, η_2, η_3) with the following $SU(2)_L$ representation.

$$\Sigma = \begin{pmatrix} \Sigma^0/\sqrt{2} & \Sigma^+ \\ \Sigma^- & \Sigma^0/\sqrt{2} \end{pmatrix}. \tag{1.6.11}$$

Where, the neutral component of the triplet fermion plays the role similar to that played by the RH neutrino in case of type I seesaw. In this seesaw model the neutrino mass is generated similarly as the type I seesaw, by the following formula

$$-m_\nu = m_D M_\Sigma m_D^T. \tag{1.6.12}$$

with $m_D = \frac{Y_\Sigma v}{\sqrt{2}}$. Interestingly, here also Lepton number is violated as the simultaneous appearance of Y_Σ and M_Σ does not assign any Lepton charges to Σ.

In this Thesis, we are also interested in the phenomenology involving a TeV scale right-handed neutrinos which is natural in inverse seesaw (ISS) models. Of special importance is the fact that these TeV scale RH neutrinos have better sensitivity of being accessed in the future colliders. The canonical type I seesaw also can accommodate TeV scale RH neutrino but with a very small Yukawa coupling of the order of 10^{-6}. But the inverse seesaw mechanism naturally accommodates a low scale RH neutrino with a larger value of Yukawa coupling, which is the driving cause of taking the ISS for explaining light neutrino mass. A bit detailed study of ISS is provided in the following subsection.

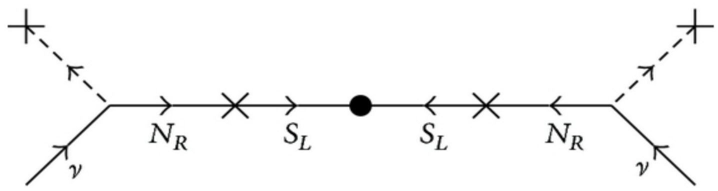

Figure 1.3: Schematic representation of inverse seesaw mechanism

1.6.4 Inverse seesaw

As mentioned earlier the ISS model offers the neutrino mass at the sub-eV scale at the cost of proposing a TeV scale RH neutrino [49–51]. To realize this scenario one needs to consider another RH fermion singlet (S) in addition to that (N_R), already taken for type I seesaw. This scenario is realized by making use of some extra symmetries, e.g., via the global lepton number symmetry. Essentially, the new fermion singlet is assigned a lepton number $L = -1$ which is opposite to that for N_R ($L = 1$). Now if the LN is conserved then the light neutrino mass matrix that this model yields has two degenerate eigenvalues, one Dirac neutrino and one massless neutrino. Since the LN symmetry is responsible for the generation of massless neutrinos we need to include a LN breaking parameter in order to generate non-zero neutrino masses. For small breaking, the neutrino masses will be small, which establish a relation between the smallness of neutrino masses with the scale where LN symmetry is broken. The figure 1.3 represents the ISS mechanism. For a three generations picture where three pairs of fermion singlets ν_R, S are added to the SM, the ISS Lagrangian in this case is given by

$$\mathcal{L} = -Y_\nu^{ij}\bar{L}_i\tilde{H}\nu_{Rj} - M_R^{ij}\bar{\nu_{Ri}^C}S_j - \frac{1}{2}\mu_R^{ij}\bar{\nu_{Ri}^C}\nu_{Rj} - \frac{1}{2}\mu_S^{ij}\bar{S_i^C}S_j + \text{H.C.} \tag{1.6.13}$$

where, Y_ν is the 3×3 neutrino Yukawa coupling matrix, M_R is a lepton number conserving complex 3×3 mass matrix, and μ_R and μ_S are Majorana complex 3×3 symmetric mass matrices that violate LN conservation by two units. After the EWSB, we obtain the complete 9×9 neutrino mass matrix as given by the following structure

$$m_\nu = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & \mu_R & M_R \\ 0 & M_R^T & \mu_S \end{pmatrix}. \tag{1.6.14}$$

It is to note that μ_R does not generate light neutrino masses at the tree level. We will set μ_R to zero for the rest of this thesis and consider a small μ_S as the only lepton number violating parameter leading to the light neutrino masses. In the mass range of our interest with μ_S, m_D, M_R, the mass matrix m_ν can be block diagonalized which leads to the following light neutrino mass formula under the ISS scheme.

$$m_{\text{light}} \approx m_D (M_R^T)^{-1} \mu_S M_R^{-1} m_D^T \tag{1.6.15}$$

1.7 Cosmological consequences of BSM physics

Neutrinos are supposed to be very tiny creatures in the sub atomic world, however they have big impact in the study of cosmos. Two foremost puzzles in modern cosmology are origin of dark matter and baryon asymmetry of the universe. It is indeed a delight to address these two issues along with the explanation of neutrino mass, within a single framework, although it a matter of choice only. And it will be even more delightful if the mechanism through which existence of neutrino mass is addressed, can also be a viable cause for the origin of the two above mentioned puzzles of cosmology. In this subsection we will briefly discuss such possibilities. As already mentioned in the earlier subsection that the presence of right handed neutrinos (RHN) are essential for generating neutrino mass beyond the standard model, thus one can say that there might be some phenomena associated with this RHN which knocks the door to cosmology.

1.7.1 Baryogenesis via leptogenesis

Cosmological and astronomical observations indicate that there is a tiny excess of matter over antimatter that is the present number of matter and antimatter are unequal. And there are strong evidences also which confirms this fact of matter excess over antimatter. This presently small but nonzero asymmetry in the amount of matter and antimatter is familiar as baryon asymmetry of the universe (BAU). The large scale structures such as galaxies, galaxy clusters, stars predominantly consist of matter rather than antimatter in appreciable measure. This fact takes us to a journey manifest for finding the precise cause behind it. Previously there was baryon symmetric universe at the epoch as suggested by many considerations, due to the fact that the early universe was radiation dominated, thus the photons always decayed to one matter-antimatter pair, hereby bringing the equality in their number. The evolution of this baryon asymmetric era from a previously baryon symmetric universe through the generation of a tiny but non-zero amount of baryon asymmetry is termed as baryogenesis. Even if we think that the present universe consists of equal numbers of matter and antimatter, there must be some annihilation process like $\mathrm{M} + \bar{\mathrm{M}} = 2\gamma$ one would expect. Unfortunately, till date we did not observe any process as such. Therefore it is claimed that the Universe is baryon asymmetric. Now the question is whether the SM of particle physics can explain the origin of this asymmetry or not! Well, the answer is NO. Although we have all the ingredients that are necessary to generate this asymmetry dynamically in an initial baryon-symmetric universe, yet it is unable to explain an observed amount of asymmetry [52]. There are two problems with SM baryogenesis. The Higgs is too heavy for the electroweak phase transition, which is to be of first order to account for a successful baryogenesis and which is of second order within the SM. Along with it, the amount of CP violated within the SM is too small to yield the observed BAU. Thereby we need to call for a Beyond Standard Model scenario for the study of baryogenesis. Andrew Sakharov in the year 1967 put forward a theory postulating the key ingredients which particle interactions and the cosmological evolution have to satisfy for having a successful baryogenesis. The criterion are as follows:

- There must be baryon number violation. A system must evolve from an initial state with $Y_{\Delta B} = 0$ to a state with $Y_{\Delta B} \neq 0$.

