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# Nomenclature

## Parameters

$L_0$	Initial length of cantilever beam (m)
$T_0$	Initial width of cantilever beam (m)
$M_0$	Initial mass of cantilever beam (kg)
$u_A$	Deflection of point A on the top fiber of the rectangular beam (mm)
$\sigma_{VM}$	von Mises stress (N/m <sup>2</sup> )
$\sigma_{VM\max}$	Maximum von Mises stress (N/m <sup>2</sup> )

## Variables

$\bar{X}$	Normalized geometry frame coordinate in $x$ -axis
$\Delta T(\bar{X})$	Bernstein polynomial of fourth order

## Abbreviations

2D	Two dimensional
FEM	Finite element method
CAD	Computer-aided design
MMA	Method of moving asymptotes
MDC	Material distribution concept
HBO	Homogenization-based optimization

