CHAPTER 7

The Inter-relational Sensitivity Among Rainfall and Other Meteorological Variables in NE India: VAR-IRF Modelling Approach

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Contents

7.1 Introduction

Rainfall is a major climate variable that has significant socioeconomic impact. Events like floods and drought is dependent on the extent of rainfall and effect the crop production as well as other production activities [1]. The key weather parameters that determine the intensity and duration of rainfall received on land are temperature, relative humidity, sea level pressure, wind speed, amount of sunshine etc. [2].

Changes in rainfall pattern, temperature, RH, wind speed and sea level pressure increase the intensity, frequency, and duration of extreme weather events like drought, flood, cyclones and heat waves [3]. Hence, it is crucial to understand the relationships between rainfall and different meteorological variables. The relations among meteorological variables can be described by using statistical models. Vector autoregression (VAR) is an important statistical tool to investigate the inter dependencies of these meteorological variables. The VAR model is an extension of univariate autoregressive process that describes the linear dependencies of variables at time t on the values of the variables at previous time points [4]. VAR methods are employed by numerous researchers to study the dynamic nature of economic factors and predict the economic growth [5, 6]. Koitsiwe and Adachi [7] examined the dynamic association among mining sector, manufacturing sector, service sector and exchange rate by using unrestricted vector autoregressive model containing impulse response function (IRF), VAR Granger causality and Variance Decomposition (VDC). The model explained the response of a specific variable to one standard deviation shock of each variable in the system. Results showed that the patterns of variation among the variables are well explained by VAR-IRF model. Yazdi and Shakouri [8] used autoregressive distributed lag method of cointegration test and vector error-correction models and found long-term relationship between CO_2 emissions, and financial growth, renewable energy consumption, and energy use. In the study, the Impulse Response Function was used under the VAR method to estimate the shocks generated by renewable energy use. Salim et.al. [9] investigated relation between climate change, research and development investment and agricultural productivity for developing economy in Bangladesh. The climate variables chosen for the study are temperature and rainfall. The traditional VAR cannot explain the time and unit specific effects for both short- and long-term effects, therefore Salim et.al. [9] applied panel heterogeneous model and Pooled Mean Group in empirical estimation.

VAR has been applied in many studies to predict future rainfall. Nugroho et.al. [10] forecasted future rainfall for five stations in Indonesia by using VAR and ARIMA model. The combinations of rainfall, humidity and temperature were used to develop the autoregressive equation. Results showed that VAR model performed better than the ARIMA model. Chapman et al. [11] applied as an extension of the linear inverse model (LIM), a first order approximation to a dynamical system in which the evolution operator is reduced to a matrix product and white Gaussian noise. Only one SST dataset, the Kaplan historical monthly mean SST gridded dataset (5^oX 5^o resolution) was considered (http://iridl.ldeo.columbia.edu/) in this study, as much of the subsurface forcing in the tropical Pacific can be modeled using only SST as a predictor. As SST is one of the earliest oceanographic variables to be recorded, SST-only models are valuable for long-term and historical studies of ENSO variability. The results showed improvements of using VAR model in forecast skill by three-months over the considered period (1981-2010). Ramli et.al [12] predicted rainfall for five years (2016-2020) using rainfall and discharge data in 2008-2015 using VAR. Based on Nash-Sutcliffe efficiency (NSE), the accuracy of the predicted data was tested for the period 2016-2017. The results obtained for this study have an NSE value of 0.9522, which is considered satisfactory. The relation among rainfall, temperature, humidity and wind speed were also studied. The impact of shocks in rainfall on other meteorological variables was examined using IRF. The variance decomposition analysis revealed that rainfall fluctuations over a period of 12 months become weaker when the shock to itself was introduced, while shocks of temperature, humidity and wind speed caused fluctuations in rainfall. In most of the studies the lag length was determined by using Akaike Information Criterion (AIC), Final Prediction Error Correction (FPE), Schwarz Information Criterion (SIC) and Hannan-Quinn Information Criterion (HQ) [13-15].

The VAR model is under stationary assumption for input variables [16]. Differentiation is used to transform non-stationary variables into stationary variables. However, many proponents oppose differentiating the non-stationary variables that are co-integrated [17]. To extract the valuable information of non-stationary variables, integration of wavelet approach and VAR approach was used in various studies [18-20]. Using wavelet decomposed variables in VAR model can offer valuable insight in sensitivity of the variables in different time scale domain. Therefore an integrated multi-resolutional approach combing wavelet and VAR was adopted in this study to examine the inter-relational sensitivity among rainfall, temperature, relative humidity, sea level pressure and wind speed.

7.2 Methodology

7.2.1 Vector Autoregressive Model

The Vector Autoregressive (VAR) model is a multivariate statistical technique used to analyse time series data that exhibit dynamic interactions. VAR model is actually a combination of several autoregressive (AR) models. These models construct a vector between the variables that impacts each other. It describes the relationship between observations on a particular variable at a given time and its own observations on similar variables at a previous time, as well as its relationship with observations on other variables.

The time series of AR of order p is denoted as:

$$Y_t = A_o + A_1 Y_{t-1} + A_2 Y_{t-2} + \cdots\cdots + A_p Y_{t-p} + \varepsilon_t$$

Here, Y_t is the current value, p is the lag length, Y_{t-1} and Y_{t-p} is observational values from $t-1$ to $t-p$, A_o is the intercept and $A_1$to A_p is regression coefficient from $t-1$ to $t-p$ and ε_t is pure white noise error term.

The VAR model used in the present study can be described as follows:

Step 1: The stationarity of individual meteorological variables were tested using augmented Dickey–Fuller (ADF) test [15].

Step 2: If data is not stationary then the differencing process was carried out. The optimum lag length has been determined on the basis of AIC, SIC and HQ test results [12].

Step 3: After determining the lag value the VAR analysis were performed to investigate the significant associations of the variable.

A VAR model of lag order ‘p’ of rainfall (RF), maximum temperature (MaxT), minimum temperature (MinT), relative humidity (RH), sea level pressure (SLP), mean sea level pressure (MSLP), wind speed (WS) can be expressed as follows:

$$\begin{bmatrix} RF_t \\ MaxT_t \\ MinT_t \\ SLP_t \\ RH_t \\ WS_t \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{bmatrix} + \begin{bmatrix} a^1_{1,1} & a^1_{1,2} & a^1_{1,3} & a^1_{1,4} & a^1_{1,5} & a^1_{1,6} \\ a^1_{2,1} & a^1_{2,2} & a^1_{2,3} & a^1_{2,4} & a^1_{2,5} & a^1_{2,6} \\ a^1_{3,1} & a^1_{3,2} & a^1_{3,3} & a^1_{3,4} & a^1_{3,5} & a^1_{3,6} \\ a^1_{4,1} & a^1_{4,2} & a^1_{4,3} & a^1_{4,4} & a^1_{4,5} & a^1_{4,6} \\ a^1_{5,1} & a^1_{5,2} & a^1_{5,3} & a^1_{5,4} & a^1_{5,5} & a^1_{5,6} \\ a^1_{6,1} & a^1_{6,2} & a^1_{6,3} & a^1_{6,4} & a^1_{6,5} & a^1_{6,6} \end{bmatrix} \begin{bmatrix} RF_{t-1} \\ MaxT_{t-1} \\ MinT_{t-1} \\ SLP_{t-1} \\ RH_{t-1} \\ WS_{t-1} \end{bmatrix} + \cdots$$

$$+ \begin{bmatrix} a^p_{1,1} & a^p_{1,2} & a^p_{1,3} & a^p_{1,4} & a^p_{1,5} & a^p_{1,6} \\ a^p_{2,1} & a^p_{2,2} & a^p_{2,3} & a^p_{2,4} & a^p_{2,5} & a^p_{2,6} \\ a^p_{3,1} & a^p_{3,2} & a^p_{3,3} & a^p_{3,4} & a^p_{3,5} & a^p_{3,6} \\ a^p_{4,1} & a^p_{4,2} & a^p_{4,3} & a^p_{4,4} & a^p_{4,5} & a^p_{4,6} \\ a^p_{5,1} & a^p_{5,2} & a^p_{5,3} & a^p_{5,4} & a^p_{5,5} & a^p_{5,6} \\ a^p_{6,1} & a^p_{6,2} & a^p_{6,3} & a^p_{6,4} & a^p_{6,5} & a^p_{6,6} \end{bmatrix} \begin{bmatrix} RF_{t-p} \\ MaxT_{t-p} \\ MinT_{t-p} \\ SLP_{t-p} \\ RH_{t-p} \\ WS_{t-p} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \\ \varepsilon_{4,t} \\ \varepsilon_{5,t} \\ \varepsilon_{6,t} \end{bmatrix}$$

The above-mentioned matrix equation can be also written as follows using regression notation.

For rainfall,

$$[RF_t] = C_1 + a^1_{1,1}[RF]_{t-1} + a^1_{1,2}[MaxT]_{t-1} + a^1_{1,3}[MinT]_{t-1} + a^1_{1,4}[SLP]_{t-1} + a^1_{1,5}[RH]_{t-1} + a^1_{1,6}[WS]_{t-1} + \cdots + a^p_{1,1}[RF]_{t-p} + a^p_{1,2}[MaxT]_{t-p} + a^p_{1,3}[MinT]_{t-p} + a^p_{1,4}[SLP]_{t-p} + a^p_{1,5}[RH]_{t-p} + a^p_{1,6}[WS]_{t-p} + \varepsilon_{p,t}$$

7.2.2 Impulse response function

An impulse response can be defined as a reaction of a system (system of equations, comprising of a multivariate autoregressive model), in response to an external change (shock). In a VAR model, all the variables depend on each other; therefore, individual coefficient estimates provide limited information on the reaction of the system to a shock [21]. Therefore, the need for impulse responses arises, which provide information about the dynamic behaviour of a VAR model. The purpose of an impulse response function is to describe the evolution of a VAR model's reaction to a shock in one or more variables [22]. Thus, impulse response analysis employs VAR models, which describes how model variables respond to one standard deviation shock of the other variables.

IRF in this study has been used to analyze the impact of shocks in the MaxT, MinT, RH, SLP, MSLP and WS on rainfall [15]. It exhibits standard error unit shocks (ε_{1t},

ε_{2t}, …, ε_{kt}) of variables over time on the endogenous variable (Y_{1t}, Y_{2t},…, Y_{kt}). Once the order of the VAR process is established, the Wald representation of Y_t based on standard error ε_t is given by

$$Y_t = \mu + \varepsilon_t + \emptyset\varepsilon_{t-1} + \emptyset\varepsilon_{t-2} + \cdots$$

Where, the $\emptyset_s$ are matrices of moving average (*nxn*) and the impulse responses to the shocks of ε_{jt} are

$$\frac{\delta Y_{i,t+s}}{\delta\varepsilon_{j,t}} = \frac{\delta Y_{i,t}}{\delta\varepsilon_{j,i-s}} = \emptyset_{ij}^{s} \quad i,j = 1,2,\ldots,n; s > 0$$

A plot of $\emptyset_{ij}^{s}$ against s is termed impulse response function (IRF) of Y_i with respect to ε_j. With n number of variables, possible numbers IRF can be n^2.

7.2.3 Coupling Wavelet and VAR model

To understand the significant associations among the meteorological variables with reference to time in different resolutions, wavelet decomposed time series data (obtained from wavelet decomposition in chapter 6) were taken as input for the VAR analysis. The individual meteorological variable was subjected to wavelet decomposition using symlet8 wavelet. As a result of the symmetrical decomposition of the data, signals at various scales were generated and used as input in the VAR model. 1σ shocks of the individual wavelet decomposed series (w1-w10) of the meteorological variables were applied on the total monthly rainfall over the NER (selected locations as per chapter 2). The response of rainfall against these shocks was then observed. A thematic representation of the working principle is given below:

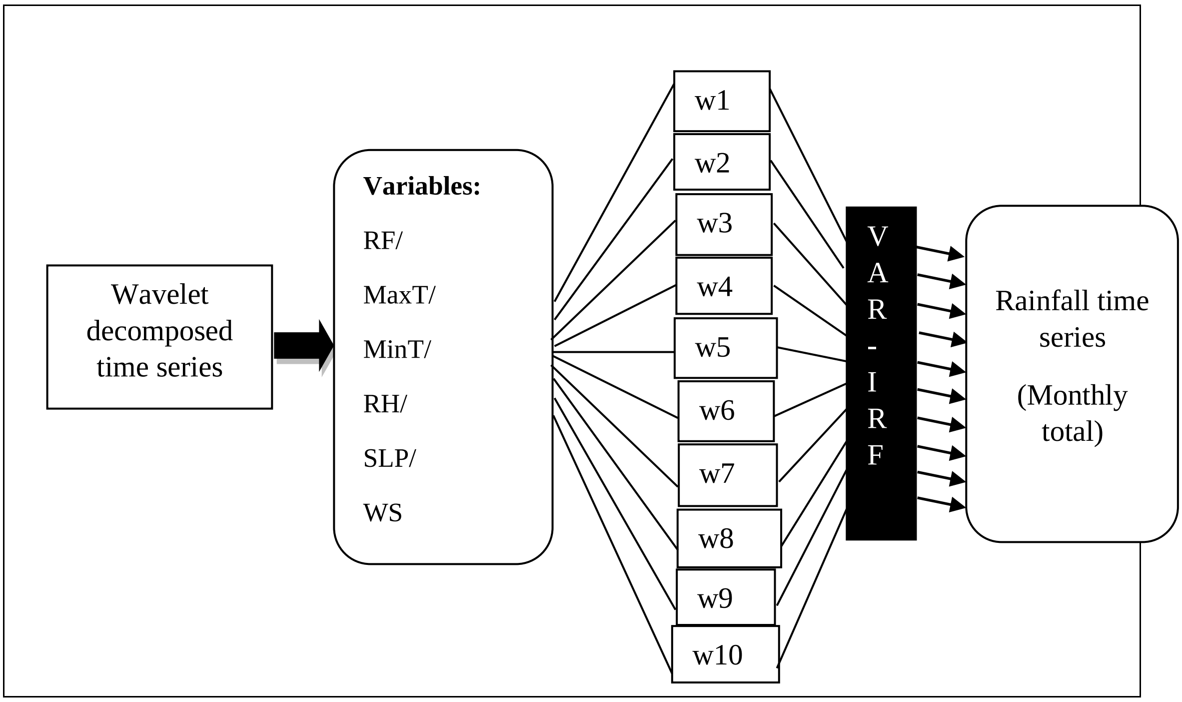


Figure 7. 1 Thematic representation of VAR-IRF model applied on the monthly total series of rainfall over NER

7.3 Results and discussions

7.3.1 VAR model estimation

The master equations for rainfall per selected locations of NER were generated from the VAR estimates considering a lag length of two. The Equations for Rainfall (RF) as an impulse towards shock of MaxT, MinT, RH, SLP and WS considering a lag length of 2 (for wavelet decomposed series w1-w10) are as follows (The boldface indicates significant values of the parameters):

