CHAPTER 6
REGRESSION MODEL

6.1 Regression analysis

Regression analysis is a statistical tool for the investigation of the relationship between a dependent variable and independent variables. The influences of various parameters like the relative density of subgrade sand (D_r), depth of placement of geocell (u), geocell pocket size (d), the height of geocell (h), the relative density of infill sand, friction angle of sand and geocell friction angle (δ_s), on the bearing capacity of the foundation bed have already been established in the previous chapters. In this chapter, an attempt is made to develop a relationship to estimate the bearing capacity as a function of these influencing parameters. A model is being created using a multiple non-linear regression method based on experimental data, where the bearing pressure is considered the dependent (response) variable, and the other parameters are treated as independent (predictors) variables.

6.1.1 Development of regression model

The following non-linear regression model was chosen to conduct the data analysis:

$$y_i = \xi_0 x_1^{\ \xi 1} x_2^{\ \xi 2} x_3^{\ \xi 3} \dots \dots \dots .. x_p^{\xi p} \tag{6.1}$$

where, i = 1, 2, 3…n, are the number of observations, the y_i are the dependent variables; $x_1, x_2, x_3, \dots, x_p$ are the independent variables, $\xi_0, \xi_1, \xi_2, \xi_3, \dots, \xi_p$ are the partial regression coefficients and p is the number of independent variables. The ξ_i coefficients are chosen so that the sum of squared residuals $\sum(y_i - \hat{y}_i)^2$ is minimized; $\hat{y}_i$ represents the predicted value. The regression analysis was carried out using the data analysis toolkit built-in Microsoft Excel®.

In order to check the suitability of the data for non-linear regression analysis, the normality test of the data was carried out using the Anderson-Darling test in Minitab software. In this test, the data is plotted in a normal probability plot, and if all the data points form a straight line, it is concluded that the data is normally distributed.

The coefficient of determination R^2 was used to assess the fitness of the regression line to the data. It is defined as follows (Chatterjee & Hadi [18]):

$$R^2 = \frac{\sum_i^{i=n}(\hat{y}_i - \bar{y})^2}{\sum_i^{i=n}(y_i - \bar{y})^2} = 1 - \frac{\sum_i^{i=n}(y_i - \hat{y}_i)^2}{\sum_i^{i=n}(y_i - \bar{y})^2} \tag{6.2}$$

Where, $\bar{y}$ is the mean of dependent variables. The closeness of R^2 to 1.0 signifies the betterment of the fit, and $R^2 = 0$ implies the absolute mismatch among the variables. In some models, having a greater number of variables will improve the R^2 spuriously. To overcome this deficiency adjustment to the coefficient R^2 was done by checking the '*adjusted R^2*' (R^2_{ddj}., Bera et al. [8]). The calculation of the adjusted coefficient of determination is performed as follows

$$R^2_{adj} = 1 - (1 - R^2)\left(\frac{n-1}{n-p}\right) \tag{6.3}$$

where n and p are the number of observations and the number of independent variables, respectively.

The standard error (E_s) was used to evaluate the effectiveness of the regression model. This value provides an accurate estimate of the variance. When the value of E_s is smaller, the predictions generated by the model will be more accurate. The E_s is calculated as follows

$$E_s = \sqrt{\frac{\sum_i^{i=n}(y_i - \hat{y}_i)^2}{n-p}} \tag{6.4}$$

F and t statistics were calculated to check whether the assumed equation was statistically significant or not. The F test was performed to test the overall significance of the regression model. It can be calculated as follows

$$F_{cal} = \frac{[R^2/(p-1)]}{[(1-R^2)/(n-p)]} \tag{6.5}$$

The procedure for determining whether to reject or accept the null hypothesis in an 'F' test is as follows:

If F_{cal} is greater than F (1-α, p-1, n-p), the null hypothesis is rejected.
If F_{cal} is less than or equal to F (1-α, p-1, n-p), the null hypothesis is accepted.

where, F (1-α, p-1, n-p) is a value selected from the F table (Table A.3, Rawlings et al. [105]) for a specified level of significance, α. In the study, a significance level of 0.05 (α) was considered. If the null hypothesis in the F-test is disproved, the effect of individual variables on the dependent variable's variation is assessed using the t-test. If the t-test results show that any of the regression coefficients are not significant, a revised equation must be created without those unimportant coefficients.