- There must be C and CP violation. In principle, the number of left-handed particles generated in any process would be different from the number of right-handed antiparticles (which are the CP conjugates of the left-handed particles): which is possible only when CP is violated. Moreover, C violation is also essential, as the generation of the left-handed particles should not compensate the generation of the left-handed antiparticles (which are the conjugates of the right-handed particles).

- Depart significantly from thermal equilibrium. The departure from equilibrium is realized when the above mentioned B-violating interaction rate is slower than the expansion rate of the universe, this fact generally does not escort the distribution of baryons and antibaryons of the universe into equilibrium. In essence, as the heavy particle decays, the decay product will move apart before it could participate in the inverse decay, causing a departure from equilibrium. In other words, before the chemical potentials of the two states become equal, they move apart from each other. Analytically one can write the out-of-equilibrium condition as $\Gamma(T) < H(T) = 1.66\sqrt{g_*}\frac{T^2}{M_{\rm Pl}}$ where Γ is the baryon-number violating interaction rate under discussion, g_* is the effective number of degrees of freedom available at temperature T and $M_{\rm Pl}$ is the Planck mass.

There are several mechanisms through which baryogenesis can be realized, viz., **GUT baryogenesis** [53–61]: where the out-of-equilibrium decay of heavy bosons create the baryon asymmetry in Grand Unified Theories; **Leptogenesis** [62]: the most popular mechanism of realizing the baryogenesis is leptogenesis, where the presence of singlet RHNs, as an ingredient of the seesaw mechanism (in particular type I and inverse) makes it possible to go through a decay, hereby creating an adequate lepton asymmetry which later on converts into baryon asymmetry through electro-weak sphaleron process. The rate of this decay process should be less than the expansion rate of the universe, to satisfy Sakharov's 3rd condition as discussed above; **Electroweak baryogenesis** [63, 64] is the scenario where departure from thermal equilibrium brought by electroweak phase transition; and finally the **Affleck-Dine mechanism** [65, 66] where the asymmetry arises in a classical scalar field which later on decays to particles. Among all these scenarios baryogenesis via Leptogenesis has gained popularity which has been picked for our work in this thesis.

The amount of baryon asymmetry has been confirmed by various cosmological observations. Big-Bang-nucleosynthesis(BBN) is one of them. In BBN observation, abundance of light elements like D, ^{3}He, ^{4}He, ^{7}Li has been predicted. The crucial time for premordial nucleosynthesis is when the thermal bath temperature falls below T $\leq$ 1 MeV. And this prediction depends on a single parameter η. One can find the BAU in two different ways given by the following equations where the difference between the number of baryons and antibaryons is normalized to the number of photons.

$$\eta = \left.\frac{n_B - n_{\bar{B}}}{n_\gamma}\right|_0 = (6.21 \pm 0.16) \times 10^{-10} \tag{1.7.1}$$

$$y_{\Delta B} = \left.\frac{\eta_B - \eta_{\bar{B}}}{s}\right|_0 = (8.75 \pm 0.23) \times 10^{-11} \tag{1.7.2}$$

where, n_B, $n_{\bar{B}}$, n_γ and s are the number densities of respectively, baryons, antibaryons, photons and entropy. The entropy density is also a function of temperature, given by $s = g_*(2\pi^2/45)T^3$ which is conserved during the expansion of the Universe. The subscript "0" means the observation of these ratios at present time. The primordial abundances of the above elements are confirmed by several observations. From those observations a range of η is found which is in agreement with all the four abundances and that in turn favors the standard hot big bang cosmology. The range can be shown as (with 95% CL)

$$4.7 \times 10^{-10} \leq \eta \leq 6.5 \times 10^{-10}, \quad 0.017 \leq \Omega_B h^2 \leq 0.024 \tag{1.7.3}$$

The other impressive choice to determine Ω_B is from measurements of the cosmic microwave background (CMB) anisotropies, the detail of which can be found in [67]. From a very recent observation such as WMAP5 data only, gives (at 68% CL) [68]

$$0.02149 < \Omega h^2 < 0.02397 \tag{1.7.4}$$

Now it is better to measure the baryon asymmetry by the Eq. (1.7.2), as in this equation the difference between numbers of baryons and antibaryons is normalized to the entropy density, since the entropy density is conserved during the expansion of the universe. From Eq. (1.7.3) and Eq. (1.7.4), one can write the BAU in terms of Y_B at 3σ level as

$$Y_{\Delta B}^{BBN} = (8.10 \pm 0.85) \times 10^{-11}, \;\; Y_{\Delta B}^{CMB} = (8.79 \pm 0.44) \times 10^{-11} \tag{1.7.5}$$

Baryogenesis via leptogenesis is a simple mechanism to explain the BAU as suggested by Fukugita and Yanagida [62]. A lepton asymmetry is dynamically generated in the lepton sector first, then it gets converted into baryon asymmetry by (B+L) violating sphaleron interactions [69] which exist in the SM. A platform to implement this mechanism can be a class of seesaw models (in particular type I in this thesis), where the presence of RH neutrinos brings out the scenario of leptogenesis via the CP-violating decay of the RH neutrinos themselves. With the growing interest of taking leptogenesis as the process of explaining the BAU several BSM frame works have shown anticipating role. Therefore leptogenesis is a mechanism of generating lepton asymmetry before the electroweak phase transition, which later on gets converted into baryon asymmetry after reprocessing by electroweak sphalerons. The relation between baryon asymmetry and lepton asymmetry is given by

$$Y_B = -\left(\frac{8n_G + 4n_H}{14n_G + 9n_H}\right)Y_L \tag{1.7.6}$$

with n_H as the number of Higgs doublets and n_G the number of fermion generations (in thermal equilibrium). The CP asymmetry generated by the decay of the lightest RHN is given by

$$\epsilon_1 = \frac{\Sigma_\alpha[\Gamma(N_1 \to Hl_\alpha)] - [\Gamma(N_1 \to \bar{H}\bar{l}_\alpha)]}{\Sigma_\alpha[\Gamma(N_1 \to Hl_\alpha) + [\Gamma(N_1 \to \bar{H}\bar{l}_\alpha)]]} \tag{1.7.7}$$

