

# Abstract

In this thesis, we find arithmetic properties of a partition function, results on vanishing coefficients in infinite series expansions and relations between sums of squares and sums of triangular numbers by using  $t$ -dissections of  $q$ -products and Ramanujan's theta functions.

Let  $PD_t(n)$  and  $PDO_t(n)$  denote the total number of tagged parts over all partitions of  $n$  with designated summands and the total number of tagged parts over all partitions of  $n$  with designated summands in which all parts are odd, respectively. We prove four conjectural congruences modulo 8 for  $PD_t$  and  $PDO_t$  posed by Lin [*J. Number Theory*, 184:216–234, 2018], In fact, we find we find the stronger versions of two congruences by obtaining the exact generating functions for  $PDO_t(8n + 6)$  and  $PDO_t(8n + 7)$ . In addition, we prove several new congruences for  $PD_t$  modulo 2 and 4.

Next, by employing two known identities for a certain quotient of  $q$ -products, we prove two interesting theorems concerning coefficients in infinite product expansions. Our theorem simplifies the proofs of vanishing coefficients in those infinite products found earlier by Hirschhorn [*Ramanujan J.*, 2018] and Tang [*Int. J. Number Theory*, 2018]. We also show that certain coefficients of some more infinite product expansions always vanish.

Finally, we prove several conjectures posed by Sun [*Acta Arith.*, 175:169–189, 2016] and Sun [*Int. J. Number Theory*, 2018] on relations between sums of squares and sums of triangular numbers. We also find some new relations.