The t-test has a set rule for determining whether to accept or reject the null hypothesis. If the calculated t-value, "t_{cal}", is greater than the t-value from the t-table (Table A.1, Rawlings et al. [105]) for a given level of significance, "α", and degrees of freedom, "n-p", then the null hypothesis is rejected. Conversely, if "t_{cal}" is less than the negative of the t-value from the t-table, the null hypothesis is again rejected. However, if "t_{cal}" falls within the range of the positive and negative t-values from the t-table, then the null hypothesis is accepted. The t-value from the t-table is determined based on the desired level of significance, "α", and the degrees of freedom, "n-p", and can be found in Table A.1, Rawlings et al. [105].

The load test results reveal that the geometric parameters of the geocell and the relative density of sand in the geocell-reinforced sand bed significantly affect the pressure-settlement behaviour of the foundation. The regression models included the following parameters: relative density (D_r), settlement of the footing ratio (s/B), placement depth of the geocell reinforcement ratio (u/B), the pocket diameter of the geocell ratio (d/B), the height of the geocell ratio (h/B), the width of the geocell reinforcement ratio (b/B). Based on the results presented in Fig. 4.7, 4.17-4.39 (Chapter 4), the non-linear regression model that best represents the behaviour of the data was selected as follows:

$$q_R = \xi_1 q_U^{\xi_2} \left(\frac{D_r}{100}\right)^{\xi_3} \left(\frac{s}{B}\right)^{\xi_4} \xi_5^{\left(\frac{u}{B}\right)} \xi_6^{\left(\frac{d}{B}\right)} \left(\frac{h}{B}\right)^{\xi_7} \left(\frac{b}{B}\right)^{\xi_8} \tag{6.6}$$

Fig. 6.1 represents the normal probability plot obtained from the Anderson-Darling test. The p-value of 0.062, which is larger than the significance level of 0.05, indicates that the data conforms to a normal distribution.

Once the values of the various coefficients have been determined, Equation (6.6) can be expressed as follows:

$$q_R = 8.58\, q_U^{0.45}\ \left(\frac{D_r}{100}\right)^{-0.24} \left(\frac{s}{B}\right)^{0.52} 0.61^{\left(\frac{u}{B}\right)} 0.62^{\left(\frac{d}{B}\right)} \left(\frac{h}{B}\right)^{0.48} \left(\frac{b}{B}\right)^{0.35} \qquad (6.7)$$

The regression statistics are presented in Table 6.1. The overall significance of the regression model is assessed by F-test; whereas, the significance of individual regression coefficients is evaluated through the t-test. From the F-distribution table, corresponding to the level of significance (α = 0.05), the value of $F_{crit,\,(7,\,56)}$ is found as 2.182, as compared to the F_{cal} = 1103 (Table 6.2). Therefore, the null hypothesis is rejected, $F_{cal} > F_{crit}$. Similarly, the null hypothesis for the t-test is rejected for $t_{crit,\,(0.025,\,56)} = 2.004 < |t_{cal}|$ (Table 6.3). This suggests that q_R is directly related to all the dependent variables.