Now the process of leptogenesis belongs to two distinct scales, high scale and low scale. By high scale leptogenesis we mean when the RHN mass is of order 10^{12} GeV or more which naturally comes from the generation of light neutrino mass by the canonical type I seesaw, whereas the second kind rules over a lower mass regime of RH neutrinos e.g., when M_R falls around a TeV. For the explanation of smallness of neutrino masses seesaw mechanisms demands the inclusion of heavy RHN. The mass of these heavy RHN needs to fall around 10^{12} GeV if the seesaw is canonical type I as already said. Now the RHN mass can stay in a lower mass regime (as around a TeV) if inverse seesaw explains the tiny neutrino mass. Thus depending on various seesaw scenarios the RHN mass scale varies. These RHNs transform as singlets under $SU(2)_L$ symmetry group. In a basis where charged lepton Yukawa couplings are diagonal, we can write the SM Lagrangian with the newly added RHN as,

$$\mathcal{L} = \mathcal{L}_{SM} + \Big(\frac{M_i}{2} N_i^2 + \lambda_{i\alpha} N_i l_\alpha H + h_\alpha H + h_\alpha H^c \bar{e}_{R\alpha} l_\alpha + h.c.\Big). \tag{1.7.8}$$

with l_α and $e_{R\alpha}$ as lepton doublet and singlet of flavor ($\alpha = e, \mu, \tau$). The newly added RHN s undergo out-of-equilibrium decay to SM leptons and Higgs via their complex Yukawa couplings, which later on play as a new source of CP violation in order to yield a nonzero lepton asymmetry in the lepton sector. The RHNs are Majorana in nature. The Majorana mass term of these RHNs indicate lepton number violation. During the decoupling of the very heavy RHN s from thermal bath, they decay to create leptons and antileptons via the Yukawa coupling

$$\mathcal{L} \subset \lambda \bar{N}_i L H + \lambda^\dagger N_i \bar{L} H^* \tag{1.7.9}$$

where λ is a 3×3 matrix containing the Yukawa coupling governing the decay of RHNs.

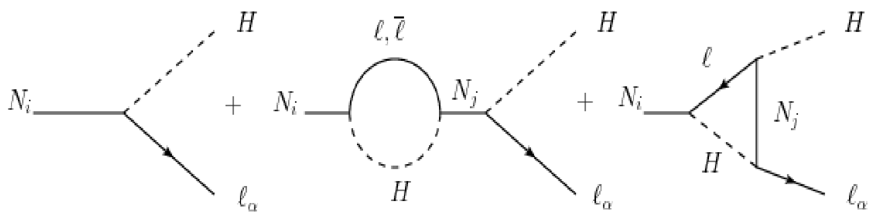

Figure 1.4: Decay modes of right handed neutrinos taking part in leptogenesis

Resonant leptogenesis

Apart from the tree level diagram as shown in figure 1.4 the two one-loop diagrams also contribute to the CP-violating lepton asymmetry. In principle, the interference of the tree-level decay amplitude with the absorptive parts of the one-loop self-energy and vertex diagrams violates CP and hence gives rise to a considerable amount of lepton asymmetry. The amount of CP violation that comes from the self-energy plot can be relatively larger than that comes from the vertex graph (see for e.g., [70–72]) through the mixing of two nearly degenerate heavy Majorana neutrinos. Even the lepton asymmetry can attain a value of order unity if two of the heavy Majorana neutrinos have a mass difference comparable to their decay widths. Generally the self energy diagram holds good when the heavy Majorana neutrino mass falls around TeV [73]. In other words, the heavy neutrino self energy effects on the CP asymmetry become dominant and hence gets resonantly enhanced. Because of this resonant enhancement of the asymmetry, this scenario of leptogenesis is termed as resonant leptogenesis. The larger the amount of lepton asymmetry, the smaller the lower bound on RHN mass.

It is said that leptogenesis is a consequence of seesaw mechanisms due to the presence of heavy Majorana fields. Now for a common search for the origin of neutrino mass and baryogenesis via leptogenesis, seesaw mechanisms take a strong hold. Resonant leptogenesis can be regarded as one of the consequences of that motivation. For resonant leptogenesis to occur, some sufficient and necessary conditions are to be satisfied, which even results into a tremendous enhancement of the leptonic asymmetry up to order unity [73]. For a pair of Majorana neutrino, one can write the conditions as

$$m_{N_i} - m_{N_j} \sim \frac{\Gamma_{N_{i,j}}}{2}, \quad \frac{|Im(Y_\nu^\dagger Y_\nu)^2_{i,j}|}{(Y_\nu^\dagger Y_\nu)_{i,i}(Y_\nu^\dagger Y_\nu)^2_{j,j}} \sim 1 \tag{1.7.10}$$

where, Γ_{N_i} are the N_i decay widths. The lepton asymmetry can be found from the following formula taken from [74, 75]

$$\epsilon_{il}^{\text{mix}} = \sum_{j\neq i} \frac{\text{Im}[Y_{\nu_{il}} Y^*_{\nu_{jl}} (Y_\nu Y_\nu^\dagger)_{ij}] + \frac{M_i}{M_j}\text{Im}[(Y_{\nu_{il}} Y^*_{\nu_{jl}} (Y_\nu Y_\nu^\dagger)_{ji}]}{(Y_\nu Y_\nu^\dagger)_{ii}(Y_\nu Y_\nu^\dagger)_{jj}} f_{ij}^{mix} \tag{1.7.11}$$

with the regulator given by,

$$f_{ij}^{mix} = \frac{(M_i^2 - M_j^2)M_i\Gamma_j}{(M_i^2 - M_j^2)^2 + M_i^2\Gamma_j^2}$$

with $\Gamma_i = \frac{M_i}{8\pi}(Y_\nu Y_\nu^\dagger)_{ii}$ as the tree level heavy-neutrino decay width. Now, there is a similar contribution ϵ_{il}^{osc} to the CP asymmetry from RH neutrino oscillation

[76, 77]. Its form is given by Eq. (1.7.11) with the replacement $f_{ij}^{mix} \rightarrow f_{ij}^{osc}$, where

$$f_{ij}^{osc} = \frac{(M_i^2 - M_j^2)M_i\Gamma_j}{(M_i^2 - M_j^2)^2 + (M_i\Gamma_i + M_j\Gamma_j)^2 \frac{\det[\mathrm{Re}(Y_\nu Y_\nu^\dagger)]}{(Y_\nu Y_\nu^\dagger)_{ii}(Y_\nu Y_\nu^\dagger)_{ii}}}$$