1. For CHR

i. $[RF] = \mathbf{631.342[MaxT_{w1}]_{t-1}}\ \mathbf{-369.196[MinT_{w1}]_{t-1}}\ -1.501[SLP_{w1}]_{t-1}\ \mathbf{-36.373[RH_{w1}]_{t-1}}\ \mathbf{--99.125[WS_{w1}]_{t-1}}\ \mathbf{+0.697[RF]_{t-1}} + 113.506[MaxT_{w1}]_{t-2} - 210.418[MinT_{w1}]_{t-2} - 11.525[SLP_{w1}]_{t-2} - 12.752[RH_{w1}]_{t-2} - 10.351[WS_{w1}]_{t-2}\ \mathbf{-0.112[RF]_{t-2}} + 401.552$

ii. $[RF] = 123.621[MaxT_{w2}]_{t-1} - 119.372[MinT_{w2}]_{t-1} + 12.902[SLP_{w2}]_{t-1} + 15.524[RH_{w2}]_{t-1}\ \mathbf{--75.959[WS_{w2}]_{t-1}}\ \mathbf{+0.547[RF]_{t-1}}\ \mathbf{+395.555[MaxT_{w2}]_{t-2}}\ \mathbf{-290.596[MinT_{w2}]_{t-2}} - 20.416[SLP_{w2}]_{t-2} + 2.878[RH_{w2}]_{t-2} - 49.217[WS_{w2}]_{t-2} + 0.045[RF]_{t-2}\ \mathbf{+394.647}$

iii. $[RF] = 9.265[MaxT_{w3}]_{t-1}\ \mathbf{+475.600[MinT_{w3}]_{t-1}} + 7.539[SLP_{w3}]_{t-1}\ \mathbf{+47.220[RH_{w3}]_{t-1}}\ \mathbf{+147.200[WS_{w3}]_{t-1}} + 0.010[RF]_{t-1} + 1.290[MaxT_{w3}]_{t-2}$ -

493.200[MinT$_{w3}$]$_{t-2}$ +28.030[SLP$_{w3}$]$_{t-2}$ -20.520[RH$_{w3}$]$_{t-2}$ -84.560[WS$_{w3}$]$_{t-2}$ +0.0530[RF]$_{t-2}$ **+905.200**

iv. [RF]= -140.644[MaxT$_{w4}$]$_{t-1}$ +585.472[MinT$_{w4}$]$_{t-1}$ +67.122[SLP$_{w4}$]$_{t-1}$ +55.845[RH$_{w4}$]$_{t-1}$ +105.682[WS$_{w4}$]$_{t-1}$ **+0.552[RF]$_{t-1}$** -237.974[MaxT$_{w4}$]$_{t-2}$ -566.338[MinT$_{w4}$]$_{t-2}$ +10.648[SLP$_{w4}$]$_{t-2}$ -44.421[RH$_{w4}$]$_{t-2}$ -40.166[WS$_{w4}$]$_{t-2}$ +0.002[RF]$_{t-2}$ **+429.854**

v. [RF]= -38.380[MaxT$_{w5}$]$_{t-1}$ +197.700[MinT$_{w5}$]$_{t-1}$ +297.600[SLP$_{w5}$]$_{t-1}$ +217.600[RH$_{w5}$]$_{t-1}$ +260.400[WS$_{w5}$]$_{t-1}$ **+0.571[RF]$_{t-1}$** -158.800[MaxT$_{w5}$]$_{t-2}$ -137.400[MinT$_{w5}$]$_{t-2}$ -242.600[SLP$_{w5}$]$_{t-2}$ -245.900[RH$_{w5}$]$_{t-2}$ -219.300[WS$_{w5}$]$_{t-2}$ -0.0140[RF]$_{t-2}$ **+428.200**

vi. [RF]= -133.600[MaxT$_{w6}$]$_{t-1}$ -737.800[MinT$_{w6}$]$_{t-1}$ +990.600[SLP$_{w6}$]$_{t-1}$ -1065.000[RH$_{w6}$]$_{t-1}$ -230.600[WS$_{w6}$]$_{t-1}$ **+0.572[RF]$_{t-1}$** -143.800[MaxT$_{w6}$]$_{t-2}$ +806.600[MinT$_{w6}$]$_{t-2}$ - 1089.000[SLP$_{w6}$]$_{t-2}$ +1055.000[RH$_{w6}$]$_{t-2}$ +397.600[WS$_{w6}$]$_{t-2}$ -0.015[RF]$_{t-2}$ **+423.000**

vii. [RF]= 15320.000[MaxT$_{w7}$]$_{t-1}$ +7086.000[MinT$_{w7}$]$_{t-1}$ -5461.000[SLP$_{w7}$]$_{t-1}$ -1175.000[RH$_{w7}$]$_{t-1}$ -1422.000[WS$_{w7}$]$_{t-1}$ **+0.577[RF]$_{t-1}$** -15770.000[MaxT$_{w7}$]$_{t-2}$ -7134.000[MinT$_{w7}$]$_{t-2}$ +5511.000[SLP$_{w7}$]$_{t-2}$ +1197.000[RH$_{w7}$]$_{t-2}$ +1438.000[WS$_{w7}$]$_{t-2}$ -0.012[RF]$_{t-2}$ **+411.600**

viii. [RF]= -19140.000[MaxT$_{w8}$]$_{t-1}$ -127.300[MinT$_{w8}$]$_{t-1}$ -3932.000[SLP$_{w8}$]$_{t-1}$ +1648.000[RH$_{w8}$]$_{t-1}$ +1495.000[WS$_{w8}$]$_{t-1}$ **+0.577[RF]$_{t-1}$** +42270.000[MaxT$_{w8}$]$_{t-2}$ +9085.000[MinT$_{w8}$]$_{t-2}$ +5983.000[SLP$_{w8}$]$_{t-2}$ -2245.000[RH$_{w8}$]$_{t-2}$ +768.700[WS$_{w8}$]$_{t-2}$ -0.012[RF]$_{t-2}$ **+453.600**

ix. [RF]= 11790.000[MaxT$_{w9}$]$_{t-1}$ -33410.000[MinT$_{w9}$]$_{t-1}$ +4260.000[SLP$_{w9}$]$_{t-1}$ +19950.000[RH$_{w9}$]$_{t-1}$ +9980.000[WS$_{w9}$]$_{t-1}$ **+0.575[RF]$_{t-1}$** +4309.000[MaxT$_{w9}$]$_{t-2}$ +9216.000[MinT$_{w9}$]$_{t-2}$ +11290.000[SLP$_{w9}$]$_{t-2}$ -18630.000[RH$_{w9}$]$_{t-2}$ -28790.000[WS$_{w9}$]$_{t-2}$ -0.0120[RF]$_{t-2}$ +302.700

x. [RF]= **-266000.000[MaxT$_{w10}$]$_{t-1}$** +132300.000[MinT$_{w10}$]$_{t-1}$ +2525.000[SLP$_{w10}$]$_{t-1}$ +20300.000[RH$_{w10}$]$_{t-1}$ -57780.000[WS$_{w10}$]$_{t-1}$ **+0.579[RF]$_{t-1}$** +247300.000[MaxT$_{w10}$]$_{t-2}$ -138000.000[MinT$_{w10}$]$_{t-2}$ -922.900[SLP$_{w10}$]$_{t-2}$ -20910.000[RH$_{w10}$]$_{t-2}$ +52530.000[WS$_{w10}$]$_{t-2}$ -0.009[RF]$_{t-2}$ **+31680000.000**

2. For DBR

i. [RF]= **34.159[MaxT$_{w1}$]$_{t-1}$** +13.903[MinT$_{w1}$]$_{t-1}$ -39.916[SLP$_{w1}$]$_{t-1}$ **-23.730[RH$_{w1}$]$_{t-1}$ -20.402[WS$_{w1}$]$_{t-1}$ +0.817[RF]$_{t-1}$** -0.606[MaxT$_{w1}$]$_{t-2}$ +16.776[MinT$_{w1}$]$_{t-2}$ +11.850[SLP$_{w1}$]$_{t-2}$ -8.085[RH$_{w1}$]$_{t-2}$ -19.520[WS$_{w1}$]t-2 **-0.201[RF]$_{t-2}$ +85.571**

ii. $[RF]= 13.223[MaxT_{w2}]_{t-1}\ \mathbf{-28.611[MinT_{w2}]_{t-1}}\ -96.424[SLP_{w2}]_{t-1}\ -3.675[RH_{w2}]_{t-1}\ +7.258[WS_{w2}]_{t-1}\ \mathbf{+0.597[RF]_{t-1}}\ +15.552[MaxT_{w2}]_{t-2}\ -9.195[MinT_{w2}]_{t-2}\ -89.619[SLP_{w2}]_{t-2}\ -1.280[RH_{w2}]_{t-2}\ -2.834[WS_{w2}]_{t-2}\ +0.013[RF]_{t-2}\ \mathbf{+87.288}$

iii. $[RF]= 30.927[MaxT_{w3}]_{t-1}\ +3.241[MinT_{w3}]_{t-1}\ +29.019[SLP_{w3}]_{t-1}\ \mathbf{+23.726[RH_{w3}]_{t-1}}\ -7.448[WS_{w3}]_{t-1}\ +0.043[RF]_{t-1}\ -20.102[MaxT_{w3}]_{t-2}\ -8.625[MinT_{w3}]_{t-2}\ +44.982[SLP_{w3}]_{t-2}\ \mathbf{-15.642[RH_{w3}]_{t-2}}\ +9.429[WS_{w3}]_{t-2}\ +0.071[RF]_{t-2}\ \mathbf{+197.991}$

iv. $[RF]= -94.377[MaxT_{w4}]_{t-1}\ +217.854[MinT_{w4}]_{t-1}\ +109.666[SLP_{w4}]_{t-1}\ +7.913[RH_{w4}]_{t-1}\ +21.740[WS_{w4}]_{t-1}\ \mathbf{+0.588[RF]_{t-1}}\ -1.614[MaxT_{w4}]_{t-2}\ -183.275[MinT_{w4}]_{t-2}\ -128.848[SLP_{w4}]_{t-2}\ -17.749[RH_{w4}]_{t-2}\ +0.080[WS_{w4}]_{t-2}\ -0.020[RF]_{t-2}\ \mathbf{+95.539}$

v. $[RF]= -77.070[MaxT_{w5}]_{t-1}\ +149.332[MinT_{w5}]_{t-1}\ +812.595[SLP_{w5}]_{t-1}\ -22.212[RH_{w5}]_{t-1}\ -68.128[WS_{w5}]_{t-1}\ \mathbf{+0.626[RF]_{t-1}}\ +63.989[MaxT_{w5}]_{t-2}\ -141.234[MinT_{w5}]_{t-2}\ -886.684[SLP_{w5}]_{t-2}\ +24.828[RH_{w5}]_{t-2}\ +69.361[WS_{w5}]_{t-2}\ -0.047[RF]_{t-2}\ \mathbf{+94.310}$

vi. $[RF]= 19.090[MaxT_{w6}]_{t-1}\ -142.200[MinT_{w6}]_{t-1}\ +2570.000[SLP_{w6}]_{t-1}\ -34.730[RH_{w6}]_{t-1}\ +71.480[WS_{w6}]_{t-1}\ \mathbf{+0.628[RF]_{t-1}}\ -3.286[MaxT_{w6}]_{t-2}\ +154.400[MinT_{w6}]_{t-2}\ -2695.000[SLP_{w6}]_{t-2}\ +43.610[RH_{w6}]_{t-2}\ -73.480[WS_{w6}]_{t-2}\ -0.047[RF]_{t-2}\ \mathbf{+93.360}$

vii. $[RF]= -3631.000[MaxT_{w7}]_{t-1}\ -1421.000[MinT_{w7}]_{t-1}\ +6545.000[SLP_{w7}]_{t-1}\ -253.500[RH_{w7}]_{t-1}\ +328.500[WS_{w7}]_{t-1}\ \mathbf{+0.627[RF]_{t-1}}\ +4078.000[MaxT_{w7}]_{t-2}\ +1677.000[MinT_{w7}]_{t-2}\ -9397.000[SLP_{w7}]_{t-2}\ +212.100[RH_{w7}]_{t-2}\ -379.000[WS_{w7}]_{t-2}\ -0.047[RF]_{t-2}\ \mathbf{+92.950}$

viii. $[RF]= 19340.000[MaxT_{w8}]_{t-1}\ +4477.000[MinT_{w8}]_{t-1}\ +21410.000[SLP_{w8}]_{t-1}\ -8746.000[RH_{w8}]_{t-1}\ +6477.000[WS_{w8}]_{t-1}\ \mathbf{+0.623[RF]_{t-1}}\ -8666.000[MaxT_{w8}]_{t-2}\ +923.000[MinT_{w8}]_{t-2}\ +28060.000[SLP_{w8}]_{t-2}\ +10260.000[RH_{w8}]_{t-2}\ -6854.000[WS_{w8}]_{t-2}\ -0.044[RF]_{t-2}\ \mathbf{+95.570}$

ix. $[RF]= -10130.000[MaxT_{w9}]_{t-1}\ -3048.000[MinT_{w9}]_{t-1}\ -11020.000[SLP_{w9}]_{t-1}\ +12110.000[RH_{w9}]_{t-1}\ -12930.000[WS_{w9}]_{t-1}\ \mathbf{+0.627[RF]_{t-1}}\ -4395.000[MaxT_{w9}]_{t-2}\ -3894[MinT_{w9}]_{t-2}\ -19910.000[SLP_{w9}]_{t-2}\ -19600.000[RH_{w9}]_{t-2} +27150.000[WS_{w9}]_{t-2}\ -0.050[RF]_{t-2}\ +148.700$

x. $[RF]= -2933.000[MaxT_{w10}]_{t-1}\ +28990.000[MinT_{w10}]_{t-1}\ +395.800[SLP_{w10}]_{t-1}\ -6682.000[RH_{w10}]_{t-1}\ -495.600[WS_{w10}]_{t-1}\ \mathbf{+0.620[RF]_{t-1}}\ +3181.000[MaxT_{w10}]_{t-2}\ \mathbf{-32410.000[MinT_{w10}]_{t-2}}\ -404.400[SLP_{w10}]_{t-2}\ +4972.000[RH_{w10}]_{t-2}\ +5009.000[WS_{w10}]_{t-2}\ -0.039[RF]_{t-2}\ +1687000.000$

