Dedicated to my parents

Shri Nripendra Chandra Bhowal

&

Smt. Sikha Bhowal

DECLARATION BY THE CANDIDATE

I, Parthajit Bhowal, hereby declare that the subject matter in this thesis entitled "Certain graphs on finite groups and their properties", is the record of work done by me, that the contents of this thesis did not form basis of the award of any previous degree to me or to the best of my knowledge to anybody else, and that the thesis has not been submitted by me for any research degree in any other university/institute.

This thesis is being submitted to the Tezpur University for the degree of Doctor of Philosophy in Mathematical Sciences.

Date:

Place: Tezpur

Signature of the Candidate

TEZPUR UNIVERSITY

Dr. Rajat Kanti Nath
Dept. of Mathematical Sciences
Napaam–784028, Assam, India

Phone(Ext.): 03712-27-5518
+91-9508840085(M)
E-mail: rknath@tezu.ernet.in

CERTIFICATE OF THE SUPERVISOR

This is to certify that the thesis entitled "*Certain graphs on finite groups and their properties*" submitted to the School of Sciences of Tezpur University in partial fulfillment for the award of the degree of Doctor of Philosophy in Mathematical Sciences is a record of research work carried out by Mr. Parthajit Bhowal under my supervision and guidance.

All help received by him from various sources have been duly acknowledged.

No part of this thesis have been submitted elsewhere for award of any other degree.

Date:
Place: Tezpur

Dr. Rajat Kanti Nath
Department of Mathematical Sciences
School of Sciences
Tezpur University, Assam

ACKNOWLEDGEMENT

With deep sense of gratitude I wish to express my sincere thanks to my supervisor Dr. Rajat Kanti Nath for his exemplary guidance throughout my Ph.D. study and related research. As my supervisor, he has constantly pushed me to stay focused on attaining my goal. I am grateful to him for his unceasing support, encouragement and motivation in all the time of research and writing of this thesis. Without his insightful inputs completion of this thesis would not have been possible. I consider it a wonderful opportunity to pursue my Ph.D. programme under his supervision and learn from his research skills. I hope this is not the end but just the beginning of a lifelong collaboration with him.

I would like to thank my doctoral committee members Prof. Debajit Hazarika and Dr. Debajit Kalita of the Department of Mathematics, Tezpur University, for reviewing my research work regularly and for all their valuable suggestions. I would also like to express my gratitude to the Department of Mathematical Sciences, Tezpur University for providing me with the opportunity to pursue a Ph.D. and access to all the research facilities. I want to convey my sincere thanks to all the faculty members and staff of the Department of Mathematical Sciences, Tezpur University. I am also thankful to both the examiners for their valuable comments and suggestions.

I would like to thank Prof. Peter J. Cameron, University of St Andrews, UK, Dr. Benjamin Sambale, Institut für Algebra, Johannes Kepler University, Germany and Dr. Deiborlang Nongsiang, Union Christian College, Meghalaya, India for their concern and suggestions towards my work. I am also thankful to Prof. Dipendra Prasad, IIT Bombay for his valuable suggestion during my evaluation of the progress of research works for upgradation from CSIR-JRF to CSIR-SRF as the external member for the assessment committee. I am thankful to Prof. Yousef Zamani, Department of Mathematics, Faculty of Sciences, Sahand University of Technology, Tabriz- Iran for his valuable corrections on one of my research article. I am no less grateful to my school and college teachers for their guidance in the beginning of my learning career.

I would also like to thank Parama Dutta, Duranta Chutia, Kuldeep Sarma and Duhuidi Terang for their help and constant encouragement. I wish to express my appreciation to my friends Monalisha, Walaa, Himangshu, Lakhyajit, Nabin Da, Ankur, Hirak, Subhajit, Kollol, Diganta, Giri Da, Subha Da, Bikram, Dharmaraj, Ajay Da, Deepak Da and all my research scholar mates who made my stay here in the campus a lively one.

At this moment of accomplishment, I would like to express my sincere thanks to Prof. Siddhartha Sankar Nath, Principal, Cachar College, Dr. Avinoy Paul, Head, Department of Mathematics, Cachar College and all esteemed colleagues of Cachar College for their support and advice.

Finally, I acknowledge the people who mean a lot to me, my parents for showing faith in me and giving me liberty to choose what I desired. I would never be able to pay back the love and affection showered upon by my parents who are much happier than me on the completion of the work. My heart felt regard goes to my sister, brother-in-law and best friend (Kankana) for their love and unconditional support. All my love goes to my little niece and nephew Mahika and Abir. There is no better feeling than spending time with them during times of difficulty while doing research. I am also grateful to all my relatives and my well-wishers for their support and being my constant source of inspiration.

Last but not the least; I would like to thank the entire fraternity of Tezpur University for providing all sorts of facilities to successfully carry out this thesis. Also, the financial support (File No. 09/796(0094)/2019-EMR-I) provided by Council of Scientific and Industrial Research (CSIR), Government of India is deeply acknowledged without which the successful realization of this thesis would not have been possible

Thank you all for your insights, guidance and infinite support!