The total CP asymmetry therefore can be written as $\epsilon_{il} = \epsilon_{il}^{mix} + \epsilon_{il}^{osc}$. One can write the final BAU as,

$$\eta_B \simeq -3 \times 10^{-2} \sum_{l,i} \frac{\epsilon_{il}}{K_l^{\mathrm{eff}} \min(z_c, z_l)} \tag{1.7.12}$$

where, $z_c = \frac{M_N}{T_c}$, $T_c \sim 149$ GeV, the critical temperature below which the sphaleron transition processes freeze-out, $z_l \simeq 1.25 \ln(25 K_l^{\mathrm{eff}})$ and $K_l^{\mathrm{eff}} = \kappa_l \sum_i K_i B_{il}$, with $K_i = \Gamma_i / H_N$ is the wash out factor. H_N is $1.66\sqrt{g^*} M_N^2 / M_{\mathrm{Pl}}$ is the Hubble expansion rate at temperature $\sim M_N$ and $g^* \simeq 106.75$. B_{il}'s are the branching ratios of the N_i decay to leptons of lth flavor: $B_{il} = \frac{|Y_{\nu_{il}}|^2}{(Y_\nu Y_\nu^\dagger)_{ii}}$. Including the RIS(Real Intermediate State) subtracted collision terms one can write the factor κ as,

$$\kappa_l = 2 \sum_{i, jj \neq i} \frac{\mathrm{Re}[Y_{\nu il} Y_{\nu jl}^* (YY^\dagger)_{ij}] + \mathrm{Im}[(Y_{\nu il} Y_{\nu jl}^*)^2]}{\mathrm{Re}[(Y^\dagger Y)_{ll} \{(YY^\dagger)_{ii} + (YY^\dagger)_{jj}\}]} \left(1 - 2i \frac{M_i - M_j}{\Gamma_i + \Gamma_j}\right)^{-1} \tag{1.7.13}$$

where, Y_ν is the Dirac Yukawa coupling matrix in a basis where RH neutrino mass is diagonal. As seen from the expression Eq. 1.7.11. It is worth noting that, during the calculation of RIS contribution since only the diagonal terms are considered in the sum, κ_l can take its maximum value and hence we can have $\kappa_l = 1 + O(\delta_l^2)$.

1.7.2 Dark matter

Starting from some astrophysical observations, such as rotation curve of spiral galaxies around the cluster by Fritz Zwicky [78, 79], inhomogeneity in cosmic microwave background radiation (CMBR) [80], or more recent observations in Bullet cluster [81] to the latest cosmology data provided by the Planck satellite [52], hint towards the existence of dark matter (DM) in the universe.

One of the main evidences for DM comes from measuring the rotation speed of galaxies. One can compute the mass of a galaxy by finding the velocity of the stars as they orbit the center of the galaxies. Had the galaxies composed of visible matter only, major portion of their masses would be concentrated in the center. But the Kepler's law says that the orbital velocities of the stars decreases as one goes to the outer edges of the galaxy, because there would be less mass. But intriguingly, astronomers observed that the orbital velocities of the stars around the center remain constant and do not decrease even if we go to a larger distance from the center of the galaxy, where there are fewer stars. And this fact implies that there must be an unseen mass in the galaxies even beyond the area containing majority of the stars.

The study of galaxy clusters gives another important observation for DM evidence. "Gravitational lensing" is one of the methods for measuring the mass of galaxy clusters. Einstein's theory of relativity tells that a massive object can bend the light which is coming from a distant source towards us, that way the object behaves like a gravitational lens. By measuring the distortion of the light, the total mass of the galaxy cluster is estimated. From this method it is found that a major portion of the galaxy clusters are composed of dark matter.

Then among the direct observational evidences, Bullet cluster gives strong confirmation regarding the existence of DM. Bullet cluster is composed of two galaxy clusters passing through each other. Now, interestingly when the two galaxies pass each other, the visible matter portions collide and slow down while the dark matter components pass each other without interacting and slowing down. This fact creates a separation between the dark and visible matter of each cluster. This separation was detected by comparing X-ray images of the luminous taken with the Chandra X-ray observatory. The dark matter components were found moving away from the center with high speeds, however the two narrower region of the ordinary matter were moving with less speeds behind them. As this evidence does not obey Newtonian mechanics, thus it is announced as a direct evidence for dark matter.

A particle description of DM is much sought after as the SM fails to provide a particle DM candidate that can satisfy all the criteria of a good DM candidate [82] and lot of exercises are performed (for a brief review, please refer to [83, 84]) to accommodate DM in extensions of SM. With the motivation of accessing an experimental verification of DM, a plethora of BSM frameworks are constructed assuming the DM to be a scalar, fermion or a vector boson and which can give rise to the correct DM phenomenology along with the possibility to be tested at several different experiments. Among them, thermal freeze-out of the weakly interacting massive particle (WIMP) [85] paradigm is the most popular BSM scenario as the correct DM relic abundance can be achieved for such a particle as it has interaction strength similar to weak interactions. This coincidence is also referred to as the *WIMP Miracle*. In terms of density parameter and $h =$ (Hubble Parameter)/100, the present dark matter abundance is conventionally reported as [52]

$$\Omega_{\rm DM} h^2 = 0.1187 \pm 0.0017 \tag{1.7.14}$$

Using the measured value of Hubble parameter, this announces that, approximately 26% of the total energy density of the present Universe being made up of DM.

1.8 Discrete flavour symmetry

Particle physics community shall ever remain indebted to Symmetry, as it plays a nontrivial role in addressing many observable phenomena associated with this ball park of particles and forces. Starting from continuous symmetries such as Lorentz, Poincare and gauge symmetries, we see that they are essential to understand several particle physics phenomena like strong, weak and electromagnetic interactions among particles. Along with these, there are discrete symmetries such as Charge conjugation(C), Parity(P) and Time reversal(T), which are also of special importance. To realize them particle physicists of different decades put forward many models.