3. For GHY

i. $[RF]= \mathbf{38.792[MaxT_{w1}]_{t-1}} -5.556[MinT_{w1}]_{t-1} -11.325[SLP_{w1}]_{t-1} \mathbf{-12.302[RH_{w1}]_{t-1}} +14.282[WS_{w1}]_{t-1} \mathbf{+0.685[RF]_{t-1}} -3.130[MaxT_{w1}]_{t-2} +13.194[MinT_{w1}]_{t-2} -5.263[SLP_{w1}]_{t-2} -6.559[RH_{w1}]_{t-2} +9.600[WS_{w1}]_{t-2} \mathbf{-0.085[RF]_{t-2}} \mathbf{+59.349}$

ii. $[RF]= 4.267[MaxT_{w2}]_{t-1} +5.194[MinT_{w2}]_{t-1} +5.451[SLP_{w2}]_{t-1} +6.754[RH_{w2}]_{t-1} +18.019[WS_{w2}]_{t-1} \mathbf{+0.546[RF]_{t-1}} \mathbf{+37.261[MaxT_{w2}]_{t-2}} \mathbf{-30.292[MinT_{w2}]_{t-2}} +23.809[SLP_{w2}]_{t-2} +5.323[RH_{w2}]_{t-2} +8.286[WS_{w2}]_{t-2} +0.037[RF]_{t-2} \mathbf{+62.283}$

iii. $[RF]= -17.476[MaxT_{w3}]_{t-1} +27.885[MinT_{w3}]_{t-1} +117.058[SLP_{w3}]_{t-1} +8.578[RH_{w3}]_{t-1} -13.446[WS_{w3}]_{t-1} +0.013[RF]_{t-1} +3.059[MaxT_{w3}]_{t-2} \mathbf{-42.238[MinT_{w3}]_{t-2}} -112.147[SLP_{w3}]_{t-2} -9.545[RH_{w3}]_{t-2} +16.768[WS_{w3}]_{t-2} +0.063[RF]_{t-2} \mathbf{+137.456}$

iv. $[RF]= -22.016[MaxT_{w4}]_{t-1} +73.554[MinT_{w4}]_{t-1} -92.738[SLP_{w4}]_{t-1} +6.495[RH]_{t-1} +9.962[WS]_{t-1} \mathbf{+0.535[RF]_{t-1}} -18.413[MaxT]_{t-2} -67.479[MinT]_{t-2} +55.652[SLP]_{t-2} -20.881[RH]_{t-2} -4.476[WS]_{t-2} +0.031[RF]_{t-2} \mathbf{+64.070}$

v. $[RF]= -80.610[MaxT_{w5}]_{t-1} +90.050[MinT_{w5}]_{t-1} -232.100[SLP_{w5}]_{t-1} +0.171[RH_{w5}]_{t-1} +70.950[WS_{w5}]_{t-1} \mathbf{+0.562[RF]_{t-1}} +70.680[MaxT_{w5}]_{t-2} -99.300[MinT_{w5}]_{t-2} +236.600[SLP_{w5}]_{t-2} +3.235[RH_{w5}]_{t-2} -78.610[WS_{w5}]_{t-2} +0.009[RF]_{t-2} \mathbf{+63.780}$

vi. $[RF]= -134.400[MaxT_{w6}]_{t-1} +14.480[MinT_{w6}]_{t-1} -523.800[SLP_{w6}]_{t-1} +3.125[RH_{w6}]_{t-1} -17.230[WS_{w6}]_{t-1} \mathbf{+0.565[RF]_{t-1}} +122.600[MaxT_{w6}]_{t-2} -25.130[MinT_{w6}]_{t-2} +666.000[SLP_{w6}]_{t-2} -6.965[RH_{w6}]_{t-2} +20.870[WS_{w6}]_{t-2} +0.009[RF]_{t-2} \mathbf{+63.720}$

vii. $[RF]= 1464.000[MaxT_{w7}]_{t-1} -37.350[MinT_{w7}]_{t-1} -4935.000[SLP_{w7}]_{t-1} +177.600[RH_{w7}]_{t-1} +348.400[WS_{w7}]_{t-1} \mathbf{+0.565[RF]_{t-1}} -1427.000[MaxT_{w7}]_{t-2} +51.840[MinT_{w7}]_{t-2} +4672.000[SLP_{w7}]_{t-2} -165.400[RH_{w7}]_{t-2} -329.500[WS_{w7}]_{t-2} +0.010[RF]_{t-2} \mathbf{+62.790}$

viii. $[RF]= 6074.000[MaxT_{w8}]_{t-1} -5855.000[MinT_{w8}]_{t-1} -9348.000[SLP_{w8}]_{t-1} +1532.000[RH_{w8}]_{t-1} -108.400[WS_{w8}]_{t-1} \mathbf{+0.566[RF]_{t-1}} -14050.000[MaxT_{w8}]_{t-2} +14090.000[MinT_{w8}]_{t-2} +20760.000[SLP_{w8}]_{t-2} -4074.000[RH_{w8}]_{t-2} +1181.000[WS_{w8}]_{t-2} +0.007[RF]_{t-2} \mathbf{+63.720}$

ix. $[RF]= 18210.000[MaxT_{w9}]_{t-1} +13600.000[MinT_{w9}]_{t-1} -320.800[SLP_{w9}]_{t-1} -1637.000[RH_{w9}]_{t-1} -194.100[WS_{w9}]_{t-1} \mathbf{+0.569[RF]_{t-1}} -5561.000[MaxT_{w9}]_{t-2} -10110.000[MinT_{w9}]_{t-2} +5786.000[SLP_{w9}]_{t-2} +4172.000[RH_{w9}]_{t-2} -5576.000[WS_{w9}]_{t-2} +0.005[RF]_{t-2} +63.930$

x. $[RF]= 1004.000[MaxT_{w10}]_{t-1} +5765.000[MinT_{w10}]_{t-1} -165.900[SLP_{w10}]_{t-1} -7852.000[RH_{w10}]_{t-1} -10070.000[WS_{w10}]_{t-1} \mathbf{+0.563[RF]_{t-1}} -7297.000[MaxT_{w10}]_{t-}$

$_{2}$ +5231.000[MinT$_{w10}$]$_{t-2}$ +425.600[SLP$_{w10}$]$_{t-2}$ +3269.000[RH$_{w10}$]$_{t-2}$ +6640.000[WS$_{w10}$]$_{t-2}$ +0.011[RF]$_{t-2}$ +307800.000

4. For KSH

i. [RF]= **64.217[MaxT$_{w1}$]$_{t-1}$** +6.080[MinT$_{w1}$]$_{t-1}$ **-109.270[SLP$_{w1}$]$_{t-1}$ -23.284[RH$_{w1}$]$_{t-1}$** +7.208[WS$_{w1}$]$_{t-1}$ **+0.703[RF]$_{t-1}$** +9.229[MaxT$_{w1}$]$_{t-2}$ +16.514[MinT$_{w1}$]$_{t-2}$ -89.747[SLP$_{w1}$]$_{t-2}$ -7.939[RH$_{w1}$]$_{t-2}$ -12.368[WS$_{w1}$]$_{t-2}$ **-0.112[RF]$_{t-2}$ +93.600**

ii. [RF]= 27.558[MaxT$_{w2}$]$_{t-1}$ -16.472[MinT$_{w2}$]$_{t-1}$ -5.123[SLP$_{w2}$]$_{t-1}$ +1.847[RH$_{w2}$]$_{t-1}$ +6.283[WS$_{w2}$]$_{t-1}$ **+0.587[RF]$_{t-1}$ +41.751[MaxT$_{w2}$]$_{t-2}$ -31.298[MinT$_{w2}$]$_{t-2}$** -19.786[SLP$_{w2}$]$_{t-2}$ -2.382[RH$_{w2}$]$_{t-2}$ +7.780[WS$_{w2}$]$_{t-2}$ +0.012[RF]$_{t-2}$ **+91.790**

iii. [RF]= -66.978[MaxT$_{w3}$]$_{t-1}$ **+74.107[MinT$_{w3}$]$_{t-1}$** +144.158[SLP$_{w3}$]$_{t-1}$ -9.131[RH$_{w3}$]$_{t-1}$ -9.558[WS$_{w3}$]$_{t-1}$ **+0.164[RF]$_{t-1}$** +32.552[MaxT$_{w3}$]$_{t-2}$ **-84.478[MinT$_{w3}$]$_{t-2}$** -64.010[SLP$_{w3}$]$_{t-2}$ -8.702[RH$_{w3}$]$_{t-2}$ +16.912[WS$_{w3}$]$_{t-2}$ +0.021[RF]$_{t-2}$ **+186.087**

iv. [RF]= -113.261[MaxT$_{w4}$]$_{t-1}$ +120.949[MinT$_{w4}$]$_{t-1}$ -214.917[SLP$_{w4}$]$_{t-1}$ -3.928[RH$_{w4}$]$_{t-1}$ +61.662[WS$_{w4}$]$_{t-1}$ **+0.565[RF]$_{t-1}$** +64.153[MaxT$_{w4}$]$_{t-2}$ -103.246[MinT$_{w4}$]$_{t-2}$ +251.143[SLP$_{w4}$]$_{t-2}$ -17.074[RH$_{w4}$]$_{t-2}$ -41.590[WS$_{w4}$]$_{t-2}$ -0.017[RF]$_{t-2}$ **+102.688**

v. [RF]= -148.686[MaxT$_{w5}$]$_{t-1}$ +115.031[MinT$_{w5}$]$_{t-1}$ +799.601[SLP$_{w5}$]$_{t-1}$ -63.755[RH$_{w5}$]$_{t-1}$ -131.134[WS$_{w5}$]$_{t-1}$ **+0.591[RF]$_{t-1}$** +139.169[MaxT$_{w5}$]$_{t-2}$ -111.575[MinT$_{w5}$]$_{t-2}$ -681.022[SLP$_{w5}$]$_{t-2}$ +60.552[RH$_{w5}$]$_{t-2}$ +142.232[WS$_{w5}$]$_{t-2}$ -0.032[RF]$_{t-2}$ **+100.732**

vi. [RF]= -679.527[MaxT$_{w6}$]$_{t-1}$ +241.404[MinT$_{w6}$]$_{t-1}$ +106.020[SLP$_{w6}$]$_{t-1}$ -26.088[RH$_{w6}$]$_{t-1}$ +269.240[WS$_{w6}$]$_{t-1}$ **+0.593[RF]$_{t-1}$** +643.091[MaxT$_{w6}$]$_{t-2}$ -242.658[MinT$_{w6}$]$_{t-2}$ +293.921[SLP$_{w6}$]$_{t-2}$ +30.468[RH$_{w6}$]$_{t-2}$ -255.575[WS$_{w6}$]$_{t-2}$ -0.032[RF]$_{t-2}$ **+100.072**

vii. [RF]= 5569.000[MaxT$_{w7}$]$_{t-1}$ +1033.000[MinT$_{w7}$]$_{t-1}$ **-27350.000[SLP$_{w7}$]$_{t-1}$** -1.375[RH$_{w7}$]$_{t-1}$ +1737.000[WS$_{w7}$]$_{t-1}$ **+0.597[RF]$_{t-1}$** -5280.000[MaxT$_{w7}$]$_{t-2}$ -997.500[MinT$_{w7}$]$_{t-2}$ **+25200.000[SLP$_{w7}$]$_{t-2}$** -14.800[RH$_{w7}$]$_{t-2}$ -1687.000[WS$_{w7}$]$_{t-2}$ -0.040[RF]$_{t-2}$ **+102.800**

viii. [RF]= -95.900[MaxT$_{w8}$]$_{t-1}$ -6208.000[MinT$_{w8}$]$_{t-1}$ -5543.000[SLP$_{w8}$]$_{t-1}$ +1058.000[RH$_{w8}$]$_{t-1}$ -+2781.000[WS$_{w8}$]$_{t-1}$ **+0.591[RF]$_{t-1}$** +4316.000[MaxT$_{w8}$]$_{t-2}$ +6110.000[MinT$_{w8}$]$_{t-2}$ -2120.000[SLP$_{w8}$]$_{t-2}$ -646.000[RH$_{w8}$]$_{t-2}$ -3204.000[WS$_{w8}$]$_{t-2}$ -0.038[RF]$_{t-2}$ **+112.200**

ix. [RF]= -41450.000[MaxT$_{w9}$]$_{t-1}$ -12360.000[MinT$_{w9}$]$_{t-1}$ -12510.000[SLP$_{w9}$]$_{t-1}$ -3993.000[RH$_{w9}$]$_{t-1}$ -19090.000[WS$_{w9}$]$_{t-1}$ **+0.603[RF]$_{t-1}$** +14330.000[MaxT$_{w9}$]$_{t-2}$

$+11320.000[MinT_{w9}]_{t-2}$ $+136.100[SLP_{w9}]_{t-2}$ $-11.130[RH_{w9}]_{t-2}$ $+18240.000[WS_{w9}]_{t-2}$ $-0.044[RF]_{t-2}$ $+130.700$

x. $[RF]=$ $-6403.000[MaxT_{w10}]_{t-1}$ $+2411.000[MinT_{w10}]_{t-1}$ $+96.170[SLP_{w10}]_{t-1}$ $-3991.000[RH_{w10}]_{t-1}$ $+2720.000[WS_{w10}]_{t-1}$ $\mathbf{+0.593[RF]_{t-1}}$ $-8489.000[MaxT_{w10}]_{t-2}$ $-2541.000[MinT_{w10}]_{t-2}$ $-63.310[SLP_{w10}]_{t-2}$ $+2258.000[RH_{w10}]_{t-2}$ $-7801.000[WS_{w10}]_{t-2}$ $-0.034[RF]_{t-2}$ $+654600.000$

