APPENDIX

Calculation of medium solution concentration at HPG exit (X_8) based on mass and heat balance in the LPG:

(a) For the VARS series configuration

From mass balance:

$\dot{m}_8 = \dot{m}_{10}$, $\dot{m}_{11} = \dot{m}_{12}$, $\dot{m}_7 - \dot{m}_8 = \dot{m}_{11}$ and $\dot{m}_8 - \dot{m}_{15} = \dot{m}_{10} - \dot{m}_{15} = \dot{m}_{14}$

$\dot{m}_8 X_8 = \dot{m}_7 X_7 = \dot{m}_7 X_4$

From heat balance at LPG:

$\dot{m}_{10} h_{10} + \dot{m}_{11} h_{11} = \dot{m}_{12} h_{12} + \dot{m}_{14} h_{14} + \dot{m}_{15} h_{15}$

$\Rightarrow \dot{m}_8 h_{10} + \dot{m}_{11} h_{11} = \dot{m}_{11} h_{12} + \dot{m}_{14} h_{14} + \dot{m}_{15} h_{15}$

$\Rightarrow \dot{m}_8 h_{10} + (\dot{m}_7 - \dot{m}_8) h_{11} = (\dot{m}_7 - \dot{m}_8) h_{12} + (\dot{m}_8 - \dot{m}_{15}) h_{14} + \dot{m}_{15} h_{15}$

$\dot{m}_8 X_8 = \dot{m}_7 X_7 = \dot{m}_7 X_4$

$\Rightarrow \left[\frac{\dot{m}_7 X_4}{X_8}\right] h_{10} - \left[\frac{\dot{m}_7 X_4}{X_8}\right] h_{11} + \dot{m}_7 h_{11} = \dot{m}_7 h_{12} - \left[\frac{\dot{m}_7 X_4}{X_8}\right] h_{12} + \left[\frac{\dot{m}_7 X_4}{X_8}\right] h_{14} - \dot{m}_{15} h_{14} + \dot{m}_{15} h_{15}$

$\Rightarrow X_8 = \left[\frac{\dot{m}_7 X_4 h_{10} - \dot{m}_7 X_4 h_{11} + \dot{m}_7 X_4 h_{12} - \dot{m}_7 X_4 h_{14}}{m_7 h_{12} - m_{15} h_{14} + m_{15} h_{15} - m_7 h_{11}}\right]$

$\Rightarrow X_8 = X_4 \left[\frac{\left(1 + \frac{h_{10} - h_{14}}{h_{12} - h_{11}}\right)}{1 + \frac{m_{15}}{m_7}\left(\frac{h_{15} - h_{14}}{h_{12} - h_{11}}\right)}\right]$

(b) For the VARS parallel configuration

From mass balance:

$\dot{m}_8 = \dot{m}_{10}$, $\dot{m}_{11} = \dot{m}_{12}$, $\dot{m}_7 - \dot{m}_8 = \dot{m}_{11}$, $\dot{m}_8 + \dot{m}_{15} = \dot{m}_{17}$ and $\dot{m}_{14} = \dot{m}_{10} - \dot{m}_{15}$

$\dot{m}_8 X_8 = \dot{m}_7 X_7$ $\dot{m}_{15} X_{15} = \dot{m}_{10} X_{10}$ and $\dot{m}_4 X_4 = \dot{m}_{17} X_{17}$; where $X_4 = X_9 = X_{10} = X_7$

Also, $D = \frac{\dot{m}_{6a}}{\dot{m}_4}$, $\dot{m}_9 = (1 - D)\dot{m}_4$, where $\dot{m}_{6a} = \dot{m}_7$

From heat balance at LPG:

$$\dot{m}_{10}h_{10} + \dot{m}_{11}h_{11} = \dot{m}_{12}h_{12} + \dot{m}_{14}h_{14} + \dot{m}_{15}h_{15}$$

$$\Rightarrow \dot{m}_{10}h_{10} + \dot{m}_{11}h_{11} = \dot{m}_{11}h_{12} + (\dot{m}_{10} - \dot{m}_{15})h_{14} + \dot{m}_{15}h_{15}$$

$$\Rightarrow \dot{m}_{11}(h_{11} - h_{12}) = \dot{m}_{10}(h_{14} - h_{10}) + \dot{m}_{15}(h_{15} - h_{14})$$