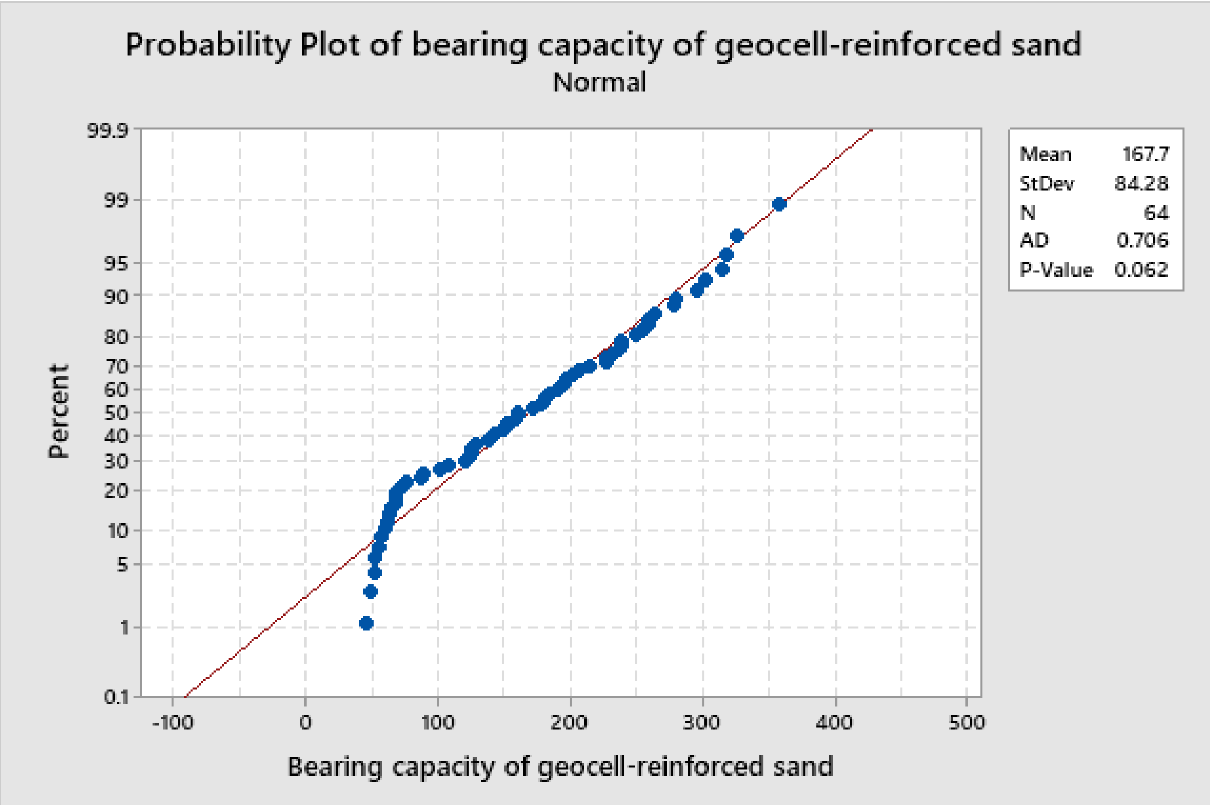

Fig. 6.1 Normal probability plot of bearing capacity of geocell-reinforced sand

Table 6.1 Regression statistics

Regression statistics	
R^2	0.993
$R^2_{Adj.}$	0.992
Standard Error	0.053
Observations	64

Table 6.2 Analysis of variance and *F*-statistic

	Degree of freedom	Sum of squares	Mean square	*F*
Regression	7	21.34	3.05	1103
Residual	56	0.15	0.00	
Total	63	21.49		

Table 6.3 *t*-statistic

Variable	Parameter	Standard error	*t*-stat	*P* -value
Intercept	$Ln(\xi_1)$ = 1.83	0.616	2.977	0.004
$Ln(q_U)$	ξ_2 = 0.54	0.167	3.203	0.002
$Ln(D_r)$/100	ξ_3 = -0.30	0.130	-2.281	0.026
$Ln(s/B)$	ξ_4 = 0.47	0.083	5.720	0.000
(u/B)	$Ln(\xi_5)$ = -0.49	0.146	-3.361	0.001
(d/B)	$Ln(\xi_6)$ = -0.48	0.024	-19.625	0.000
$Ln(h/B)$	ξ_7 = 0.48	0.025	19.402	0.000
$Ln(b/B)$	ξ_8 = 0.35	0.037	9.356	0.000

6.1.2 Validation of the model

To validate the regression model represented by Equation (6.7), the model's results were compared with sixty-four experimental ultimate bearing capacity results that were not used in the model's development. Fig. 6.2 presents the correlation of experimental and corresponding predicated bearing pressures. The fitted data points are corresponds to *s/B* = 2, 6, 10 and 14; whereas, the validated data points are corresponds to 1, 4, 8 and 12 of *s/B*. Depending on the overall performance and scattering of values, the proposed models can be judged good to estimate the foundation behaviour.

Table 6.4 presents the validation results along with the absolute error obtained. As can be seen in Table 6.4, 70% of the errors are in the range of 0 to $\pm$ 5% while the highest absolute error obtained for the proposed model was 9%. Thus, the values calculated by the equation are considered to be a good approximation of the experimentally measured values. Further, the value of R^2 was 0.993 (Table 6.1), which indicated that the estimation obtained with the model was quite reasonable. The study established that the variables considered in the model have a significant effect on q_R. Using the regression model, equation (6.7), a preliminary calculation of the bearing capacity of square foundations supported on sand bed reinforced with woven geotextile-based geocell can be done.