5. For TUL

i. $[RF]=$ $\mathbf{33.081[MaxT_{w1}]_{t-1}}$ $\mathbf{-31.127[MinT_{w1}]_{t-1}}$ $+12.221[SLP_{w1}]_{t-1}$ $\mathbf{-9.063[RH_{w1}]_{t-1}}$ $+5.410[WS_{w1}]_{t-1}$ $\mathbf{+0.586[RF]_{t-1}}$ $+1.884[MaxT_{w1}]_{t-2}$ $-7.329[MinT_{w1}]_{t-2}$ $+13.247[SLP_{w1}]_{t-2}$ $\mathbf{-5.969[RH_{w1}]_{t-2}}$ $-9.835[WS_{w1}]_{t-2}$ $-0.012[RF]_{t-2}$ $\mathbf{+51.875}$

ii. $[RF]=$ $20.899[MaxT_{w2}]_{t-1}$ $-4.856[MinT_{w2}]_{t-1}$ $-15.305[SLP_{w2}]_{t-1}$ $\mathbf{+8.650RH_{w2}]_{t-1}}$ $-1.303[WS_{w2}]_{t-1}$ $\mathbf{+0.436[RF]_{t-1}}$ $\mathbf{+24.924[MaxT_{w2}]_{t-2}}$ $\mathbf{-23.857[MinT_{w2}]_{t-2}}$ $-4.052[SLP_{w2}]_{t-2}$ $+3.611[RH_{w2}]_{t-2}$ $-6.744[WS_{w2}]_{t-2}$ $\mathbf{+0.101[RF]_{t-2}}$ $\mathbf{+56.595}$

iii. $[RF]=$ $\mathbf{-30.451[MaxT_{w3}]_{t-1}}$ $-4.518[MinT_{w3}]_{t-1}$ $-18.353[SLP_{w3}]_{t-1}$ $\mathbf{+8.741[RH_{w3}]_{t-1}}$ $+7.377[WS_{w3}]_{t-1}$ $+0.050[RF]_{t-1}$ $+14.622[MaxT_{w3}]_{t-2}$ $-1.386[MinT_{w3}]_{t-2}$ $+7.690[SLP_{w3}]_{t-2}$ $\mathbf{-10.481[RH_{w3}]_{t-2}}$ $-4.775[WS_{w3}]_{t-2}$ $\mathbf{+0.102[RF]_{t-2}}$ $\mathbf{+103.351}$

iv. $[RF]=$ $7.554[MaxT_{w4}]_{t-1}$ $+62.285[MinT_{w4}]_{t-1}$ $-94.262[SLP_{w4}]_{t-1}$ $+6.038[RH_{w4}]_{t-1}$ $+18.155[WS_{w4}]_{t-1}$ $\mathbf{+0.418.000[RF]_{t-1}}$ $-44.061[MaxT_{w4}]_{t-2}$ $-56.492[MinT_{w4}]_{t-2}$ $+52.924[SLP_{w4}]_{t-2}$ $-11.156[RH_{w4}]_{t-2}$ $+9.502[WS_{w4}]_{t-2}$ $\mathbf{+0.093[RF]_{t-2}}$ $\mathbf{+59.319}$

v. $[RF]=$ $-95.880[MaxT_{w5}]_{t-1}$ $+46.392[MinT_{w5}]_{t-1}$ $-151.071[MSLP_{w5}]_{t-1}$ $-6.737[RH_{w5}]_{t-1}$ $+212.912[SLP_{w5}]_{t-1}$ $+137.234[WS_{w5}]_{t-1}$ $\mathbf{+0.448[RF]_{t-1}}$ $+100.295[MaxT_{w5}]_{t-2}$ $-52.599[MinT_{w5}]_{t-2}$ $+187.564[MSLP_{w5}]_{t-2}$ $+9.551[RH_{w5}]_{t-2}$ $-248.971[SLP_{w5}]_{t-2}$ $-124.357[WS_{w5}]_{t-2}$ $+0.078[RF]_{t-2}$ $\mathbf{+57.723}$

vi. $[RF]=$ $-316.86[MaxT_{w6}]_{t-1}$ $-205.747[MinT_{w6}]_{t-1}$ $+188.81[SLP_{w6}]_{t-1}$ $+56.996[RH_{w6}]_{t-1}$ $+41.744[WS_{w6}]_{t-1}$ $\mathbf{+0.448[RF]_{t-1}}$ $+293.766[MaxT_{w6}]_{t-2}$ $+198.399[MinT_{w6}]_{t-2}$ $-175.300[SLP_{w6}]_{t-2}$ $-56.995[RH_{w6}]_{t-2}$ $-45.937[WS_{w6}]_{t-2}$ $+0.078[RF]_{t-2}$ $\mathbf{+57.533}$

vii. $[RF]=$ $2516.000[MaxT_{w7}]_{t-1}$ $+1017.000[MinT_{w7}]_{t-1}$ $-788.200[SLP_{w7}]_{t-1}$ $-136.800[RH_{w7}]_{t-1}$ $+319.200[WS_{w7}]_{t-1}$ $\mathbf{+0.444[RF]_{t-1}}$ $-2324.000[MaxT_{w7}]_{t-2}$ $-980.500[MinT_{w7}]_{t-2}$ $+380.700[SLP_{w7}]_{t-2}$ $+136.100[RH_{w7}]_{t-2}$ $-274.900[WS_{w7}]_{t-2}$ $+0.074[RF]_{t-2}$ $\mathbf{+59.470}$

viii. $[RF]=$ $-10130[MaxT_{w8}]_{t-1}$ $-6679.000[MinT_{w8}]_{t-1}$ $+6378.000[SLP_{w8}]_{t-1}$ $+1476.000[RH_{w8}]_{t-1}$ $-628.700[WS_{w8}]_{t-1}$ $\mathbf{+0.439[RF]_{t-1}}$ $-5118.000[MaxT_{w8}]_{t-2}$ -

372.100$[MinT_{w8}]_{t-2}$ +2131.000$[SLP_{w8}]_{t-2}$ -713.700$[RH_{w8}]_{t-2}$ +1941.000$[WS_{w8}]_{t-2}$ +0.081$[RF]_{t-2}$ **+57.230**

ix. [RF]= 1029.000$[MaxT_{w9}]_{t-1}$ +1309.000$[MinT_{w9}]_{t-1}$ -1862.000$[SLP_{w9}]_{t-1}$ +532.800$[RH_{w9}]_{t-1}$ -8265.000$[WS_{w9}]_{t-1}$ **+0.450$[RF]_{t-1}$** -3292.000$[MaxT_{w9}]_{t-2}$ -5979.000$[MinT_{w9}]_{t-2}$ +3023.000$[SLP_{w9}]_{t-2}$ +505.400$[RH_{w9}]_{t-2}$ +7120.000$[WS_{w9}]_{t-2}$ +0.073$[RF]_{t-2}$ +66.310

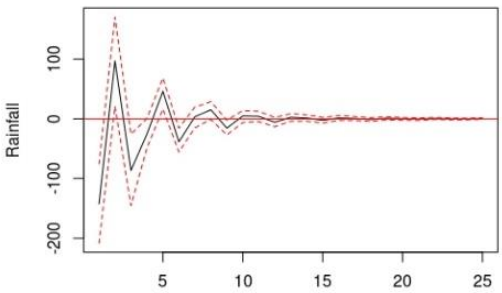
x. [RF]= -5484.000$[MaxT_{w10}]_{t-1}$ -1181.000$[MinT_{w10}]_{t-1}$ +107.000$[SLP_{w10}]_{t-1}$ -3927.000$[RH_{w10}]_{t-1}$ +6806.000$[WS_{w10}]_{t-1}$ **+0.451$[RF]_{t-1}$** +4651.000$[MaxT_{w10}]_{t-2}$ +1475.000$[MinT_{w10}]_{t-2}$ -306.700$[SLP_{w10}]_{t-2}$ +3766.000$[RH_{w10}]_{t-2}$ -7604.000$[WS_{w10}]_{t-2}$ +0.081$[RF]_{t-2}$ +365400.000

7.3.2 IRF approach

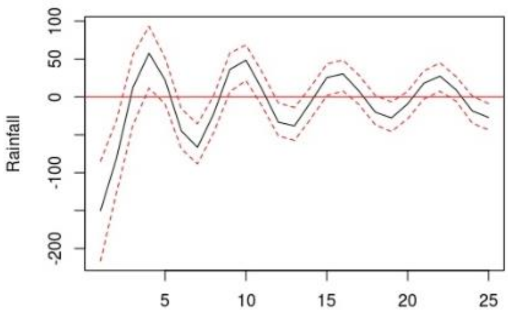
The Responses of rainfall as impulse towards shocks of selected meteorological variables (wavelet decomposed series w_1-w_{10}) at different locations of NER are described with the help of the IRFs location wise in this section. The IRFs were plotted for lags up to 25th months. In each plot the response of rainfall to a shock is represented by the solid black line and the confidence interval (95% significance level) is plotted in red dotted lines. The magnitude of response is represented in the vertical axis, while the horizontal axis shows the duration after the initial shock (lag) in months.

At CHR

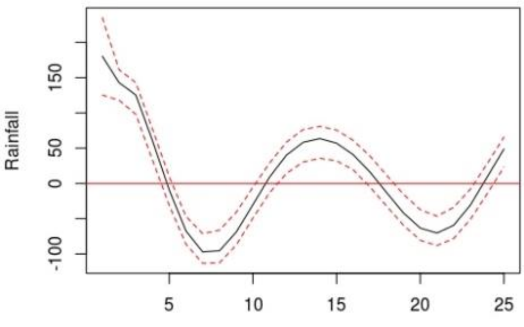
As evident in Figure 7.2, varied response of rainfall was observed upon shocks in different resolutions of MaxT (w1-w10). A shock in the MaxT(w1-w2) led to initial increase in rainfall from its present value and after attaining peak after certain months (~2 months in w1 and 4 months in w2 respectively), the response again starts decreasing and attained negative minimum. Rainfall responded in a cyclical manner afterwards, upon further shocks given to these two series and finally the effect settles after the 12th months in case of w1, while this cyclical mode continued on to an indefinite period in case of w2. The IRF exhibited fluctuating patterns up to w4 of MaxT, however, these fluctuations seemed to low down from w1-w4 and finally subsided in w5. The reason behind the fluctuations was because of the high frequency noise prevailing in the high pass filter series generated as a result of wavelet decomposition (see last part of the section 6.2.2 in chapter 6; where the order of frequency in the wavelet decomposed series was w1>w10). This pattern in the IRFs was constant in case of impulse in w1-w4 of each meteorological variable throughout the studied locations. In can be seen that when a shock is given to w5 of MaxT, rainfall responded positively at the 1st month, after which its positive response decreased. The IRF converged near to zero after six months of receiving the shock in MaxT, became negative afterwards up to 18th months lag. After the 18th months lag, the response in rainfall reached zero, beyond which it started increasing positively, but stayed near zero and stabilised for an indefinite period. The response of rainfall to shock in MaxT was stabilised after this 6-month lag in case of w7 too, while in case of w8 the response was stabilised after 12 months lag. However, in case of w6, rainfall responded negatively at 1 month, after which its negative response started declining and stabilised after 6 months. The w8-w10 series were the low frequency noise containing series (see last part of the section 6.2.2 in chapter 6; where the order of frequency in the wavelet decomposed series was w1>w10), hence the response of rainfall in the IRFs can be regarded as containing biasness (in case of all the variables per site).



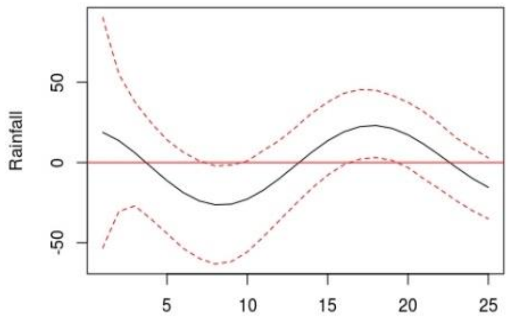
$MaxT_{w1}$



$MaxT_{w2}$



$MaxT_{w3}$



$MaxT_{w4}$

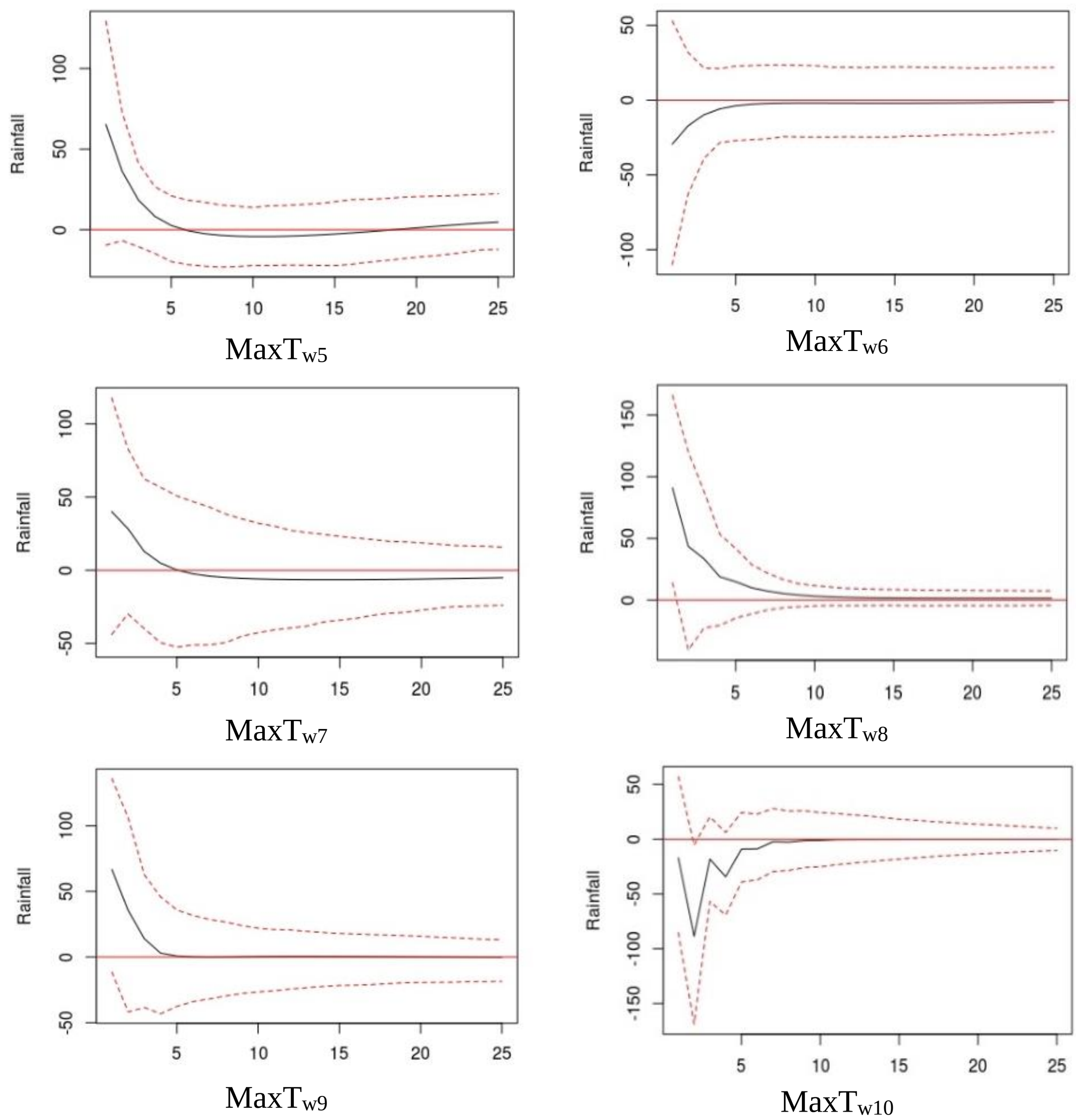
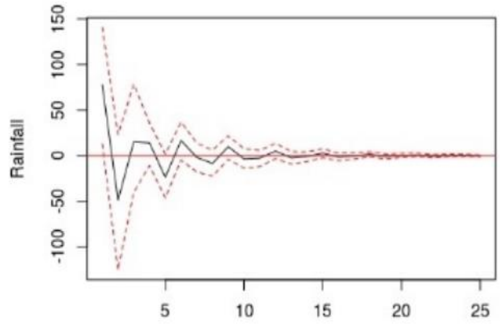
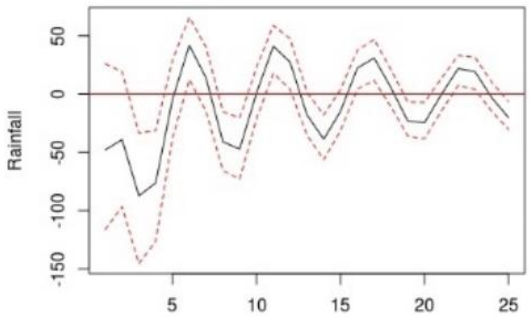


Figure 7. 2 Response of rainfall towards MaxT (w1-w10) at CHR

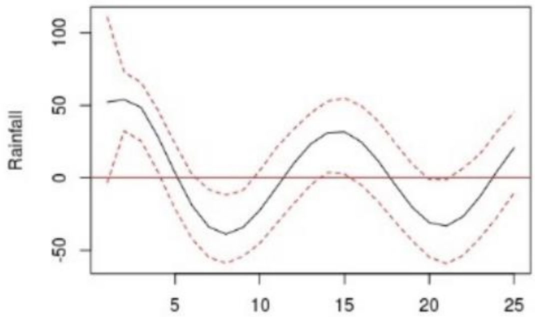
Response structure of rainfall to shocks in all the MinT (Figure 7.3) was like that of MaxT in case of w1-w4. The immediate response by rainfall at the shock was positive in case of w5, which rapidly declined and reached zero after six months. This response started increasing positively after 12 months, however it became stabilised afterwards soon. In case of w6 and w7 however, the initial response of rainfall was negative, which stabilised at zero after six months same as that in case of w5.



