

# A

## APPENDIX

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### A.1 Elements of the type II Seesaw mass matrix (case B) and type I Seesaw mass matrix (Case C):

$$S_{11} = (c_{12}^2 c_{13}^2 - X c_{12}^2{}^{\mu\tau}) m_1 + e^{2i(\beta-\delta)} s_{13}^2 m_3 + (c_{13}^2 s_{12}^2 - X s_{12}^2{}^{\mu\tau}) e^{2i\alpha} m_2 \quad (\text{A.1})$$

$$\begin{aligned} S_{12} = & \left( -c_{12} c_{13} c_{23} s_{12} - c_{12}^2 c_{13} s_{13} s_{23} e^{i\delta} + X c_{12}^{\mu\tau} c_{23}^{\mu\tau} s_{12}^{\mu\tau} \right) m_1 + \\ & \left( -c_{13} s_{12} c_{12} c_{23} e^{2i\alpha} - c_{13} s_{12}^2 s_{13} s_{23} e^{i(2\alpha+\delta)} + X c_{12}^{\mu\tau} c_{23}^{\mu\tau} s_{12}^{\mu\tau} e^{2i\alpha} \right) m_2 + \\ & \left( c_{13} s_{13} s_{23} e^{i(2\beta-\delta)} \right) m_3 \end{aligned} \quad (\text{A.2})$$

$$\begin{aligned} S_{13} = & \left( c_{12}^2 c_{13} c_{23} s_{13} e^{i\delta} + s_{12} s_{23} c_{12} c_{13} - X c_{12}^{\mu\tau} s_{12}^{\mu\tau} s_{23}^{\mu\tau} \right) m_1 + \\ & \left( -c_{13} s_{12} c_{23} s_{12} s_{13} e^{i(2\alpha+\delta)} - X c_{12}^{\mu\tau} s_{12}^{\mu\tau} s_{23}^{\mu\tau} e^{2i\alpha} \right) m_2 + \\ & \left( e^{i(2\beta-\delta)} c_{13} c_{23} s_{13} \right) m_3 \end{aligned} \quad (\text{A.3})$$

$$\begin{aligned} S_{21} = & \left( -c_{12} c_{13} c_{23} s_{12} - c_{12}^2 c_{13} s_{13} s_{23} e^{i\delta} + X c_{12}^{\mu\tau} c_{23}^{\mu\tau} s_{12}^{\mu\tau} \right) m_1 + \\ & \left( c_{13} s_{12} c_{12} c_{23} e^{2i\alpha} - s_{12}^2 s_{13} s_{23} c_{13} e^{i(2\alpha+\delta)} + X c_{12}^{\mu\tau} c_{23}^{\mu\tau} s_{12}^{\mu\tau} e^{2i\alpha} \right) m_2 \\ & \left( e^{i(2\beta-\delta)} c_{13} s_{23} s_{13} \right) m_3 \end{aligned} \quad (\text{A.4})$$

$$\begin{aligned} S_{22} = & \left( \left( c_{23} s_{12} - e^{i\delta} c_{12} s_{13} s_{23} \right)^2 - X c_{23}^2{}^{\mu\tau} s_{12}^2{}^{\mu\tau} \right) m_1 + \\ & \left( -X c_{12}^2{}^{\mu\tau} c_{23}^2{}^{\mu\tau} + \left( -c_{12} c_{23} - e^{i\delta} s_{12} s_{13} s_{23} \right)^2 \right) m_2 e^{2i\alpha} + \\ & \left( c_{13}^2 s_{23}^2 - X s_{23}^2{}^{\mu\tau} e^{2i\beta} \right) m_3 \end{aligned} \quad (\text{A.5})$$