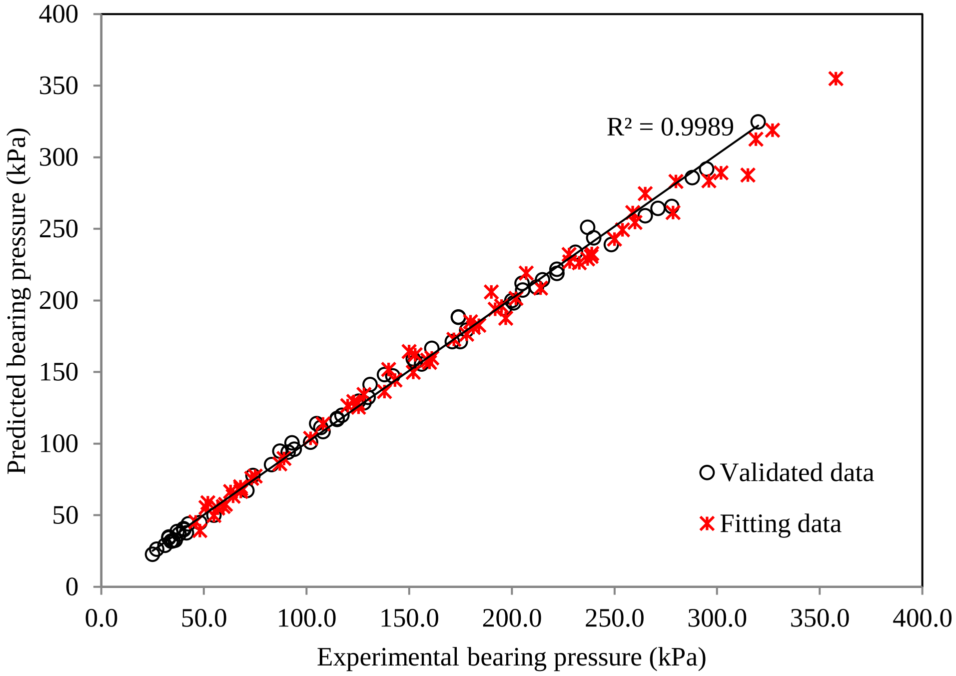

Fig. 6.2 Variation of observed and predicted bearing pressures

Table 6.4 Verification of regression model

q_U	$D_r/100$	s/B	u/B	d/B	h/B	b/B	Experimental, q_R	Predicted, q_R	Percentage of error
16.5	0.35	1.0	0.1	0.5	0.66	3	33.0	35.0	6.00
35.3	0.35	4.0	0.1	0.5	0.66	3	102.0	101.2	0.79
51.2	0.35	8.0	0.1	0.5	0.66	3	171.0	171.3	0.19
64.0	0.35	12.0	0.1	0.5	0.66	3	231.0	233.8	1.23
26.7	0.7	1.0	0.1	0.5	0.66	3	38.0	36.8	3.04
59.0	0.7	4.0	0.1	0.5	0.66	3	108.0	108.5	0.43
82.0	0.7	8.0	0.1	0.5	0.66	3	178.1	179.5	0.76
98.0	0.7	12.0	0.1	0.5	0.66	3	248.5	239.1	3.79
32.2	0.9	1.0	0.1	0.5	0.66	3	41.5	37.8	8.90
78.0	0.9	4.0	0.1	0.5	0.66	3	115.0	117.0	1.70