$$\Rightarrow (\dot{m}_7 - \dot{m}_8)(h_{11} - h_{12}) = \dot{m}_{10}(h_{14} - h_{10}) + \dot{m}_{10}\left(\frac{X_4}{X_{15}}\right)(h_{15} - h_{14})$$

$$\Rightarrow \left(D\dot{m}_4 - \frac{X_7}{X_8}\dot{m}_7\right)(h_{11} - h_{12}) = \dot{m}_{10}\left[(h_{14} - h_{10}) + \dot{m}_{10}\left(\frac{X_4}{X_{15}}\right)(h_{15} - h_{14})\right]$$

$$\Rightarrow \left(1 - \frac{X_7}{X_8}\right) = \left[\frac{1-D}{D}\right]\left[\frac{(h_{14} - h_{10}) + \left(\frac{X_4}{X_{15}}\right)(h_{15} - h_{14})}{(h_{11} - h_{12})}\right]$$

$$\Rightarrow X_8 = \left[\frac{DX_7}{D - (1-D)\left(\frac{X_{15}(h_{14} - h_{10}) + X_4(h_{15} - h_{14})}{X_{15}(h_{11} - h_{12})}\right)}\right]$$

(c) For the VARS reverse parallel configuration

From mass balance:

$\dot{m}_4 = \dot{m}_5 = \dot{m}_6$, $\dot{m}_8 = \dot{m}_9 = \dot{m}_{10}$, $\dot{m}_7 - \dot{m}_8 = \dot{m}_{11} = \dot{m}_{12}$, $\dot{m}_{15} = \dot{m}_{15a} + \dot{m}_{15c}$, $\dot{m}_6 - \dot{m}_{15} = \dot{m}_{14}$

$\dot{m}_{17} = \dot{m}_{10} + \dot{m}_{15c}$

$\dot{m}_8 X_8 = \dot{m}_7 X_7$, $\dot{m}_4 X_4 = \dot{m}_6 X_6 = \dot{m}_{15} X_{15}$ and $\dot{m}_4 X_4 = \dot{m}_{17} X_{17}$; where $X_4 = X_6$ and $X_{15} = X_7$

Also, $D = \frac{\dot{m}_{15a}}{\dot{m}_{15}}$, $\dot{m}_{15c} = (1-D)\dot{m}_{15}$; where $\dot{m}_{15a} = \dot{m}_7$

From heat balance at LPG:

$$\dot{m}_6 h_6 + \dot{m}_{11}h_{11} = \dot{m}_{12}h_{12} + \dot{m}_{14}h_{14} + \dot{m}_{15}h_{15}$$

$$\Rightarrow \dot{m}_{11}(h_{11} - h_{12}) = (\dot{m}_6 - \dot{m}_{15})h_{14} + \dot{m}_{15}h_{15} - \dot{m}_6 h_6$$

$$\Rightarrow (\dot{m}_7 - \dot{m}_8)(h_{11} - h_{12}) = \dot{m}_6(h_{14} - h_6) + \dot{m}_{15}(h_{15} - h_{14})$$

$$\Rightarrow \left[1 - \frac{X_{15}}{X_8}\right](h_{11} - h_{12}) = \frac{X_{15}\dot{m}_{15}}{X_4\dot{m}_{15a}}(h_{14} - h_6) + \frac{\dot{m}_{15}}{\dot{m}_{15a}}(h_{15} - h_{14})$$

$$\Rightarrow \left[\frac{X_{15}}{X_8}\right](h_{11} - h_{12}) = (h_{11} - h_{12}) - \frac{1}{D}\left[\frac{X_{15}}{X_4}\right](h_{14} - h_6) + \frac{1}{D}(h_{15} - h_{14})$$

$$\Rightarrow \left[\frac{1}{X_8}\right] = \left[\frac{1}{X_{15}}\right] - \frac{1}{D}\left[\frac{1}{X_4}\left(\frac{h_{14} - h_6}{h_{11} - h_{12}}\right) + \frac{1}{X_{15}}\left(\frac{h_{15} - h_{14}}{h_{11} - h_{12}}\right)\right]$$

$$\Rightarrow X_8 = \left[\frac{1}{\left[\frac{1}{X_{15}}\right] - \frac{1}{D}\left[\frac{1}{X_4}\left(\frac{h_{14} - h_6}{h_{11} - h_{12}}\right) + \frac{1}{X_{15}}\left(\frac{h_{15} - h_{14}}{h_{11} - h_{12}}\right)\right]}\right]$$