Several continuous symmetry groups such as SU(N) and SO(N) are found to play vital role in explaining the masses of elementary particles. The non Abelian continuous symmetry groups are also termed as the lie groups of particle physics. The Standard Model is a collection of $SU(3)_c \times SU(2)_L \times U(1)_Y$ Lie groups, which is popularly known as Glashow-Weinberg-Salam model. It does not accommodate neutrino mass as the SM Higgs can not give mass to the neutrinos due to the absence of the right handed neutrinos within it. There are several extensions of the SM, where the SM gauge groups are augmented with one or more symmetry groups. In essence, those symmetry groups beautifully accommodate some extra right handed neutrinos in particular, and holds a concrete theory explaining the existence of massive neutrinos. In this context some non-Abelian discrete symmetry groups (please see e.g., [86] for a detailed analysis) took a strong hold of the entire scenario, with the introduction of the RHNs along with some additional scalar fields. In this thesis we build a few new models and modify some of earlier in the light of some non-Abelian discrete flavour symmetry groups. We extensively use S_4 and A_4 symmetry groups to explain the neutrino phenomenology in this thesis. On the other hand some sub groups such as Z_N of the bigger groups e.g., S_N makes it possible to control the desired and permitted Yukawa couplings in model building beyond the standard model. A class of Z_N groups are in extensive use in this context. This class of Z_N groups are even helpful to shade light on the dark sector, specially in stabilizing a potential dark matter candidate in a particular model. In this regard the non-Abelian groups are of special importance as they can simultaneously accommodate neutrino mass and a stable dark matter candidate under a proper charge assignment to the particle contents of a model. In the following subsections we will briefly discuss the properties of S_4 and A_4 symmetry groups.

1.8.1 The group S_4 and its properties

S_N is a group of permutation of N objects, that is all possible kinds of permutation among these N number of objects form a group called S_N. Order of S_N goes as N!. The S_N group has two one-dimensional representations, one is trivial singlet, which is by definition invariant under all the elements (symmetric representation), the other is pseudo singlet, that is, symmetric under the even permutation-elements but anti-symmetric under the odd permutation-elements. Depending on the number N, there are variants of groups starting from S_1, S_2 and so on. According to the definition of S_N, there is only one element in the group S_1. Similarly S_2 has got two elements formed by the permutation of two objects, which is nothing but the group Z_2 and Abelian. Therefore we can start with the immediate bigger group formed by the permutation of three objects familiar as S_3 with order $3! = 6$. S_3 is isomorphic to the symmetry group of a equilateral triangle. Following the same definition for S_N, S_4 consists of all permutations among four objects, x_1, x_2, x_3, x_4. The generalized transformation among the positions of four objects, one can write as

$$x_1, x_3, x_2, x_4 \rightarrow x_i, x_j, x_k, x_l$$

The order of S_4 is equal to $4! = 24$. There are two generators of S_4 familiar as S and T which satisfy

$$T^4 = S^3 = e,\ TS^2T = S \tag{1.8.1}$$

All of the S_4 elements can be written as products of these two generators. There are five in-equivalent irreducible representations of S_4, among which there are two singlets 1 and $1'$, one doublet 2 and two triplets 3 and $3'$. S and T have different structures, depending on which irreducible representation we are considering singlet, doublet or triplet. The representations are given as follows:

$$a, b \sim 1_1, \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \sim 2, \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}, \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \sim 3, \begin{pmatrix} a'_1 \\ a'_2 \\ a'_3 \end{pmatrix}, \begin{pmatrix} b'_1 \\ b'_2 \\ b'_3 \end{pmatrix} \sim 3'.$$

$$(A)_3 \times (B)_3 = (A \cdot B)_1 + \begin{pmatrix} A \cdot \Sigma \cdot B \\ A \cdot \Sigma^* \cdot B \end{pmatrix}_2 + \begin{pmatrix} \{A_y B_z\} \\ \{A_z B_x\} \\ \{A_x B_y\} \end{pmatrix}_3 + \begin{pmatrix} [A_y B_z] \\ [A_z B_x] \\ [A_x B_y] \end{pmatrix}_{3'}. \tag{1.8.2}$$

$$
\begin{aligned}
A \cdot B &= A_x B_x + A_y B_y + A_z B_z \\
\{A_i B_j\} &= A_i B_j + B_j A_i \\
[A_i B_j] &= A_i B_j - A_j B_j \\
A \cdot \Sigma \cdot B &= A_x B_x + \omega A_y B_y + \omega^2 A_z B_z \\
A \cdot \Sigma^* \cdot B &= A_x B_x + \omega^2 A_y B_y + \omega A_z B_z .
\end{aligned}
\tag{1.8.3}
$$

Later on for simplicity, we can replace $3 \to 3_1$, $3' \to 3_2$, $1 \to 1_1$, $1' \to 1_2$. The tensor products of S_4 that has been used in the present analysis are given below .

$$
3 \times 1_1 = 3, 3 \times 1_2 = 3_2, 3_2 \times 1_2 = 3, 2 \times 1_2 = 2.
$$

$$
\begin{aligned}
2 \otimes 2 &= 1_1 \oplus 1_2 \oplus 2, \\
3_1 \otimes 3_1 &= 1_1 \oplus 2 \oplus 3_1 \oplus 3_2.
\end{aligned}
$$

The Clebsch-Gordon coefficients for the product of two triplets can be written from [86] as follows.

$$
\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}_{3_1} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{3_1} = (a_1 b_1 + a_2 b_2 + a_3 b_3)_{1_1} \oplus \begin{pmatrix} 1/\sqrt{2}(a_2 b_2 - a_3 b_3) \\ 1/\sqrt{6}(-2a_1 b_1 + a_2 b_2 + a_3 b_3) \end{pmatrix}_2 \oplus
$$
$$
\begin{pmatrix} a_2 b_3 + a_3 b_2 \\ a_1 b_3 + a_3 b_1 \\ a_1 b_2 + a_2 b_1 \end{pmatrix}_{3_1} \oplus \begin{pmatrix} a_3 b_2 - a_2 b_3 \\ a_1 b_3 - a_3 b_1 \\ a_2 b_1 - a_1 b_2 \end{pmatrix}_{3_2} .
$$

$$
\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}_{3_2} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{3_2} = (a_1 b_1 + a_2 b_2 + a_3 b_3)_{1_1} \oplus \begin{pmatrix} 1/\sqrt{2}(a_2 b_2 - a_3 b_3) \\ 1/\sqrt{6}(-2a_1 b_1 + a_2 b_2 + a_3 b_3) \end{pmatrix}_2 \oplus
$$
$$
\begin{pmatrix} a_2 b_3 + a_3 b_2 \\ a_1 b_3 + a_3 b_1 \\ a_1 b_2 + a_2 b_1 \end{pmatrix}_{3_1} \oplus \begin{pmatrix} a_3 b_2 - a_2 b_3 \\ a_1 b_3 - a_3 b_1 \\ a_2 b_1 - a_1 b_2 \end{pmatrix}_{3_2} .
$$