115.2	0.9	8.0	0.1	0.5	0.66	3	200.0	200.0	0.02
137.0	0.9	12.0	0.1	0.5	0.66	3	278.0	265.7	4.43
26.7	0.7	1.0	0.25	0.5	0.66	3	33.0	34.2	3.67
59.0	0.7	4.0	0.25	0.5	0.66	3	93.0	100.7	8.29
82.0	0.7	8.0	0.25	0.5	0.66	3	161.0	166.6	3.50
98.0	0.7	12.0	0.25	0.5	0.66	3	222.0	222.0	0.00
26.7	0.7	1.0	0	0.5	0.66	3	37.0	38.7	4.63
59.0	0.7	4.0	0	0.5	0.66	3	105.0	114.0	8.53
82.0	0.7	8.0	0	0.5	0.66	3	174.0	188.6	8.37
98.0	0.7	12.0	0	0.5	0.66	3	237.0	251.2	5.99
26.7	0.7	1.0	0.1	0.33	0.66	3	40.2	40.0	0.59
59.0	0.7	4.0	0.1	0.33	0.66	3	115.0	117.6	2.30
82.0	0.7	8.0	0.1	0.33	0.66	3	196.5	194.7	0.94
98.0	0.7	12.0	0.1	0.33	0.66	3	265.0	259.3	2.14
26.7	0.7	1.0	0.1	0.75	0.66	3	36.0	32.7	9.18
59.0	0.7	4.0	0.1	0.75	0.66	3	94.0	96.2	2.39
82.0	0.7	8.0	0.1	0.75	0.66	3	152.0	159.2	4.77
98.0	0.7	12.0	0.1	0.75	0.66	3	205.0	212.1	3.49
26.7	0.7	1.0	0.1	1.00	0.66	3	31.0	29.0	6.41
59.0	0.7	4.0	0.1	1.00	0.66	3	83.0	85.4	2.90
82.0	0.7	8.0	0.1	1.00	0.66	3	131.0	141.3	7.87
98.0	0.7	12.0	0.1	1.00	0.66	3	174.0	188.2	8.19
26.7	0.7	1.0	0.1	1.50	0.66	3	25.0	22.8	8.62
59.0	0.7	4.0	0.1	1.50	0.66	3	71.0	67.2	5.28
82.0	0.7	8.0	0.1	1.50	0.66	3	107.0	111.3	3.99
98.0	0.7	12.0	0.1	1.50	0.66	3	138.0	148.2	7.41
26.7	0.7	1.0	0.1	0.50	0.33	3	27.0	26.4	2.16
59.0	0.7	4.0	0.1	0.50	0.33	3	74.0	77.8	5.09
82.0	0.7	8.0	0.1	0.50	0.33	3	128.0	128.7	0.52
98.0	0.7	12.0	0.1	0.50	0.33	3	175.0	171.4	2.05
26.7	0.7	1.0	0.1	0.50	0.50	3	35.0	32.2	7.86
59.0	0.7	4.0	0.1	0.50	0.50	3	87.0	94.9	9.12
82.0	0.7	8.0	0.1	0.50	0.50	3	153.0	157.1	2.66
98.0	0.7	12.0	0.1	0.50	0.50	3	212.0	209.2	1.30
26.7	0.7	1.0	0.1	0.50	1.00	3	48.0	45.0	6.30
59.0	0.7	4.0	0.1	0.50	1.00	3	130.0	132.4	1.85
82.0	0.7	8.0	0.1	0.50	1.00	3	222.0	219.1	1.32
98.0	0.7	12.0	0.1	0.50	1.00	3	295.0	291.8	1.07
26.7	0.7	1.0	0.1	0.50	1.25	3	55.0	50.1	8.98
59.0	0.7	4.0	0.1	0.50	1.25	3	142.0	147.4	3.79
82.0	0.7	8.0	0.1	0.50	1.25	3	240.0	243.8	1.60
98.0	0.7	12.0	0.1	0.50	1.25	3	320.0	324.8	1.51
26.7	0.7	1.0	0.1	0.50	0.66	2	34.1	32.0	6.25
59.0	0.7	4.0	0.1	0.50	0.66	2	91.1	94.1	3.31
82.0	0.7	8.0	0.1	0.50	0.66	2	156.1	155.7	0.24
98.0	0.7	12.0	0.1	0.50	0.66	2	205.4	207.4	1.00
26.7	0.7	1.0	0.1	0.50	0.66	4	40.1	40.7	1.62
59.0	0.7	4.0	0.1	0.50	0.66	4	117.2	120.0	2.35
82.0	0.7	8.0	0.1	0.50	0.66	4	201.0	198.5	1.26
98.0	0.7	12.0	0.1	0.50	0.66	4	271.3	264.4	2.54
26.7	0.7	1.0	0.1	0.50	0.66	5	42.5	44.1	3.67
59.0	0.7	4.0	0.1	0.50	0.66	5	125.3	129.7	3.51
82.0	0.7	8.0	0.1	0.50	0.66	5	215.0	214.6	0.19
98.0	0.7	12.0	0.1	0.50	0.66	5	288.0	285.9	0.74