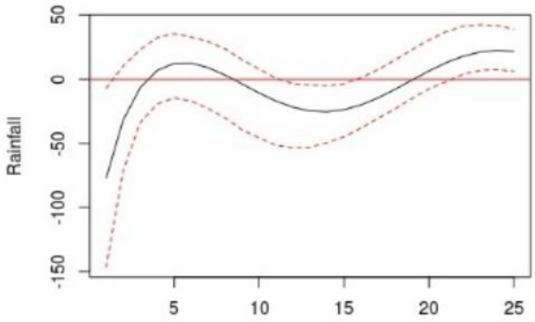
MinT$_{w1}$



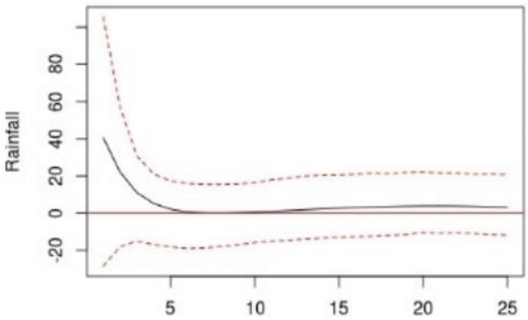
MinT$_{w2}$



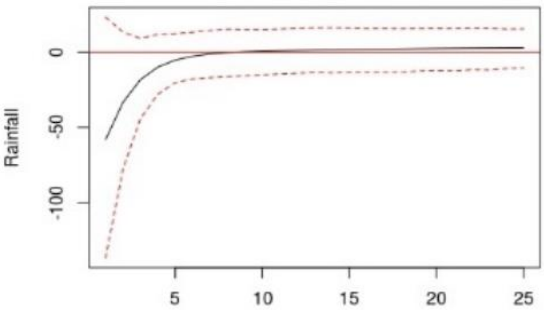
MinT$_{w3}$



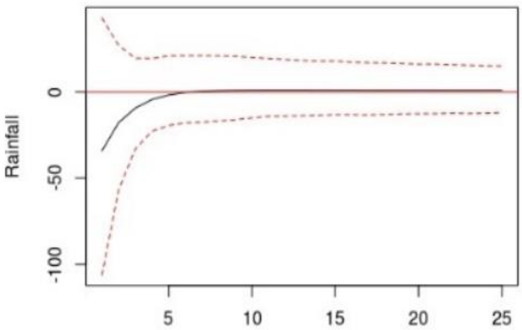
MinT$_{w4}$



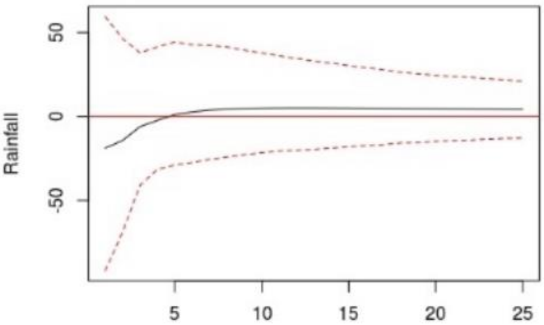
MinT$_{w5}$



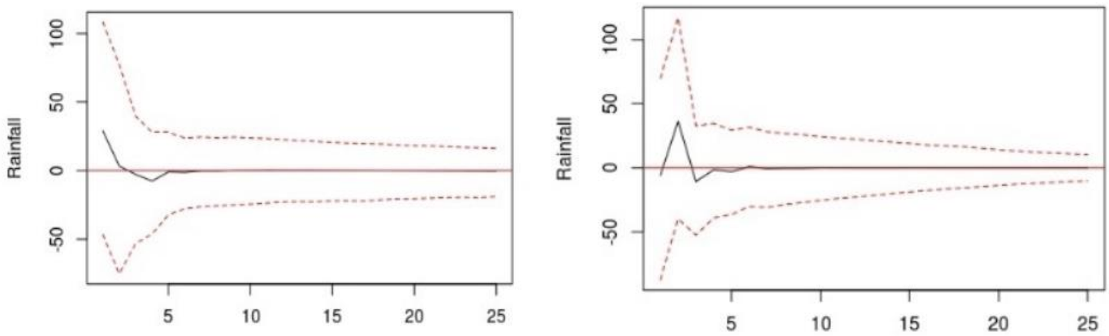
MinT$_{w6}$



MinT$_{w7}$



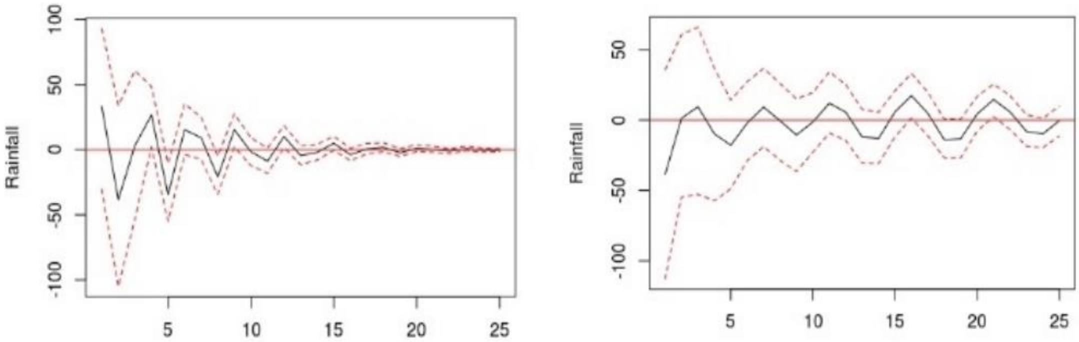
MinT$_{w8}$



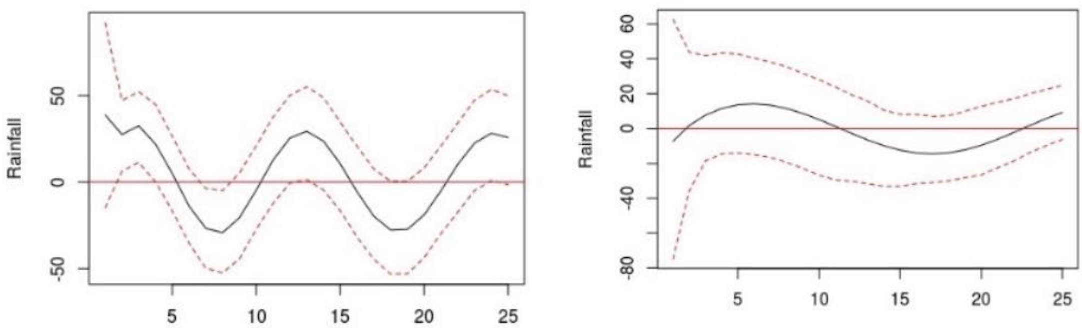
MinT$_{w9}$ MinT$_{w10}$

Figure 7. 3Response of rainfall towards MinT (w1-w10) at CHR

In case of RH, the response of rainfall (Figure 7.4) was initially positive, but it slowly declined and attained zero at 10 months lag, after which the response continued declining in the same slow way along the negative vertical axis and was stabilised at indefinite period. In case of w6, the initial response of rainfall was negative; however, the negative response started declining and after the 20th months the response died out as zero. In w7, the initial negative response of rainfall became zero after four months.



RH$_{w1}$ RH$_{w2}$



RH$_{w3}$ RH$_{w4}$

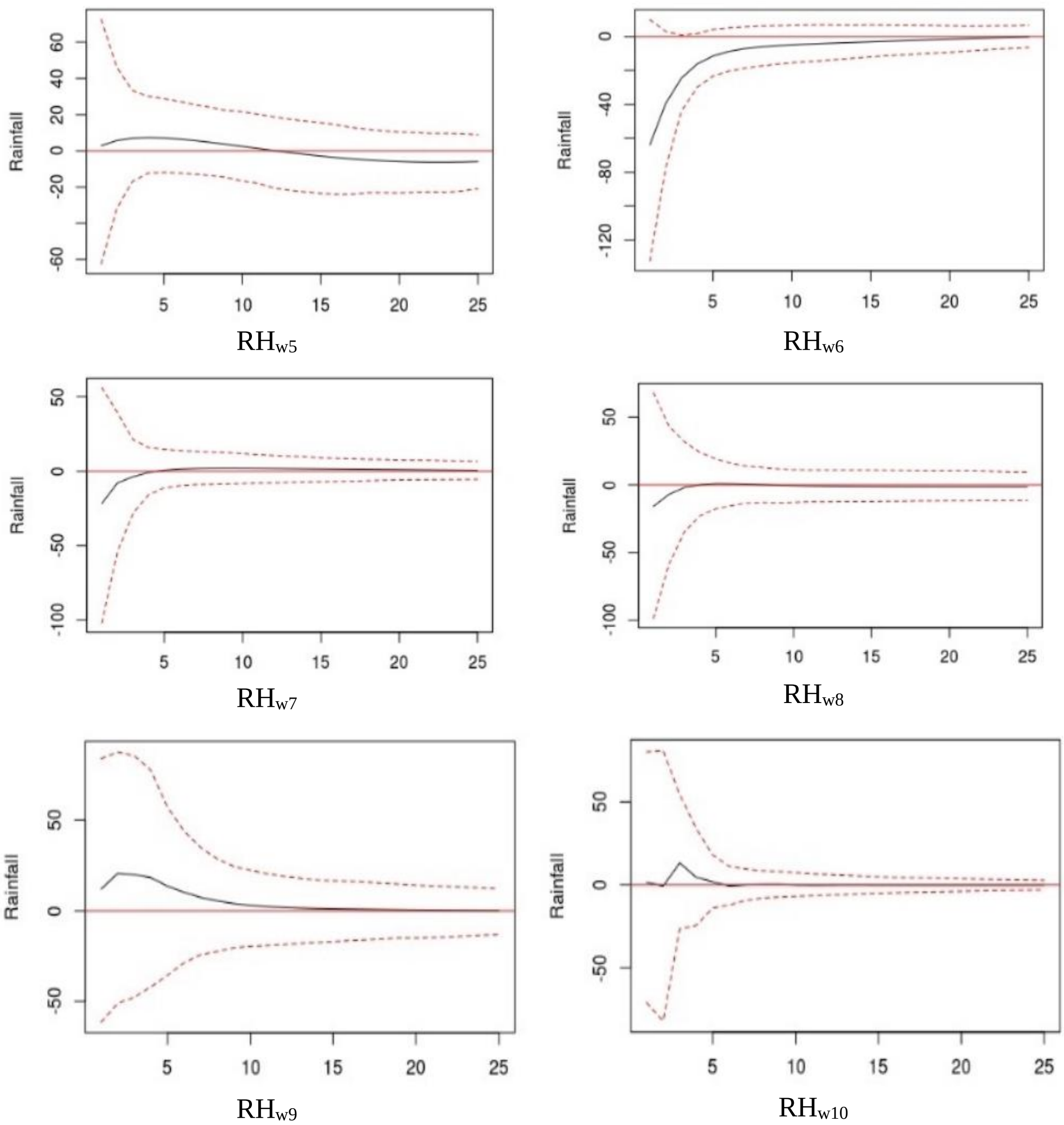
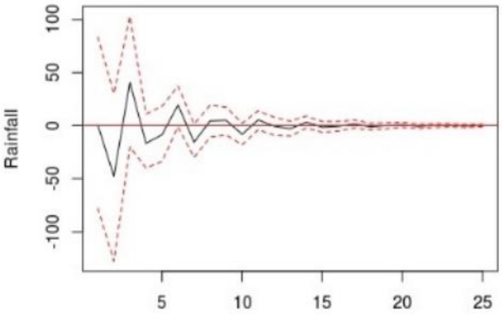
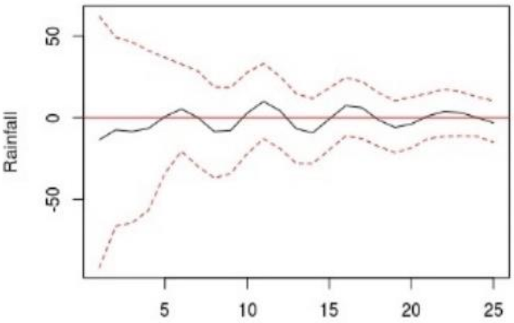


Figure 7. 4 Response of rainfall towards RH(w1-w10) at CHR

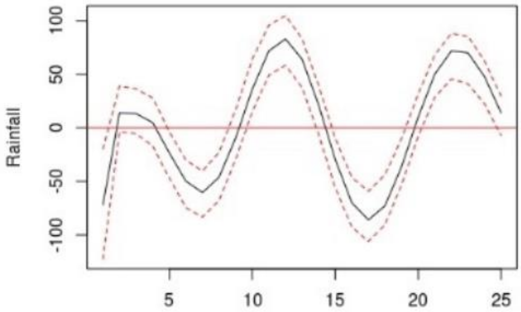
In case of SLP (Figure 7.5), the response of RF was positive to the initial shock in w5-w6, and this response became zero at 6 and 9 months respectively. In case of w5 the response further on stayed negative to the 18^{th} months lag, after which a slight positive increase was observed. However, the increasing positive response stabilised near zero same as in w5 of MaxT. In case of w7, the initial response was negative, and it reached its negative maximum at 2.4/2.5 months, after which the response shifted toward the positive vertical axis and attained zero after 12 months.



