DECLARATION OF THE CANDIDATE

I, **Bikram Bir**, hereby declare that the subject matter in this thesis entitled **"Finite Element Analysis of a Linear Viscoelastic Fluid Flow Model: Oldroyd Model of Order One"** is the record of work done by me, that the contents of this thesis did not form the basis of the award of any previous degree to me or to the best of my knowledge to anybody else, and that the thesis has not been submitted by me for any research degree in any other university/ institute. This thesis is being submitted by me to Tezpur University for the degree of Doctor of Philosophy in the department of Mathematical Sciences.

Place: Tezpur

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CERTIFICATE OF THE SUPERVISOR

This is to certify that the thesis entitled **"Finite Element Analysis of a Linear Viscoelastic Fluid Flow Model: Oldroyd Model of Order One"** submitted to the School of Sciences, Tezpur University in partial fulfillment for the award of the degree of Doctor of Philosophy in Mathematical Sciences is a record of research work carried out by **Mr. Bikram Bir** under my supervision and guidance.

All help received by him from various sources have been duly acknowledged.

No part of this thesis has been submitted elsewhere for award of any other degree.

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Place: Tezpur **Bikram Bir**

Date:

Dedicated to
my Parents and my Brother
for their love, sacrifice and devotion

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