$$\begin{aligned}
 S_{23} = & \left( (-c_{12}c_{23}s_{13}e^{i\delta} + s_{12}s_{23}) (-c_{23}s_{12} - e^{i\delta}c_{12}s_{13}s_{23}) + Xc_{23}^{\mu\tau}s_{12}^2s_{23}^{\mu\tau} \right) m_1 + \\
 & \left( (-e^{i\delta}c_{23}s_{12}s_{13} + c_{12}s_{23}) (-c_{12}c_{23} - e^{i\delta}s_{12}s_{13}s_{23}) + Xc_{12}^2c_{23}^{\mu\tau}s_{23}^{\mu\tau} \right) m_2 e^{2i\alpha} + \\
 & \left( c_{13}^2c_{23}s_{23}e^{2i\beta} - c_{23}^{\mu\tau}s_{23}^{\mu\tau} \right) m_3
 \end{aligned} \tag{A.6}$$

$$\begin{aligned}
 S_{31} = & \left( c_{12}^2c_{13}c_{23}s_{13}e^{i\delta} + s_{12}s_{23}c_{12}c_{13} - Xc_{12}^{\mu\tau}s_{12}^{\mu\tau}s_{23}^{\mu\tau} \right) m_1 + \\
 & \left( c_{13}s_{12}^2e^{i\delta}c_{23}s_{13} + c_{12}s_{23}c_{13}s_{12}e^{2i\alpha} - Xc_{12}^{\mu\tau}s_{12}^{\mu\tau}s_{23}^{\mu\tau} \right) m_2 e^{2i\alpha} + \\
 & \left( e^{2i\beta-i\delta}c_{13}c_{23}s_{13} \right) m_3
 \end{aligned} \tag{A.7}$$

$$\begin{aligned}
 S_{32} = & \left( (-e^{i\delta}c_{12}c_{23}s_{13} + s_{12}s_{23}) (-c_{23}s_{12} - e^{i\delta}c_{12}s_{13}s_{23}) + c_{23}^{\mu\tau}s_{12}^2s_{23}^{\mu\tau} \right) m_1 \\
 & \left( (-e^{i\delta}c_{23}s_{12}s_{13} + c_{12}s_{23}) (-c_{12}c_{23} - e^{i\delta}s_{12}s_{13}s_{23}) + Xc_{12}^2c_{23}^{\mu\tau}s_{23}^{\mu\tau} \right) e^{2i\alpha} m_2 \\
 & \left( c_{13}^2c_{23}s_{23} - Xc_{23}^{\mu\tau}s_{23}^{\mu\tau} \right) e^{2i\beta} m_3
 \end{aligned} \tag{A.8}$$

$$\begin{aligned}
 S_{33} = & \left( (-e^{i\delta}c_{12}c_{23}s_{13} + s_{12}s_{23})^2 - Xs_{12}^2s_{23}^2s_{23}^{\mu\tau} \right) m_1 + \\
 & \left( (-e^{i\delta}c_{23}s_{12}s_{13} + c_{12}s_{23})^2 - Xc_{12}^2c_{23}^2s_{23}^{\mu\tau} \right) e^{2i\alpha} m_2 + \\
 & \left( c_{13}^2c_{23}^2 - c_{23}^2s_{23}^{\mu\tau} \right) e^{2i\beta} m_3
 \end{aligned} \tag{A.9}$$

Where,  $c_{ij}^{\mu\tau} = \cos \theta_{ij}^{\mu\tau}$ ,  $s_{ij}^{\mu\tau} = \sin \theta_{ij}^{\mu\tau}$  represents the mixing angles for  $\mu - \tau$  symmetric neutrino mass matrix (TBM, BM, HM, GRM).

## A.2 Determination of $M_D$ :

From type I SS term,  $M_\nu^I \approx -M_D M_{RR}^{-1} M_D^T$

Again,  $M_\nu^I = U_{(TBM)} U_{Maj} X M_\nu^{(diag)} U_{Maj}^T U_{(TBM)}^T$

$$M_{RR} = \frac{1}{\gamma} \left( \frac{v_R}{M_{WL}} \right)^2 M_\nu^{II} \quad (\text{A.10})$$

Where,  $M_\nu^{II} = U_{PMNS} M_\nu^{(diag)} U_{PMNS}^T - U_{(TBM)} U_{Maj} X M_\nu^{(diag)} U_{Maj}^T U_{(TBM)}^T$ . Considering,  $X=0.5$ ,  $M_{WL} = 80$  GeV,  $v_R = 5$  TeV (for one case only) and expressing  $M_\nu^{(diag)}$  in terms of lightest neutrino mass,  $m_1(m_3)$  for NH (IH), we obtained  $M_{RR}$  varying the Majorana phases  $\alpha$  and  $\beta$  from 0 to  $2\pi$  and lightest neutrino mass from  $10^{-3}$  to  $10^{-1}$ .