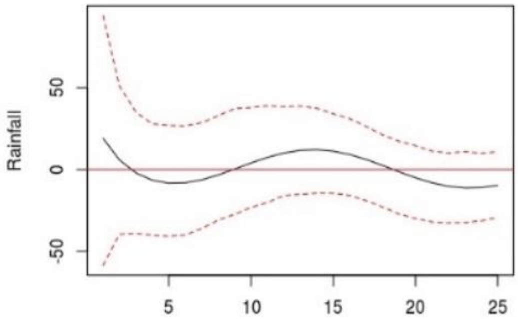
SLP_{w1}



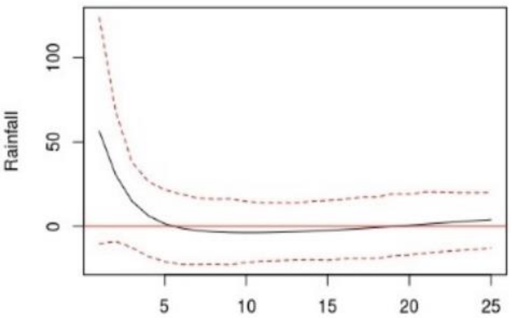
SLP_{w2}



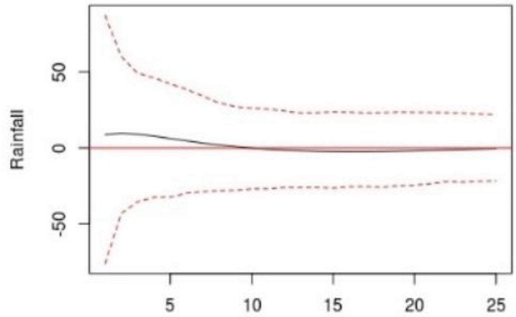
SLPw3



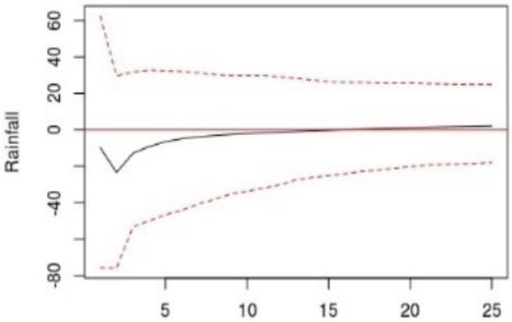
SLP_{w4}



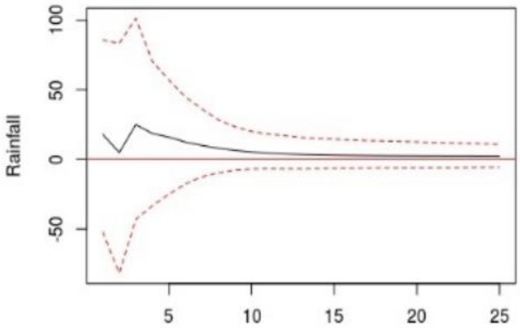
SLP_{w5}



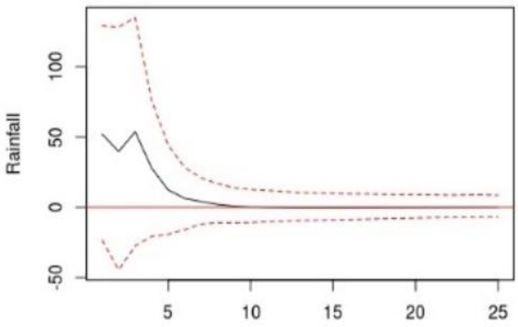
SLP_{w6}



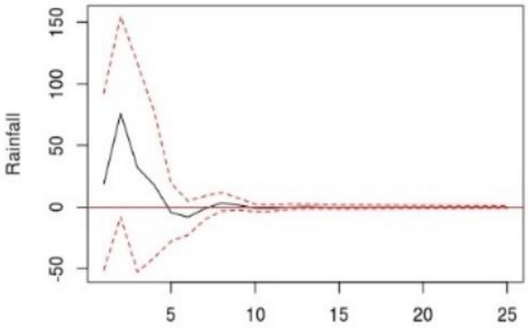
SLP_{w7}



SLP_{w8}



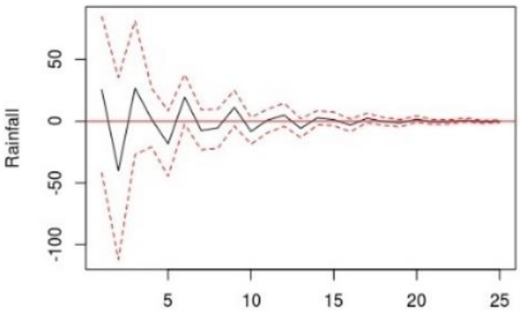
SLP_{w9}



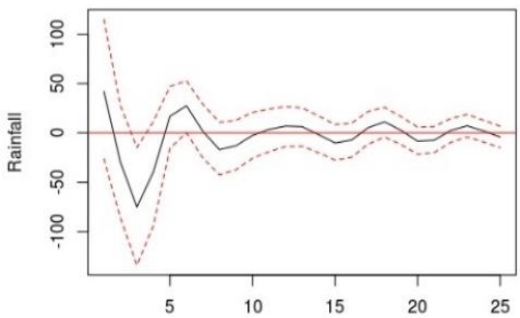
SLP_{w10}

Figure 7. 5 Response of rainfall towards SLP (w1-w10) at CHR

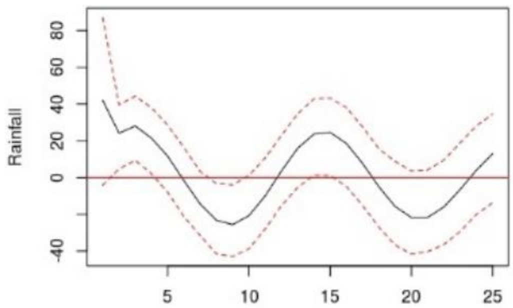
In case of WS (Figure 7.6), the response of rainfall to w5-w6 was positive at the 1st months lag, after which it eventually decreased and became stable after 5th-6th months in case of w6. On the other hand, the response of rainfall became negative after the 18th months lag and stabilised soon after. In case of w7 however, rainfall responded negatively after receiving the initial shock, but the response died quickly within five months.



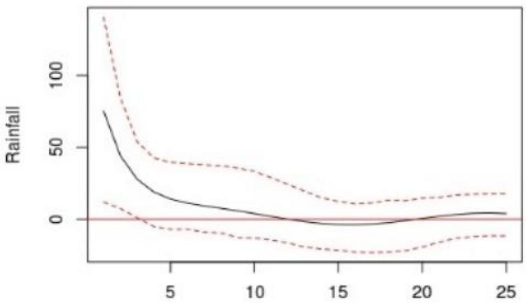
WS_{w1}



WS_{w2}



WS_{w3}



WS_{w4}

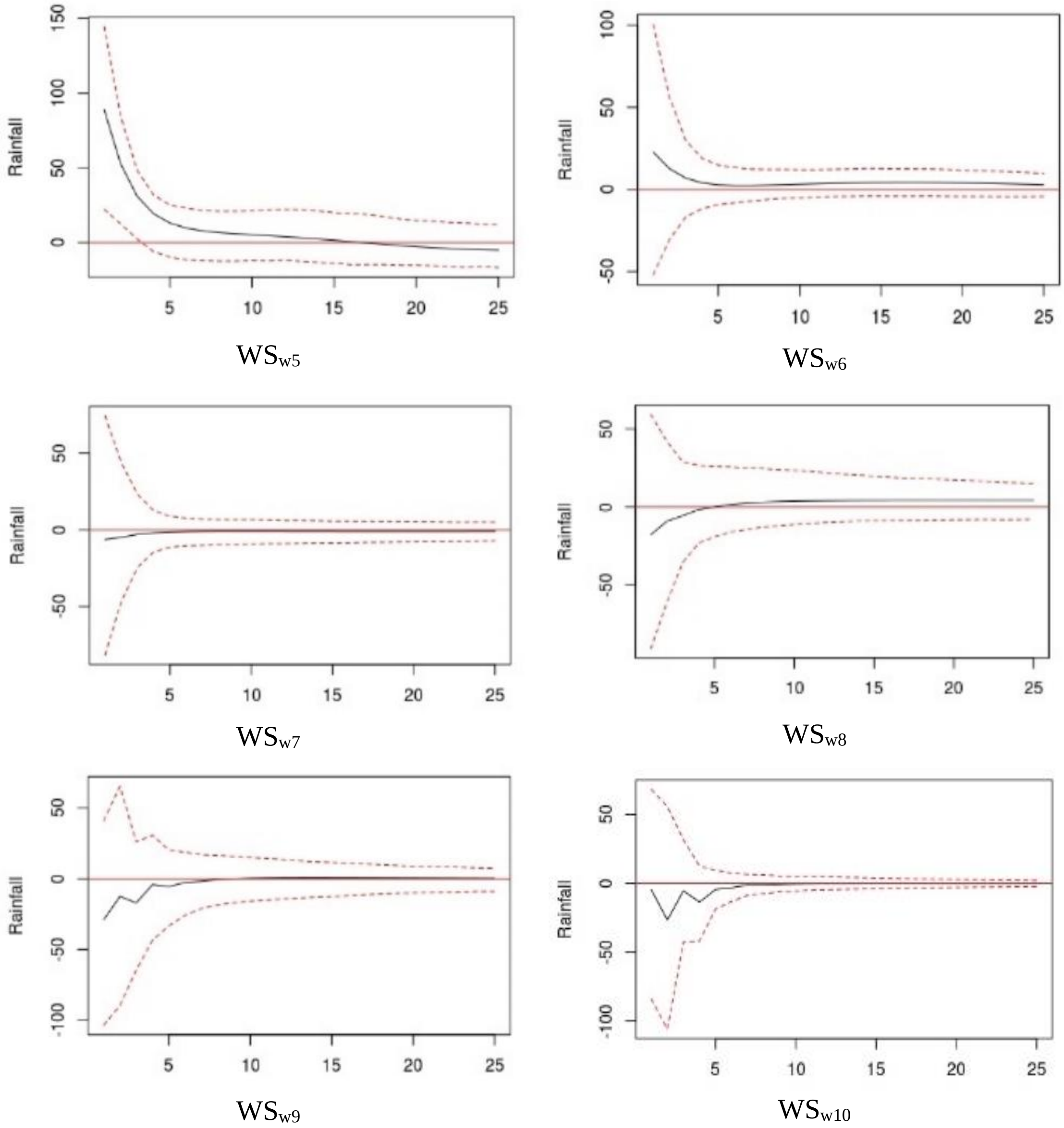
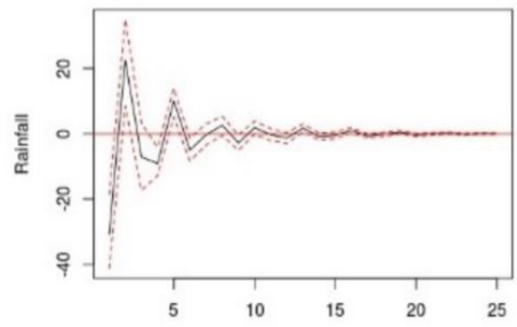


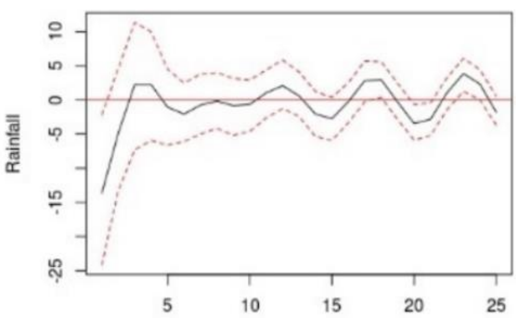
Figure 7. 6 Response of rainfall towards WS (w1-w10) at CHR

At DBR

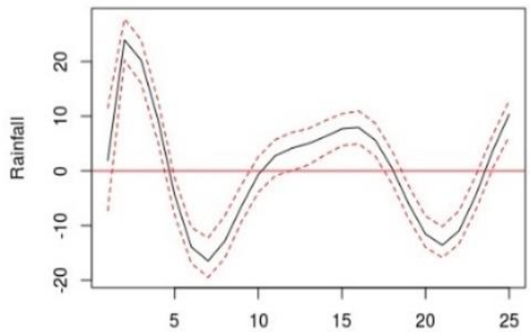
In case of MaxT (Figure 7.7), rainfall responded positively to the initial shocks and this response settled eventually after 5-6 months (w5-w7). The IRFs in case of the other high and low frequency noise containing series were similar to that in CHR.