Parthajit Bhowal

List of symbols

$\sqcup$	union of disjoint sets/graphs
$\times$	direct product
$\rtimes$	semidirect product
$\mathbb{N}$	set of natural numbers
G	any finite group
$Z(G)$	center of G
$C_G(x)$	centralizer of x in G
x^G	conjugacy class of x in G
x^g	gxg^{-1} for some $g \in G$
$\langle x, y \rangle$	subgroup generated by the elements x and y
$\langle H, K \rangle$	subgroup generated by the set H and K
$\lceil x \rceil$	greatest integer less than or equal to x
$o(x)$	order of x
$\mathrm{Nil}_G(x)$	$\{y \in G : \langle x, y \rangle$ is nilpotent$\}$
$\mathrm{Nil}(G)$	$\{x \in G : \langle x, y \rangle$ is nilpotent for all $y \in G\}$
$\mathrm{Sol}_G(x)$	$\{y \in G : \langle x, y \rangle$ is solvable$\}$
$\mathrm{Sol}(G)$	$\{x \in G : \langle x, y \rangle$ is solvable for all $y \in G\}$
$\mathrm{Pr}(G)$	commuting probability
$P_s(G)$	solvability degree
Γ	simple undirected graph
$V(\Gamma)$	vertex set of graph Γ
$d(u, v)$	distance between two vertices u and v
$\mathrm{diam}(\Gamma)$	diameter of Γ
$\deg_\Gamma(x)$	degree of a vertex in the graph Γ
$\deg(\Gamma)$	$\{\deg_\Gamma(x) : x \in V(\Gamma)\}$
$\mathrm{Nbd}_\Gamma(x)$	neighbourhood of the vertex x in the graph Γ
$N_\Gamma[S]$	$S \cup (\bigcup_{x \in S} \mathrm{Nbd}_\Gamma(x))$

$\Gamma[S]$	induced subgraph of Γ on S
$\lambda(\Gamma)$	domination number of Γ
$\alpha(\Gamma)$	independence number of Γ
$\omega(\Gamma)$	clique number of Γ
$\kappa(\Gamma)$	smallest number of vertices whose removal disconnects Γ
$\chi(\Gamma)$	chromatic number of Γ
$\gamma(\Gamma)$	genus of Γ
K_n	complete graph of n vertices
$K_{m,n}$	complete bipartite graph
$K_{m,m,m}$	complete tripartite graph
$\mathrm{Spec}(\Gamma)$	spectrum of Γ
$\text{L-spec}(\Gamma)$	Laplacian spectrum of Γ
$\text{Q-spec}(\Gamma)$	signless Laplacian spectrum of Γ
$E(\Gamma)$	energy of Γ
$LE(\Gamma)$	Laplacian energy of Γ
$LE^+(\Gamma)$	signless Laplacian energy of Γ
$\mathcal{C}(G)$	commuting graph of G
$\mathcal{NC}(G)$	non-commuting graph of G
$\mathcal{N}(G)$	nilpotent graph of G
$\mathcal{NN}(G)$	non-nilpotent graph of G
$\mathcal{S}(G)$	solvable graph of G
$\mathcal{NS}(G)$	non-solvable graph of G
$\mathcal{CCC}(G)$	commuting conjugacy class graph of G
$\mathcal{NCC}(G)$	nilpotent conjugacy class graph of G
$\mathcal{SCC}(G)$	solvable conjugacy class graph of G
$\mathbb{Z}_n$	cyclic group of order n
S_n	symmetric group of degree n
A_n	alternating group of degree n

D_{2n}	$\langle x, y : x^n = y^2 = 1, yxy = x^{-1} \rangle$, the dihedral group of order $2n$
Q_{4m}	$\langle x, y : x^{2m} = 1, x^m = y^2, y^{-1}xy = x^{-1} \rangle$, the generalized quaternion group of order $4m$
$SG(16, 3)$	$\langle a, b : a^4 = b^4 = 1, ab = b^{-1}a^{-1}, ab^{-1} = ba^{-1} \rangle$
$SL(2, q)$	special linear group of degree 2 over the field of order q
$PSL(3, 2)$	projective special linear group of degree three over the field of order 2
$Sz(2)$	$\langle a, b : a^5 = b^4 = 1, b^{-1}ab = a^2 \rangle$, the suzuki group of order 20
QD_{2^n}	$\langle a, b : a^{2^{n-1}} = b^2 = 1, bab^{-1} = a^{2^{n-2}-1} \rangle$, the quasidihedral group of order 2^n
$GL(2, q)$	general linear group of degree 2 over the field of order q
V_{8n}	$\langle a, b : a^{2n} = b^4 = 1, b^{-1}ab^{-1} = bab = a^{-1} \rangle$
SD_{8n}	$\langle a, b : a^{4n} = b^2 = 1, bab = a^{2n-1} \rangle$, the semidihedral group of order $8n$
$U_{(n,m)}$	$\langle x, y : x^{2n} = y^m = 1, x^{-1}yx = y^{-1} \rangle$
M_{2nk}	$\langle a, b : a^n = b^{2k} = 1, bab^{-1} = a^{-1} \rangle$
U_{6n}	$\langle a, b : a^{2n} = b^3 = 1, a^{-1}ba = b^{-1} \rangle$
$G(p, m, n)$	$\langle x, y : x^{p^m} = y^{p^n} = [x, y]^p = 1, [x, [x, y]] = [y, [x, y]] = 1 \rangle$
$\mathcal{M}_{16}$	$\langle a, b : a^8 = b^2 = 1, bab = a^5 \rangle$