We have considered  $M_D$  as,

$$M_D = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_2 & a_4 & a_5 \\ a_3 & a_5 & a_6 \end{bmatrix}, \quad (\text{A.11})$$

which is symmetric. Equating both sides of type I seesaw equation and solving for  $a_1, a_2, a_3, a_4, a_5, a_6$ , we obtain the matrix elements of one of the  $M_D$  of the form,

$$M_D = \begin{bmatrix} 24776.2 + 122368.i & 70524.8 + 76561.i & -12687.1 + 21472.4i \\ 70524.8 + 76561.i & 14308.4 + 138730.i & -45802.3 - 46293.4i \\ -12687.1 + 21472.4i & -45802.3 - 46293.4i & 87313.6 + 158166.i \end{bmatrix}, \quad (\text{A.12})$$

which we have implemented for our further analysis.

### Elements of the type II Seesaw mass matrix:

$$S_{11} = \left( c_{12}^2 c_{13}^2 - X c_{12}^2 {}^{TBM} \right) m_1 + e^{2i(\beta-\delta)} s_{13}^2 m_3 + \left( c_{13}^2 s_{12}^2 - X s_{12}^2 {}^{TBM} \right) e^{2i\alpha} m_2 \quad (\text{A.13})$$

$$\begin{aligned} S_{12} = & \left( -c_{12} c_{13} c_{23} s_{12} - c_{12}^2 c_{13} s_{13} s_{23} e^{i\delta} + X c_{12}^{TBM} c_{23}^{TBM} s_{12}^{TBM} \right) m_1 + \\ & \left( -c_{13} s_{12} c_{12} c_{23} e^{2i\alpha} - c_{13} s_{12}^2 s_{13} s_{23} e^{i(2\alpha+\delta)} + X c_{12}^{TBM} c_{23}^{TBM} s_{12}^{TBM} e^{2i\alpha} \right) m_2 + \\ & \left( c_{13} s_{13} s_{23} e^{i(2\beta-\delta)} \right) m_3 \end{aligned} \quad (\text{A.14})$$

$$\begin{aligned}
 S_{13} = & \left( c_{12}^2 c_{13} c_{23} s_{13} e^{i\delta} + s_{12} s_{23} c_{12} c_{13} - X c_{12}^{TBM} s_{12}^{TBM} s_{23}^{TBM} \right) m_1 + \\
 & \left( -c_{13} s_{12} c_{23} s_{12} s_{13} e^{i(2\alpha+\delta)} - X c_{12}^{TBM} s_{12}^{TBM} s_{23}^{TBM} e^{2i\alpha} \right) m_2 + \\
 & \left( e^{i(2\beta-\delta)} c_{13} c_{23} s_{13} \right) m_3
 \end{aligned} \tag{A.15}$$

$$\begin{aligned}
 S_{21} = & \left( -c_{12} c_{13} c_{23} s_{12} - c_{12}^2 c_{13} s_{13} s_{23} e^{i\delta} + X c_{12}^{TBM} c_{23}^{TBM} s_{12}^{TBM} \right) m_1 + \\
 & \left( c_{13} s_{12} c_{12} c_{23} e^{2i\alpha} - s_{12}^2 s_{13} s_{23} c_{13} e^{i(2\alpha+\delta)} + X c_{12}^{TBM} c_{23}^{TBM} s_{12}^{TBM} e^{2i\alpha} \right) m_2 \\
 & \left( e^{i(2\beta-\delta)} c_{13} s_{23} s_{13} \right) m_3
 \end{aligned} \tag{A.16}$$