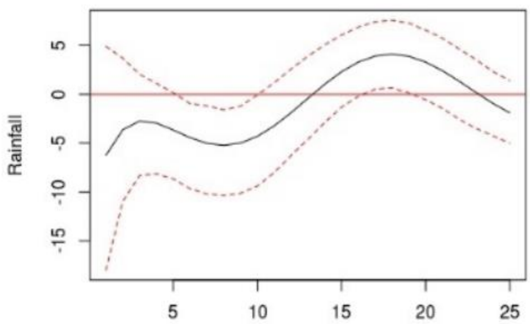
$MaxT_{w1}$



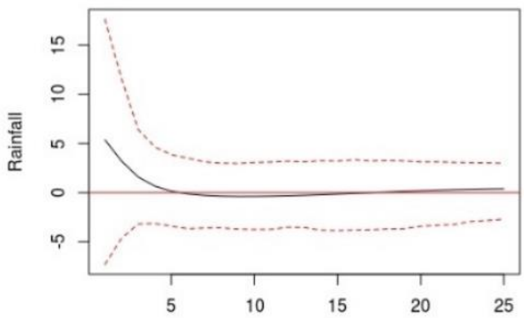
$MaxT_{w2}$



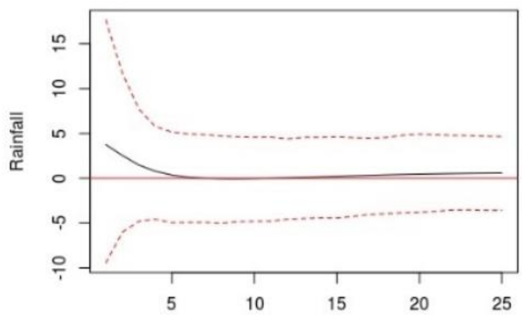
$MaxT_{w3}$



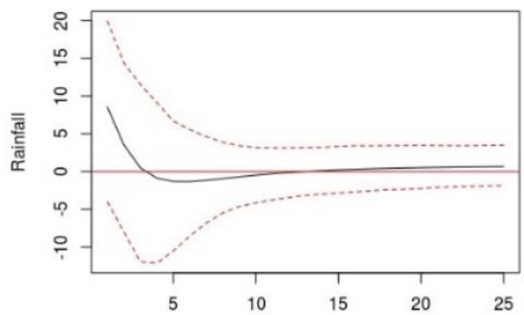
$MaxT_{w4}$



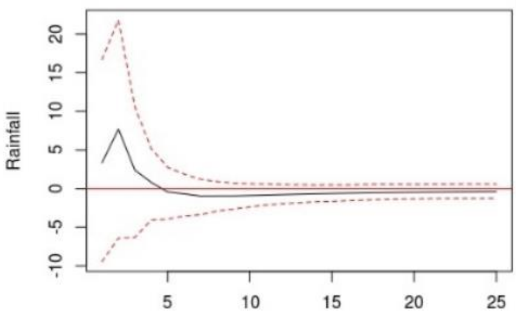
$MaxT_{w5}$



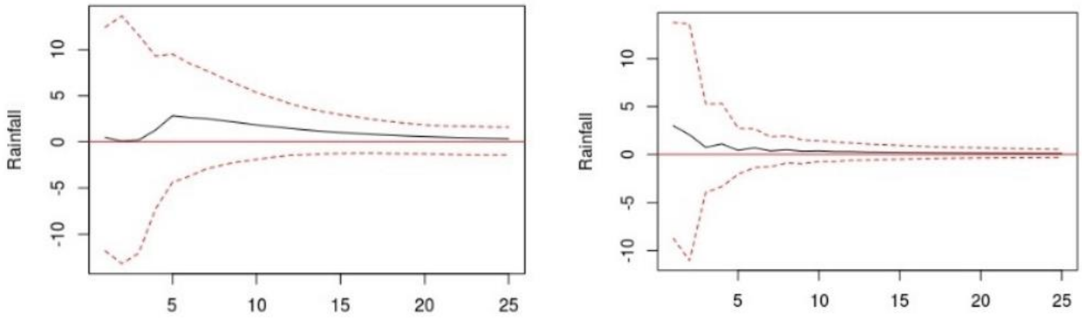
$MaxT_{w6}$



$MaxT_{w7}$



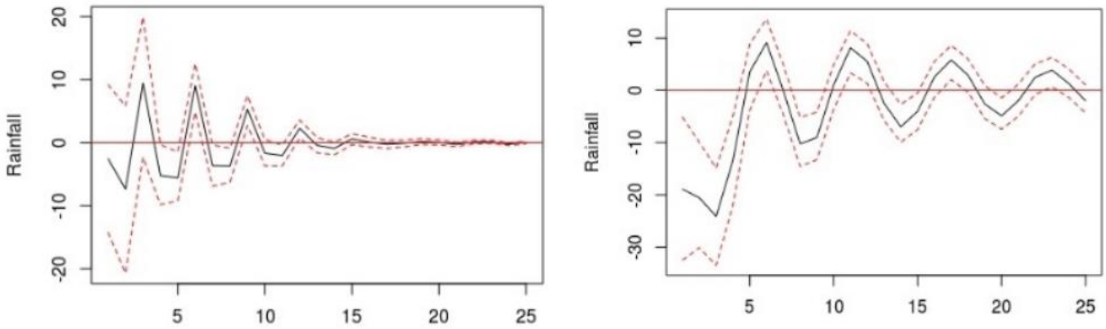
$MaxT_{w8}$



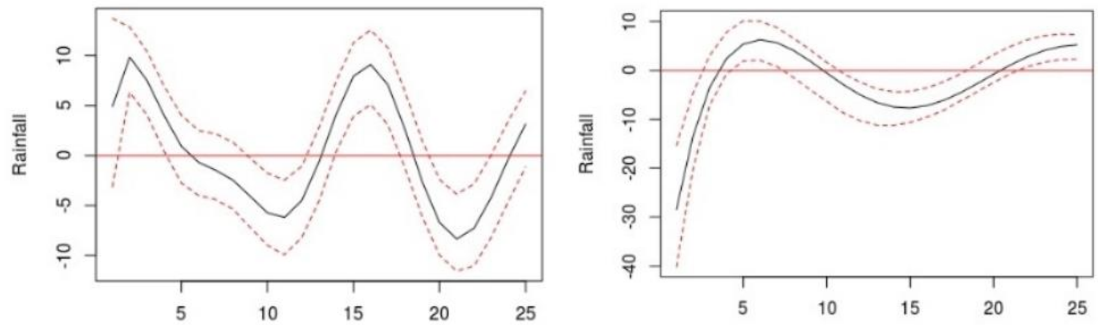
$MaxT_{w9}$ $MaxT_{w10}$

Figure 7. 7 Response of rainfall towards MaxT (w1-w10) at DBR

As depicted in the IRFs for rainfall to MinT (Figure 7.8), the response of rainfall to the initial shock was negative, which eventually declined and diminished after 6th-7th months in case of w6. In case of w5 and w7 the response became stable after 18th and 12th months (in case of w7 the initial response was positive, which declined eventually and became negative after the second lag till 12th lag).



$MinT_{w1}$ $MinT_{w2}$



$MinT_{w3}$ $MinT_{w4}$

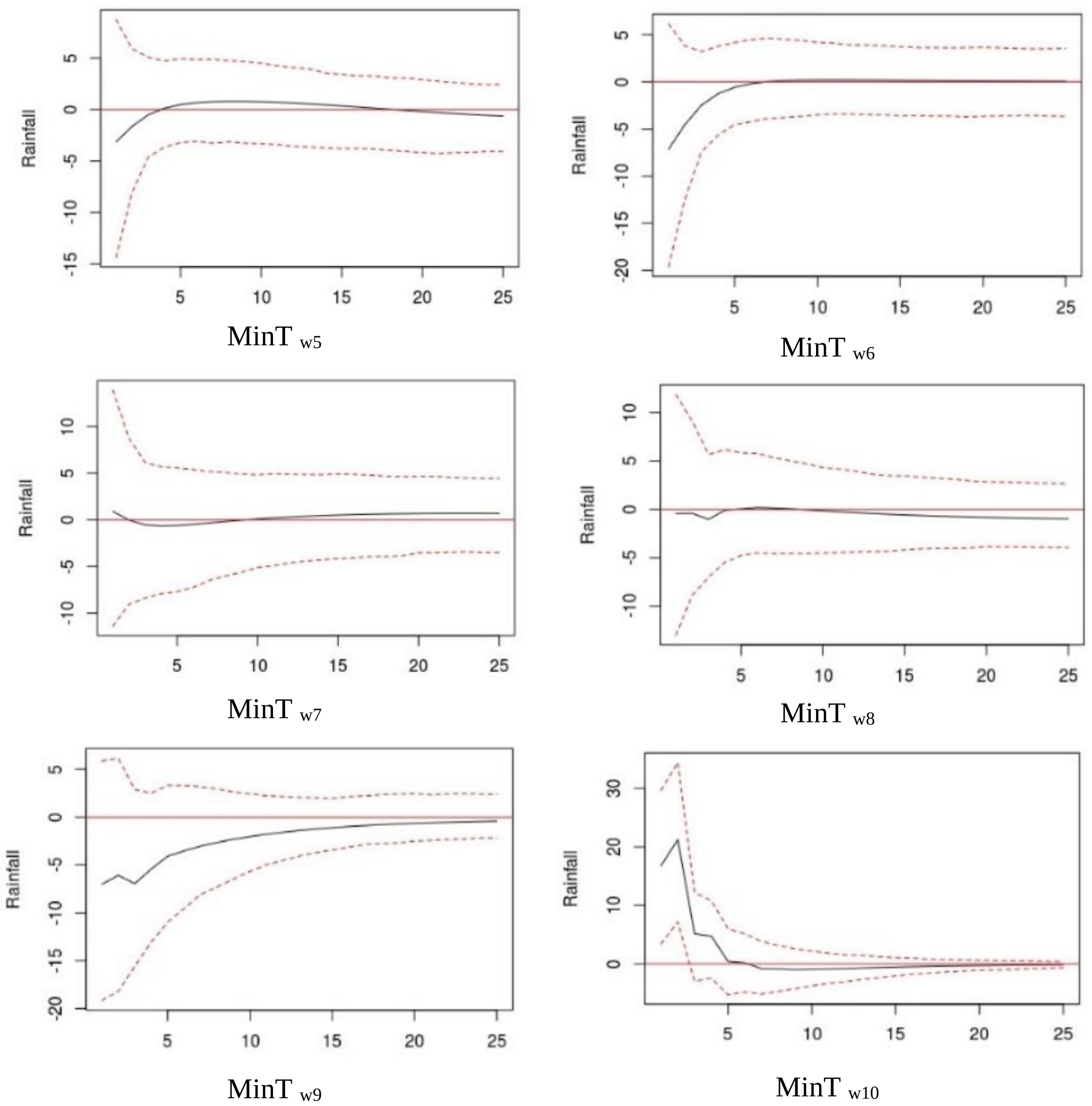
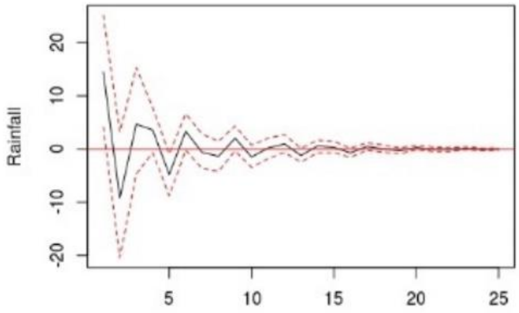
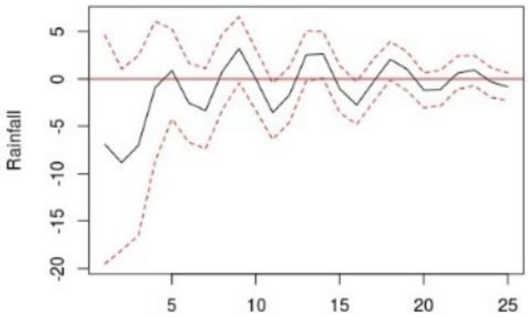


Figure 7. 8 Response of rainfall towards MinT (w1-w10) at DBR

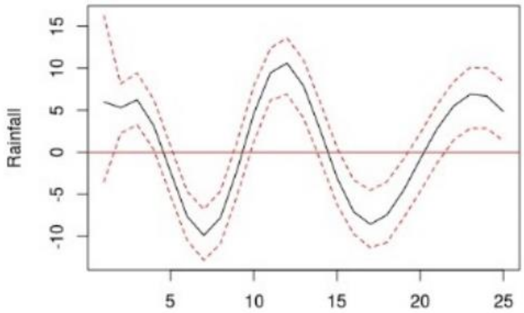
In case of RH (Figure 7.9) the initial response of rainfall to impulse in RH (w1-w10) was negative in w5 and w7, which eventually settled after 5-6th lag. However, rainfall was not found to show any response toward shock in w6 at all.



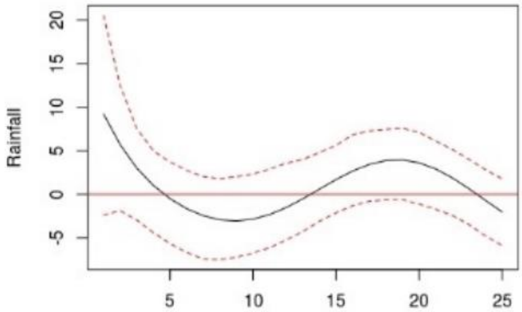
RH_{w1}



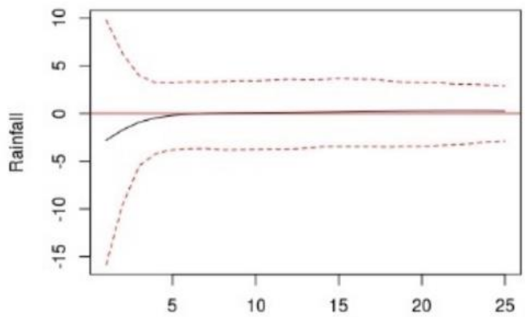
RH_{w2}



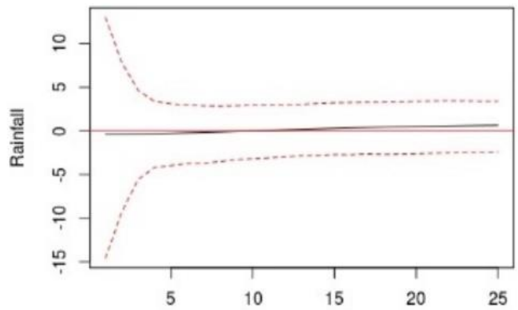
RH_{w3}



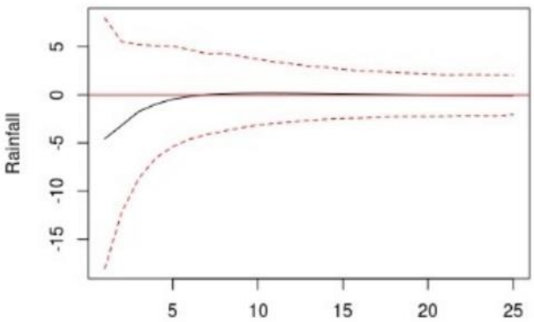
RH_{w4}



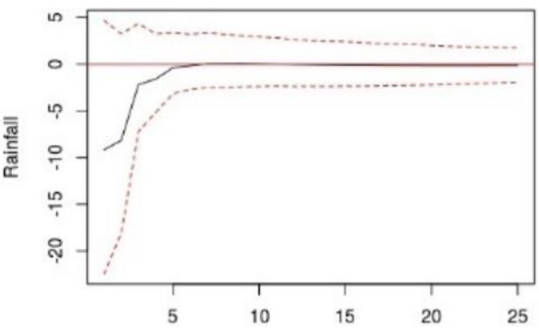
RH_{w5}



RH_{w6}



RH_{w7}



RH_{w8}

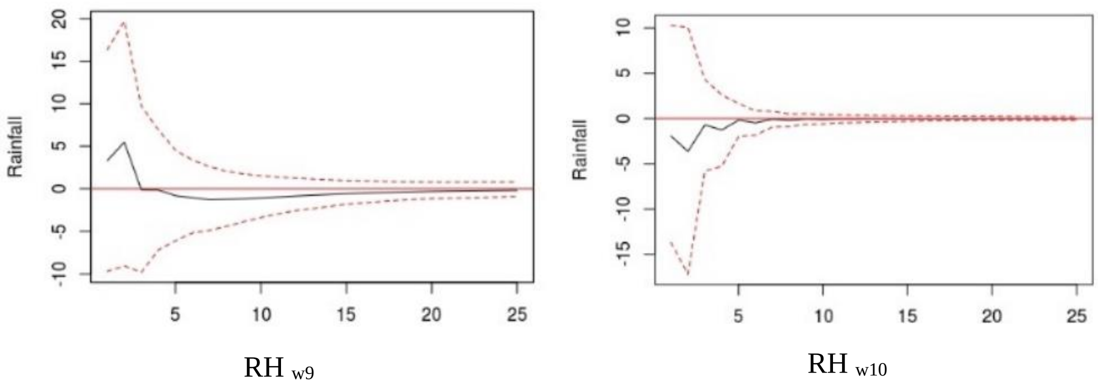
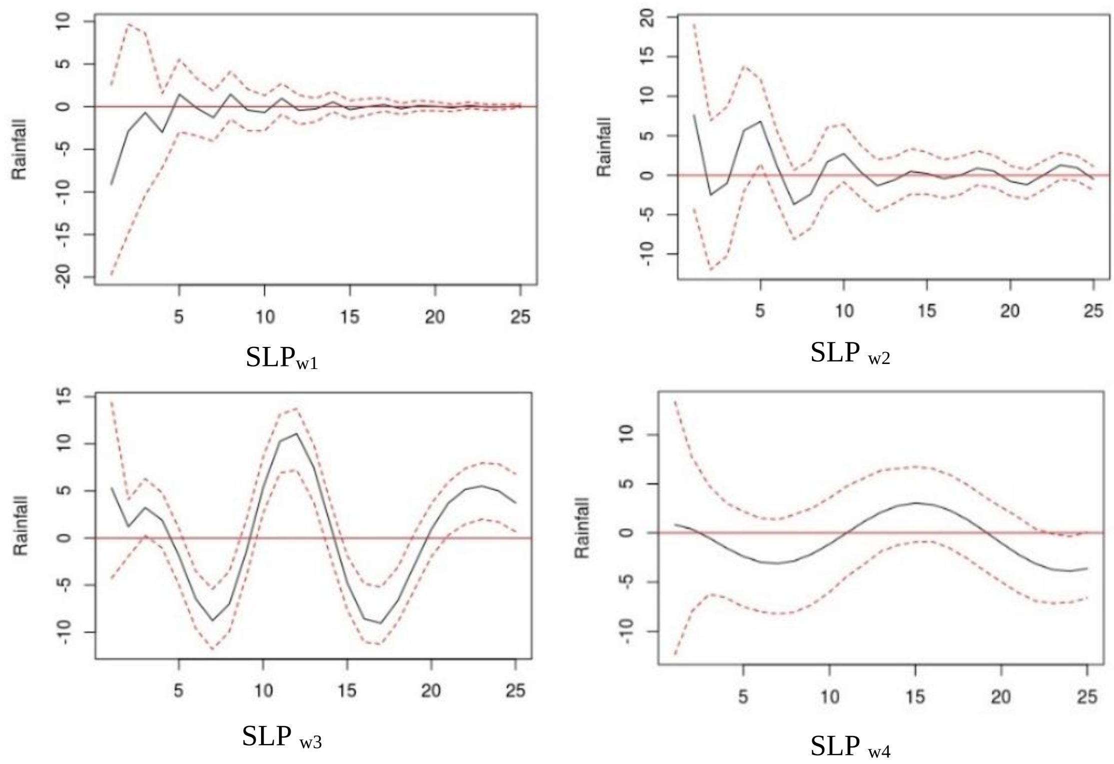