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}_{3_1} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{3_2} = (a_1b_1 + a_2b_2 + a_3b_3)_{1_2} \oplus \begin{pmatrix} 1/\sqrt{6}(2a_1b_1 - a_2b_2 - a_3b_3) \\ 1/\sqrt{2}(a_2b_2 - a_3b_3) \end{pmatrix}_2 \oplus$$
$$\begin{pmatrix} a_3b_2 - a_2b_3 \\ a_1b_3 - a_3b_1 \\ a_2b_1 - a_1b_2 \end{pmatrix}_{3_1} \oplus \begin{pmatrix} a_2b_3 + a_3b_2 \\ a_1b_3 + a_3b_1 \\ a_1b_2 + a_2b_1 \end{pmatrix}_{3_2} .$$

1.8.2 The group A_4 and its properties

A_4 consists of all even permutations of S_4 with order equal to $4!/2 = 12$. The A_4 group is the symmetry of a tetrahedron. Thus, the A_4 group is often denoted as T. A_4 has four conjugacy classes and hence four irreducible representations, among which there are three singlets and one triplet. The group has got two generators namely **S** and **T**. The triplet multiplication rules of A_4 that has been used in this thesis are given below. There are two sets of Clebsch Gordan coefficients involved in the triplet product rules. One has been prepared by the **S**-diagonal basis, i.e. when the generator **S** is diagonal, and another is built from T-diagonal basis, i.e. when the generator **T** is diagonal (for more details see [87–89]).
The representations are given as follows

$$a, b \sim 1, \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}, \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \sim 3.$$

$$\mathbf{1} \otimes \mathbf{1} = \mathbf{1}, \quad \mathbf{1'} \otimes \mathbf{1'} = \mathbf{1''}, \quad \mathbf{1'} \otimes \mathbf{1''} = \mathbf{1}$$
$$\mathbf{1''} \otimes \mathbf{1''} = \mathbf{1'}, \quad \mathbf{1'} \otimes \mathbf{3} = \mathbf{3}, \quad \mathbf{1''} \otimes \mathbf{3} = \mathbf{3}$$
$$\mathbf{3} \otimes \mathbf{3} = \mathbf{1} \otimes \mathbf{1'} \otimes \mathbf{1''} \otimes \mathbf{3_a} \otimes \mathbf{3_s}$$

where a and s in the subscript corresponds to anti-symmetric and symmetric parts respectively. Denoting two triplets as (a_1, a_2, a_3) and (b_1, b_2, b_3) respectively, their direct product can be decomposed into the direct sum mentioned above as

$$\mathbf{1} \backsim \mathbf{a_1 b_1 + a_2 b_3 + a_3 b_2}$$

$$\mathbf{1'} \backsim \mathbf{a_3b_3 + a_1b_2 + a_2b_1}$$

$$\mathbf{1''} \backsim \mathbf{a_2b_2 + a_1b_3 + a_3b_1}$$

$$\mathbf{3_s} \backsim (\mathbf{2a_1b_1 - a_2b_3 - a_3b_2, 2a_3b_3 - a_1b_2 - a_2b_1, 2a_2b_2 - a_1b_3 - a_3b_1})$$

$$\mathbf{3_a} \backsim (\mathbf{a_2b_3 - a_3b_2, a_1b_2 - a_2b_1, a_1b_3 - a_3b_1})$$

The above product rules are built by considering the triplet representation in a basis where the generator **T** is diagonal. Moreover we also have another set of Clebsch Gordan coefficients for the triplet product rule, considering the triplets in a basis where **S** is diagonal instead **T**. They are as follows.

$$\mathbf{1} \backsim \mathbf{a_1b_1 + b_2b_2 + a_3b_3}$$

$$\mathbf{1'} \backsim \mathbf{a_1b_1} + \omega^{\mathbf{2}}\mathbf{a_2b_2} + \omega\mathbf{a_3b_3}$$

$$\mathbf{1''} \backsim \mathbf{a_1b_1} + \omega\mathbf{a_2b_2} + \omega^{\mathbf{2}}\mathbf{a_3b_3}$$

$$\mathbf{3} \backsim (\mathbf{a_2b_3, a_3b_1, a_1b_2})$$

$$\mathbf{3} \backsim (\mathbf{a_3b_2, a_1b_3, a_2b_1})$$

Bibliography

[1] Mohapatra, R. N. and Smirnov, A. Y. Neutrino mass and New Physics. *Annual Review of Nuclear and Particle Science*, 56:569-628, 2006.

[2] Mohapatra, R. N. and others. Theory of neutrinos: A white paper. *Reports on Progress in Physics* 70:1757-1867, 2007.

[3] King, S. F. Neutrino mass models. *Nuclear Physics Proceedings Supplements*, 118:267, 2003.

[4] Altarelli, G. and Feruglio, F. Models of neutrino masses and mixings. *New Journal of Physics*, 6:106, 2004.

[5] Reines, F., and Cowan, C. L. Free anti-neutrino absorption cross-section.1: Measurement of the free anti-neutrino absorption cross-section by protons. *Physical Review* , 113(1):273, 1959.

[6] Davis, R. Jr., Harmer, D. S., and Hoffman, K. C. Search for neutrinos from the sun. *Physical Review Letters*, 20(21):1205, 1968.

[7] Cleveland, B. T., et al. Measurement of the solar electron neutrino flux with the Homestake chlorine detector. *Astrophysical Journal*, 496:505, 1998.

[8] Hirata, K. S., et al. Observation of a small atmospheric ν_e/ν_μ ratio in Kamiokande. *Physics Letters B*, 280:146-152, 1992.

[9] Becker-Szendy, R., et al. The Electron-neutrino and muon-neutrino content of the atmospheric flux. *Physical Review D*, 46:3720, 1992.

[10] Bilenky, S. M. and Pontecorvo, B. Lepton Mixing And Neutrino Oscillations. *Physics Reports* , 41:225-261, 1978.

[11] Fukuda, Y., et al. Evidence for oscillation of atmospheric neutrinos. *Physical Review Letters*, 81:1562, 1998.

[12] Ashie, Y., et al. A Measurement of atmospheric neutrino oscillation parameters by SUPER-KAMIOKANDE I, *Physical Review D*, 71:112005, 2005.

[13] Eguchi, K., et al. First results from KamLAND: Evidence for reactor antineutrino disappearance. *Physical Review Letters*, 90:021802, 2003.

[14] Araki, T. Measurement of neutrino Oscillation with KamLAND:Evidence of spectral distortion. *Physical Review Letters*, 94:081801, 2005.

[15] Aliu, E., et al. Evidence for muon neutrino oscillation in an accelerator-based experiment. *Physical Review Letters*, 94:081802, 2005.