6.1.3 Comparison of regression model with analytical methodologies

The ultimate bearing capacity of laboratory test results, as well as the predictions made using the regression equation established in this study, were compared with the analytical methodologies developed by Koerner [64] and Avesani Neto et al. [5]. The theoretical bearing capacity for the geocell was computed by using Koerner [64] and Avesani Neto et al. [5] method for different soil parameters corresponding to 35%, 70% & 90% relative density of the Brahmaputra sand of this study (*Annexure 3*). The results of the comparison can be seen in Fig. 6.3, where the graph of q_R vs. D_r for square footing supported on geocell-reinforced sand beds are presented. As seen from the plot, the predictions made using the Koerner method [64] and the regression model established in this study closely match the laboratory measurements. The other method, Avesani Neto et al. [5] overestimated the performance of reinforced sand beds. The deviations observed in the case of Avesani Neto et al. [5] can be explained by the 'wide slab' effect (Huang and Tatsuoka, [48]). The "wide slab" effect indicates that the behaviour of reinforced soil closely resembles that of unreinforced soil with a footing that has an additional embedment depth equal to the depth of the reinforced zone. The additional bearing capacity provided by the lateral forces produced by the embedment depth can be evaluated by the 'wide slab' effect, according to which, the footing with embedment depth has a base wider than the original footing. Contrarily, in the case of the Koerner method [64], although the embedment depth of the footing is considered, the wide slab effect is not incorporated in the analytical model. According to the response illustrated by the q_R with the various parameters studied, the laboratory results and regression equation are within the range of values predicted by various analytical approaches. Hence, the regression model developed in Equation 6.7 provides a satisfactory approximation for predicting the bearing capacity of the soil reinforced with geocell.

This model can be useful for the preliminary design of square foundations supported by a sand-geocell system. However, the regression model developed in this study is applicable to square footings supported on geocell-reinforced sand, with the following conditions of parameters:

$35\% \leq D_r \leq 90\%$, $0 \leq s/B \leq 0.14$, $0 \leq u/B \leq 0.25$, $0.33 \leq d/B \leq 1.5$, $0.33 \leq h/B \leq 1.25$, $2 \leq d/B \leq 5$.