$$\begin{aligned}
 S_{22} = & \left( \left( c_{23} s_{12} - e^{i\delta} c_{12} s_{13} s_{23} \right)^2 - X c_{23}^2 s_{12}^2 \right) m_1 + \\
 & \left( -X c_{12}^2 c_{23}^2 + \left( -c_{12} c_{23} - e^{i\delta} s_{12} s_{13} s_{23} \right)^2 \right) m_2 e^{2i\alpha} + \\
 & \left( c_{13}^2 s_{23}^2 - X s_{23}^2 e^{2i\beta} \right) m_3
 \end{aligned} \tag{A.17}$$

$$\begin{aligned}
 S_{23} = & \left( \left( -c_{12} c_{23} s_{13} e^{i\delta} + s_{12} s_{23} \right) \left( -c_{23} s_{12} - e^{i\delta} c_{12} s_{13} s_{23} \right) + X c_{23}^{TBM} s_{12}^2 s_{23}^{TBM} \right) m_1 + \\
 & \left( \left( -e^{i\delta} c_{23} s_{12} s_{13} + c_{12} s_{23} \right) \left( -c_{12} c_{23} - e^{i\delta} s_{12} s_{13} s_{23} \right) + X c_{12}^2 c_{23}^{TBM} s_{23}^{TBM} \right) m_2 e^{2i\alpha} + \\
 & \left( c_{13}^2 c_{23} s_{23} e^{2i\beta} - c_{23}^{TBM} s_{23}^{TBM} \right) m_3
 \end{aligned} \tag{A.18}$$

$$\begin{aligned}
 S_{31} = & \left( c_{12}^2 c_{13} c_{23} s_{13} e^{i\delta} + s_{12} s_{23} c_{12} c_{13} - X c_{12}^{TBM} s_{12}^{TBM} s_{23}^{TBM} \right) m_1 + \\
 & \left( c_{13} s_{12}^2 e^{i\delta} c_{23} s_{13} + c_{12} s_{23} c_{13} s_{12} e^{2i\alpha} - X c_{12}^{TBM} s_{12}^{TBM} s_{23}^{TBM} \right) m_2 e^{2i\alpha} + \\
 & \left( e^{2i\beta-i\delta} c_{13} c_{23} s_{13} \right) m_3
 \end{aligned} \tag{A.19}$$

$$\begin{aligned}
 S_{32} = & \left( \left( -e^{i\delta} c_{12} c_{23} s_{13} + s_{12} s_{23} \right) \left( -c_{23} s_{12} - e^{i\delta} c_{12} s_{13} s_{23} \right) + c_{23}^{TBM} s_{12}^2 s_{23}^{TBM} \right) m_1 \\
 & \left( \left( -e^{i\delta} c_{23} s_{12} s_{13} + c_{12} s_{23} \right) \left( -c_{12} c_{23} - e^{i\delta} s_{12} s_{13} s_{23} \right) + X c_{12}^2 c_{23}^{TBM} s_{23}^{TBM} \right) e^{2i\alpha} m_2 \\
 & \left( c_{13}^2 c_{23} s_{23} - X c_{23}^{TBM} s_{23}^{TBM} \right) e^{2i\beta} m_3
 \end{aligned} \tag{A.20}$$

$$\begin{aligned}
 S_{33} = & \left( \left( -e^{i\delta} c_{12} c_{23} s_{13} + s_{12} s_{23} \right)^2 - X s_{12}^2 s_{23}^2 \right) m_1 + \\
 & \left( \left( -e^{i\delta} c_{23} s_{12} s_{13} + c_{12} s_{23} \right)^2 - X c_{12}^2 c_{23}^2 \right) e^{2i\alpha} m_2 + \\
 & \left( c_{13}^2 c_{23}^2 - c_{23}^2 \right) e^{2i\beta} m_3
 \end{aligned} \tag{A.21}$$

Where,  $c_{ij}^{TBM} = \cos \theta_{ij}^{TBM}$ ,  $s_{ij}^{TBM} = \sin \theta_{ij}^{TBM}$  represents the mixing angles for TBM neutrino mass matrix.