[16] Michael, D., et al. Observation of muon neutrino disappearance with the MINOS detectors and NuMI neutrino beam. *Physical Review Letters*, 97:191801, 2006.

[17] Abe, K., et al. Indication of Electron Neutrino Appearance from an Accelerator-produced Off-axis Muon Neutrino Beam. *Physical Review Letters*, 107(4):041801, 2011.

[18] Abe, Y., et al. Indication for the disappearance of reactor electron antineutrino in the Double Chooz experiment. *Physical Review Letters*, 108(13):131801, 2012.

[19] An, F. P., et al. Observation of electron anti-neutrino disappearance at Daya Bay. *Physical Review Letters*, 108(17):171803, 2012.

[20] Ahn, J. K., et al. Observation of Reactor Electron Antineutrino Disappearance in the RENO experiment. *Physical Review Letters*, 108(19):191802, 2012.

[21] Weinberg, S. A Model of Leptons. *Physical Review Letters*, 19:1264-1266, 1967.

[22] Weinberg, S. Effects of a neutral intermediate boson in semileptonic processes. *Physical Review D*, 5:1412-1417, 1972.

[23] Gross, D.J. and Wilczek, F. Asymptotically Free Gauge Theories. *Physical Review D*, 8:3633-3652, 1973.

[24] Gonzalez-Garcia, M. C. and Maltoni, M. Phenomenology with massive neutrinos. *Physics Reports* , 460(1):1-129, 2008.

[25] Coleman, S. R. and Weinberg, E. J. Radiative Corrections as the origin of Spontaneous symmetry Breaking. *Physical Review D*, 7:1888-1910, 1973.

[26] Drees, M. An Introduction to supersymmetry. arXiv: hep-ph/9611409, 1996.

[27] Martin, S. P. A Supersymmetry primer. *Advanced Series on Directions in High Energy Physics* , 18:1-98, 1998.

[28] Vissani, F. Do experiments suggest a hierarchy problem?. *Physical Review D*, 57:7027-7030, 1998.

[29] Casas, J. A., Espinosa, J. R., and Hidalgo, I. Implications for new physics from fine-tuning arguments. 1. Application to SUSY and seesaw cases. *Journal of High Energy Physics*, 11:057, 2004.

[30] Abada, A., Biggio, C., Bonnet, F., Gavela, M. B., and Hambye, T. Low energy effects of neutrino masses. *Journal of High Energy Physics*, 12:061, 2007.

[31] Farina, M., Pappadopulo, D., and Strumia, A. A modified naturalness principle and its experimental tests. *Journal of High Energy Physics*, 08: 022, 2013.

[32] Patrignani, C., et al. [Particle Data Group collaboration]. Review of Particle Physics. *Chinese Physics*, C40:100001, 2016.

[33] Abe, Y., et al. Indication of Reactor $\bar{\nu}_e$ Disappearance in the Double Chooz Experiment. *Physical Review Letters*, 108(13):131801, 2012.

[34] Abe, Y., et al. Improved measurements of the neutrino mixing angle θ_{13} with the Double Chooz detector. *Journal of High Energy Physics*, 10:086, 2014.

[35] Abe, Y., et al. Background-independent measurement of θ_{13} in Double Chooz. *Physics Letters B*, 735:51-56, 2014.

[36] Cahn, R. N., et al. *White Paper: Measuring the Neutrino Mass Hierarchy.* Retrieved on 21st July 2013 from https://arxiv.org/abs/1307.5487.

[37] Gandhi, R., Ghoshal, P., Goswami, S., Mehta, P., Sankar, S. U., and Shalgar, S. Mass Hierarchy Determination via future Atmospheric Neutrino detectors. *Physical Review D*, 76:073012, 2007.

[38] Gandhi, R., Ghoshal, P., Goswami, S., and Sankar, S. U. Mass Hierarchy Determination for $\theta_{13} = 0$ and atmospheric neutrinos. *Modern Physics Letters A* , 25:2255-2266, 2010.

[39] Dighe, A., Goswami, S., and Ray, S. 2540 km: Bimagic Baseline for neutrino oscillation parameters. *Physical Review Letters* , 105:261802, 2010.

[40] Minkowski, P., $\mu \to e\gamma$ at a Rate of One Out of 10^9 Muon Decays?. *Physics Letters B*, 67:421, 1977.

[41] Mohapatra, R. N. and Senjanovic, G. Neutrino Mass and Spontaneous Parity Violation. *Physical Review Letters*, 44:912, 1980.

[42] Weinberg, S., Baryon and Lepton Nonconserving Processes. *Physical Review Letters*, 43:1566, 1979.

[43] Babu, K.S., Leung, C. N., and Pantaleone, J. T. Renormalization of the neutrino mass operator. *Physics Letters B*, 319:191, 1993.

[44] Wetterich, C. Neutrino masses and the scale of B-L violation. *Nuclear Physics*, 187(2):343-375, 1981.

[45] Schechter, J. and Valle, J.W.F. Neutrino Decay and Spontaneous Violation of Lepton Number, *Physical Review D*, 25:774, 1982.

[46] Brahmachari, B. and Mohapatra, R. N. Unified explanation of the solar and atmospheric neutrino puzzles in a minimal supersymmetric SO(10) model. *Physical Review D*, 58:015001, 1998.

[47] Foot, R. , Lew, H., He, X. G., and Joshi, G. C. Seesaw Neutrino Masses Induced by a Triplet of Leptons. *Zeitschrift fÃijr Physik C Particles and Fields* , 44:441, 1989.

[48] Ma, E., Pathways to naturally small neutrino masses, *Physical Review Letters*, 81:1171, 1998.

[49] Bernabeu, J., Santamaria, A., Vidal, J., Mendez, A., and Valle, J. W. F. Lepton flavour non-conservation at high energies in a superstring inspired standard model, *Physics Letters B*, 187: 303, 1987.

[50] Hooft, G.'t., Itzykson, C., Jaffe, A., Lehmann, H., Mitter, P. K., Singer, I. M., and Stora, R. Recent Developments in Gauge Theories. In *Recent Developments in Gauge Theories*, volume B59, pages 1-, Nato Advanced Study Institute, Cargese, France, August 26 - September 8, 1979, Springer.

[51] Mohapatra, R. N. and Valle, J. W. F. Neutrino mass and baryon-number nonconservation in superstring models. *Physical Review D*, 34(5):1642, 1986.

[52] Ade, P. A. R., et al. [Planck Collaboration]. Planck 2015 results. XIII. Cosmological parameters. *Astronomy and Astrophysics* , 594:A13, 2016.

