Abstract

In this thesis, we find several new exact generating functions and congruences for various partition functions by using dissections of q-products, Ramanujan's theta function identities and some identities for the Rogers-Ramanujan continued fraction. Several of these functions are related to Ramanujan/Watson mock theta functions.

Let $Q(n)$ denote the number of partitions of a nonnegative integer into distinct (or, odd) parts. We find exact generating functions for $Q(5n+1)$, $Q(25n+1)$ and $Q(125n+26)$. We also deduce some congruences modulo 5 and 25.

Recently, Andrews, Dixit and Yee introduced partition functions associated with Ramanujan/Watson third order mock theta functions $\omega(q)$ and $\nu(q)$. We study the exact generating functions for those partition functions and the associated smallest parts functions and deduce several new congruences modulo powers of 5. We also find an exact generating function for the second order mock theta function $\eta(q)$ and deduce some congruences modulo 5 and 25.

We also find new generating function representations for $p_3(5n+1)$, $b_4(5n+3)$ and $a_4(5n)$, where $p_3(n)$, $b_4(n)$, and $a_4(n)$ are the number of 2-color partitions of n in which one color appears only in parts that are multiples of 3, the number of 4-regular partitions of n, and the number of 4-cores of n, respectively.

On page 3 of his lost notebook, Ramanujan defines the Appell-Lerch sum

$$\phi(q) := \sum_{n=0}^{\infty} \frac{(-q;q)_{2n} q^{n+1}}{(q;q^2)_{n+1}^2},$$

which is connected to some of his sixth order mock theta functions. In 2012, S. H. Chan studied the co-efficients $a(n)$ defined by $\sum_{n=1}^{\infty} a(n) q^n := \phi(q)$ and proposed

some conjectural congruences. In our work, we find a representation of the generating function of $a(10n+9)$ in terms of q-products. As corollaries, we deduce the congruences $a(50n+19) \equiv a(50n+39) \equiv a(50n+49) \equiv 0 \pmod{25}$ as well as $a(1250n+250r+219) \equiv 0 \pmod{125}$, where $r = 1$, 3, and 4. The first three congruences were conjectured by S. H. Chan, whereas the congruences modulo 125 are new. We also prove two conjectural congruences of S. H. Chan for the coefficients of two more Appell-Lerch sums.