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List of published papers:

1. Bhowal, P., Nongsiang, D. and Nath, R. K. Solvable graphs of finite groups. *Hacettepe Journal of Mathematics and Statistics*, 49(6):1955–1964, 2020.
2. Bhowal, P. and Nath, R. K. Spectral aspects of commuting conjugacy class graph of finite groups. *Algebraic Structures and Their Applications*, 8(2):67–118, 2021.
3. Bhowal, P. and Nath, R. K. Genus of commuting conjugacy class graph of finite groups. *Algebraic Structures and Their Applications*, 9(1):93–108, 2022.
4. Bhowal, P., Nongsiang, D. and Nath, R. K. Non-Solvable Graphs of Groups. *Bulletin of the Malaysian Mathematical Sciences Society*, 45:1255–1272, 2022.

List of communicated papers:

1. Bhowal, P., Cameron, P. J., Nath, R. K. and Sambale, B. Solvable conjugacy class graph of groups, Preprint.
2. Bhowal, P. and Nath, R. K. Genus of commuting graphs of certain finite groups, Preprint.
3. Bhowal, P. and Nath, R. K. Spectrum and energies of commuting conjugacy class graph of a finite group, Preprint.

Lists of conferences attended:

1. Participated and presented a paper in the National Conference on Recent Advancements in Science and Technology (RAST) organized by Union Christian College through the Science Society (UCC) in collaboration with NESAC, PA Sangma Foundation and College of Post-Graduate Studies (CAU), Umiam, Meghalaya, India, during 12th-14th June, 2019.
2. Participated and presented a paper in the International Conference on Analysis, Algebra, Combinatorics and their Application (ICAACA-2020) organized by Department of Mathematics, Jadavpur University, Kolkata-700032, India, during January 20-22, 2020.
3. Participated and presented a paper in the International Conference on Advances in Mathematics, Science and Technology (ICAMST-2020) Organized by Department of Mathematics, Rajiv Gandhi University (A Central University) Rono Hills, Doimukh – 791112, Arunachal Pradesh, INDIA, during 1-3 September, 2020.
4. Participated and presented a paper in the International Conference on Present Scenario in Mathematical Sciences (ICPSMS 2020) Organized by Department of Mathematics, Karnatak University's Karnatak Arts College, Dharwad, India during 12-13 September, 2020.
5. Participated and presented a paper in the 6th Biennial International Group Theory Conference - 2021 (6BIGTC-2021) organized by Department of Mathematics, Vellore Institute of Technology, Vellore, Tamil Nadu, India during 4-6 march, 2021.
6. Participated and presented a paper in the International Conference on Emerging trends in Pure and Applied Mathematics (Blended Mode) organized by Department of Applied Sciences in association with Department of Mathematical Sciences, Tezpur University, India during 12-13 march, 2022.