[53] Ignatiev, A.Yu., Krasnikov, N. V., Kuzmin, V. A., and Tavkhelidze, A. N. Universal CP noninvariant superweak interaction and baryon asymmetry of the universe. *Physics Letters B*, 76:436-438, 1978.

[54] Yoshimura, M. Unified gauge theories and the baryon number of the universe, *Physical Review Letters*, 41:281-284, 1978.

[55] Toussaint, D., Treiman, S. B., Wilczek, F., and Zee., A. Matter-antimatter accounting, thermodynamics, and black hole radiation. *Physical Review D*, 19(4):1036-1045, 1979.

[56] Dimopoulos, S. and Susskind, L., On the Baryon Number of the Universe. *Physical Review D*, 18:4500, 1978.

[57] Ellis, J. R., Gaillard, M. K., and Nanopoulos, D. V. Baryon Number Generation in Grand Unified Theories. *Physics Letters B*, 80:360, 1979.

[58] Weinberg, S. Cosmological Production of Baryons. *Physical Review Letters*, 42:850, 1979.

[59] Yoshimura, M. Origin of cosmological baryon asymmetry. *Physics Letters B*, 88:294, 1979.

[60] Barr, S. M. , Segre, G., and Weldon, H. A. The Magnitude of the Cosmological Baryon Asymmetry. *Physical Review D*, 20:2494, 1979.

[61] Nanopoulos, D.‘V. and Weinberg, S. Mechanisms for Cosmological Baryon Production, *Physical Review D*, 20:2484, 1979.

[62] Fukugita, M. and Yanagida, T. Baryogenesis without grand unification. *Physics Letters B*, 174:45-47, 1986.

[63] Rubakov, V. A. and Shaposnikov, M. E. Electroweak baryon number nonconservation in the early universe and in high-energy collisions. *Uspekhi Fizicheskikh Nauk* , 166:493-537, 1996.

[64] Troden, M. Electroweak baryogenesis. *Review of Modern Physics* , 71:1463-1500, 1999.

[65] Ian, A. and Michael, D. A new mechanism for baryogenesis. *Nuclear Physics B*, 249:361-380, 1985.

[66] Dine, M., Randall, L., and Thomas, S. D. Baryogenesis from at directions of the supersymmetric standard model. *Nuclear Physics B*, 458:291-326, 1996.

[67] Hu, W and Dodelson, S. Cosmic Microwave Background Anisotropies. *Annual Review of Astronomy and Astrophysics*, 40: 171-216, 2002.

[68] Larson, D. Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Power Spectra and WMAP-Derived Parameters, *The Astrophysical JournalSupplement Series* , 192(2):16, 2011.

[69] Kuzmin, V. A., Rubakov, V. A., and Shaposhnikov, M. E. On anomalous electroweak baryon-number non-conservation in the early universe. *Physics Letters B*, 155(1-2):36-42, 1985.

[70] Flanz, M., Paschos, E. A., and Sarkar. U. Baryogenesis from a Lepton Asymmetric Universe. *Physics Letters B* 345(3):248, 1995.

[71] Flanz, M., Paschos, E. A., Sarkar. U., and Weiss, J. Baryogenesis Through Mixing of Heavy Majorana Neutrinos. *Physics Letters B*, 389(4):693, 1996.

[72] Covi, L., Roulet, E., and Vissani, F. CP violating decays in leptogenesis scenarios. *Physics Letters B*, 384(1-4):169, 1996.

[73] Pilaftsis, A. CP Violation and Baryogenesis due to Heavy Majorana Neutrinos. *Physical Review D*, 56: 5431, 1997.

[74] Deppisch, F. F. and Pilaftsis, A. Lepton Flavour Violation and theta(13) in Minimal Resonant Leptogenesis. *Physical Review D*, 83:076007, 2011.

[75] Bambhaniya, G., Dev, P.S.B., Goswami, S., Khan, S. and Rodejohann, W. Naturalness, Vacuum Stability and Leptogenesis in the Minimal Seesaw Model. *Physical Review D*, 95(9):095016, 2017.

[76] Kartavtsev, A., Millington, P., and Vogel, H. Lepton asymmetry from mixing and oscillations. *Journal of High Energy Physics*, 06:066, 2016.

[77] Dev, P. S. B., Millington, P. Pilaftsis, A. and Teresi, D. Corrigendum to "Flavour Covariant Transport Equations: an Application to Resonant Leptogenesis". *Nuclear Physics B*, 897:740, 2015.

[78] Zwicky, F. Spectral displacement of extra galactic nebulae. *Helvetica Physica Acta*, 6:110, 1933.

[79] Rubin, V. Dark matter in spiral galaxies. *Scientific American*, 248:96, 1983.

[80] Schlegel, D. J., Finkbeiner, D. P., and Davis, M. Maps of Dust Infrared Emission for Use in Estimation of Reddening and Cosmic Microwave Background Radiation Foregrounds. *The Astrophysical Journal*, 500(2):525, 1998.

[81] Markevitch, M., et al. Direct Constraints on the Dark Matter Self-Interaction Cross Section from the Merging Galaxy Cluster 1E 0657-56. *The Astrophysical Journal*, 606:819, 2004.

[82] Taoso, M., Bertone, G., and Masiero, A. Dark Matter Candidates: A Ten-Point Test. *Journal of Cosmology and Astroparticle Physics*, 03:022, 2008.

[83] Bertone, G., Hooper, D., and Silk, J., Particle Dark Matter: Evidence, Candidates and Constraints, *Physical Review D*, 405:279, 2005.

[84] Jungman, G., Kamionkowski, M., and Griest, K. Supersymmetric dark matter. *Physics Reports*, 267(5-6):195-373, 1996.

[85] Kolb, E. W. and Turner, M. S. *The Early Universe*. Elsevier Inc., San Diego, CA. pages 1-547, (1990).

[86] Ishimori, H., Kobayashi, T., Ohki, H., Shimizu, Y., Okada, H., and Tanimoto, M. Non-Abelian Discrete Symmetries in Particle Physics. *Progress of Theoretical Physics Supplement*, 183:1, 2010.

[87] Altarelli, G. and Feruglio, F. Discrete Flavor Symmetries and Models of Neutrino Mixing. *Reviews of Modern Physics*, 82:2701, 2010.

[88] King, S. F. and Luhn, C. Trimaximal neutrino mixing from vacuum alignment in A_4 and S_4 models, *Journal of High Energy Physics*, 09:042, 2011.

[89] King, S. F. and Luhn, C. Neutrino Mass and Mixing with Discrete Symmetry, *Reports on Progress in physics*, 76:056201, 2013.