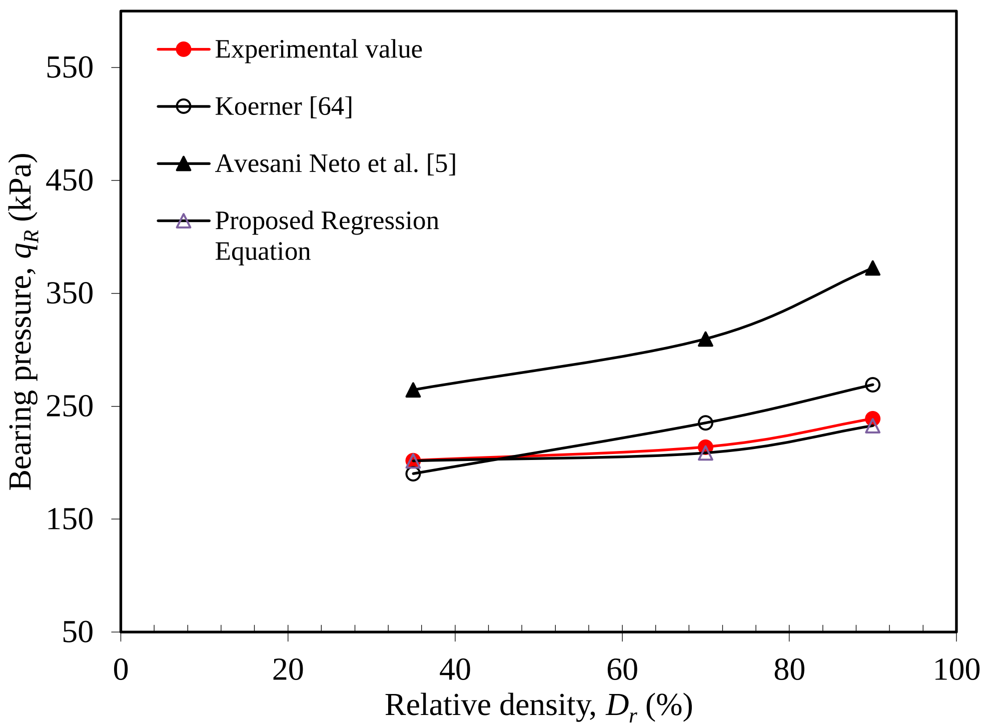

Fig. 6.3 Ultimate bearing capacity of reinforced sand, q_R versus relative density for different analytical methodologies, measured values, and proposed regression equation

6.2 Scale effects

Small-scale model test results are prone to scale effects. However, the scale effect can be reduced by carefully controlling the test parameters (Raja and Shukla [101]). The particle size of the foundation soil, the size of the model footing, and the geocell size contribute to the scale effect in model tests on sand (Cerato and Lutenegger [17, Shadmand et al. [112], Tavakoli Mehrjardi [128]). In the case of prototype footing, the width of footing (B) is usually very large in comparison to the mean particle size (d_{50}) of the soil. However, in model footing, although the width of the footing is small, d_{50}/B ratio may still be low enough to nullify the scale effect due to particle size. Cerato and Lutenegger [17] suggested that the bearing capacity does not get affected if d_{50}/B is kept smaller than 1/200. Considering this recommendation, the particle size effect will be negligible as the ratio of d_{50}/B for this study is 1/250. The response of a granular foundation under a small model footing is not directly representative of that under prototype footing because of the different mean stresses under footings with different widths at a given relative density [17].

Tests performed by Cerato and Lutenegger [17] on the sand with footing sizes ranging from 0.0254 m to 0.914 m showed an appreciable scale effect up to a footing width of 0.25 m after which the scale effect was significantly reduced.

Results of a study on load-settlement characteristics of large square footings (150 mm to 600 mm) investigated by performing large-scale loading tests on unreinforced and geocell-reinforced (d/B = 1.66 to 0.416, h/B = 1 to 0.25, u/B = 0.33 to 0.083, a tensile strength of geocell material = 15 kN/m) granular soils have shown that the effect of footing width on scale effect has the same trend at 65% relative density for both unreinforced and reinforced conditions [112]. This indicates that the model footing size has a much more dominant role than geocell parameters in the scale effect of the tests conducted. The parameters of the present study (d/B = 0.5, h/B = 0.66, b/B = 3, u/B = 0.1, tensile strength of geocell material = 24 kN/m, relative density 70%) are in the same range as seen in this study.

In order to have a closer representation of behaviour of full-scale footing, the model-scale tests are suggested to be performed on a looser state of sand than the density of the sand in the full-scale test [17]. In view of this, in the current study, the model tests were carried out in medium-dense sand. The relative density of the sand bed in the present study was adopted at 70% so as to avoid punching shear failure of the unreinforced sand. The foundation soil particle size and relative density, size of the model footing plate, and geocell dimensions adopted in the present study are sufficient enough to reduce, but not eliminate, the scale effect on the performance of the model tests. Using the results of this small-scale test for larger footings is not conservative and this fact should be considered in the design and construction of footings.

6.3 Practical implications of this study

One of the most important outcomes of this study is providing practical guidelines in terms of choosing geocell geometric dimensions. By conducting series of small-scale model tests the influence of different parameters were identified and the comprehensive practical implications were presented in terms bearing capacity and settlement. Also, an empirical equation was proposed to predict the bearing capacity and settlement of square footing supported on geocell-reinforced sand beds. The current study suggests the maximum initial static load level of repeated loading in a geocell reinforced sand. It was shown that by reinforcing the sand with geocells of optimum geometric dimension, the improvement factor could be increased by about 1.9 to 4 to that of unreinforced sand. Geocells were found to have a profound influence on improving resiliency (i.e., increasing the resilient modulus) and decreased the corresponding settlement by about 56% compared to unreinforced sand. In summary the results obtained in the present study provides a basis to support the behaviour of the static and repeatable loads in several areas, such as oil or water storage tank, service road, and parking yard.