Figure 7. 9Response of rainfall towards RH (w1-w10) at DBR

In case of SLP, the initial response of rainfall (Figure 7.10) was positive (w5, w6 and w8), which decreased and became stable after the 8-9th lag in w5 and w6, whereas the initial positive response reached peak at 3rd lag and again declined to be stable at indefinite time in w8. The same happened in case of w7 also, though the initial response of rainfall was negative.



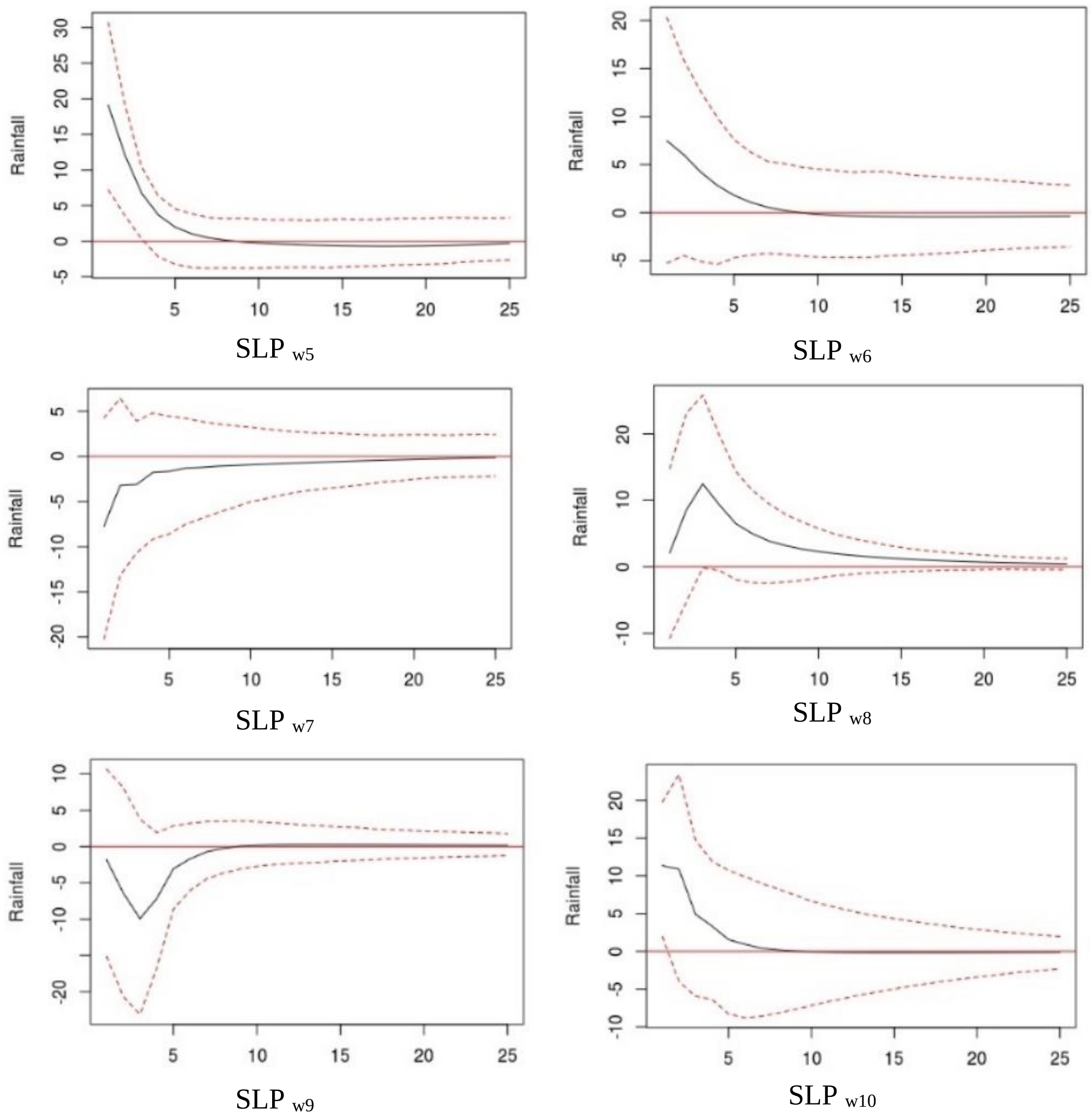
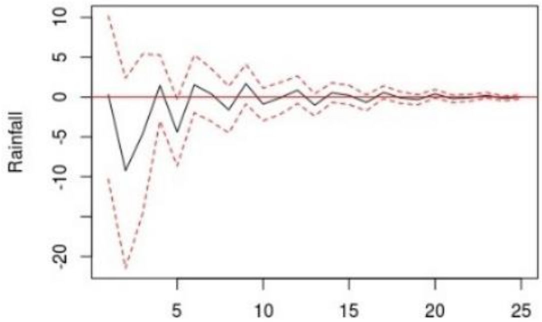
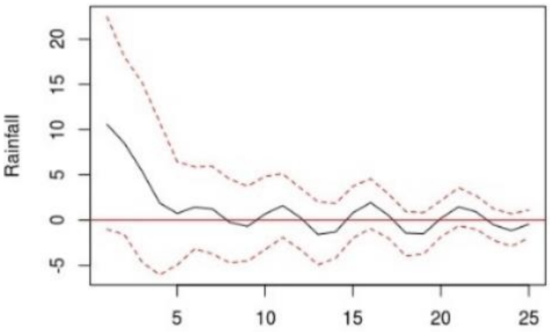


Figure 7. 10 Response of rainfall towards SLP (w1-w10) at DBR

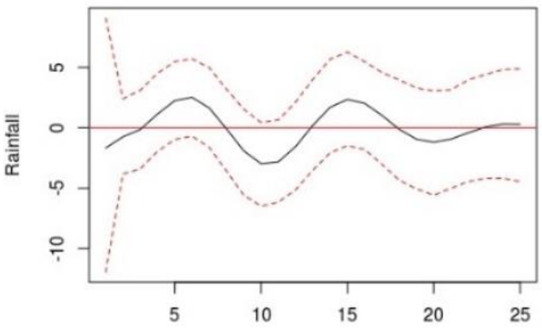
The response of rainfall to impulse in WS (w5-w7) as depicted in Figure 7.11 was positive, which declined and settled after 5-6th and 7-8th lag in w5 and w6-w7 respectively.



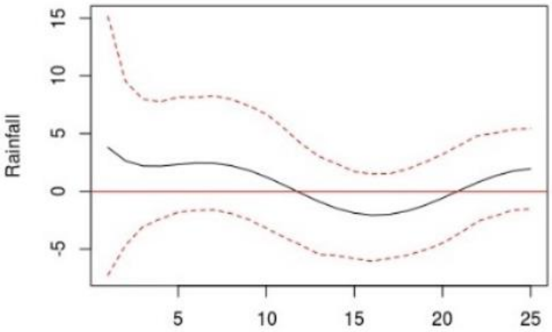
WS_{w1}



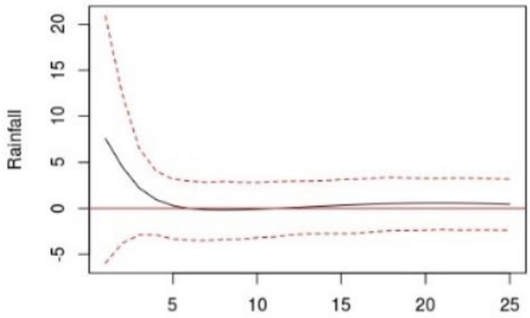
WS_{w2}



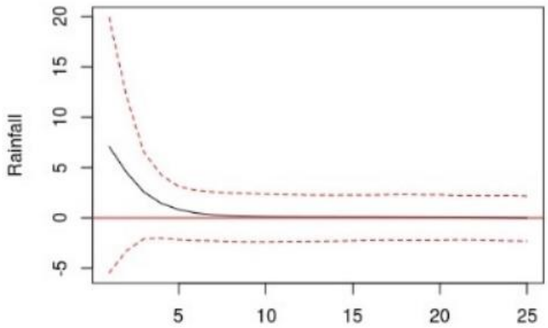
WS_{w3}



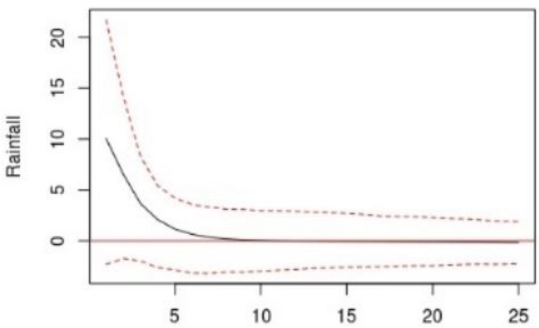
WS_{w4}



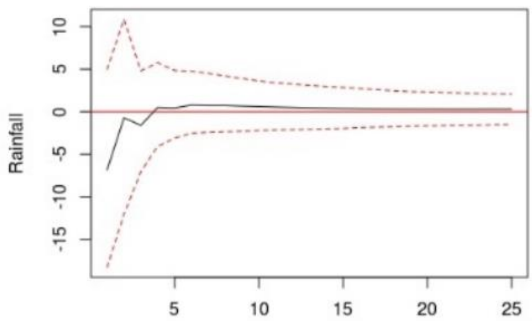
WS_{w5}



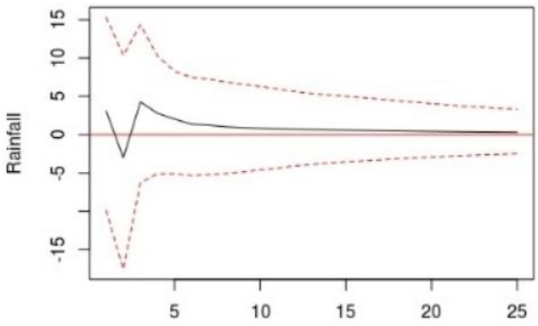
WS_{w6}



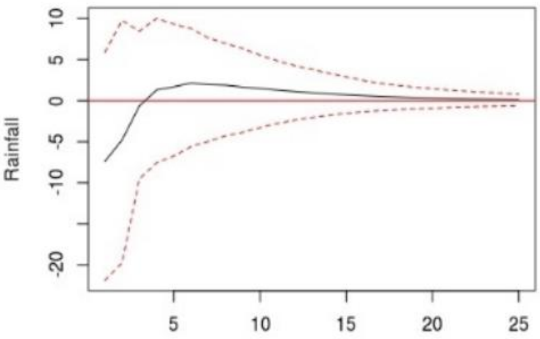
WS_{w7}



WS_{w8}



WS $_{w9}$

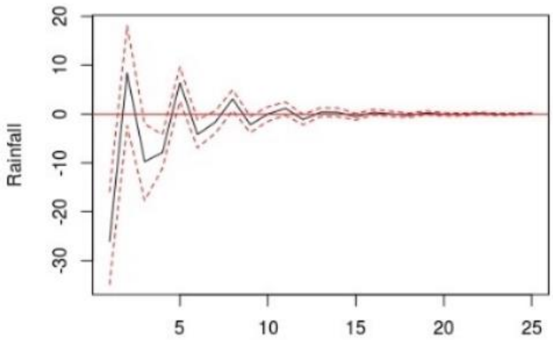


WS $_{w10}$

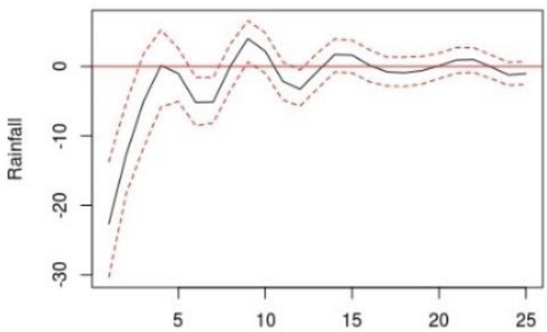
Figure 7. 11 Response of rainfall towards WS (w1-w10) at DBR

At GHY

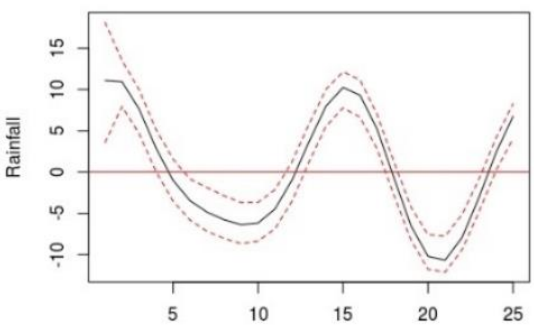
The response of rainfall to shocks in MaxT series of different resolutions are presented in Figure 7.12. As evident from the IRFs, the response of rainfall was positive, which declined and became zero after 3-4 months and became negative. The response stayed negative for the next few months, after which it became positive after 15-16 months and for an indefinite period became stable. In w6 however, the initial response was negative, and it became stable after 8-10 months. The response in case of w7 was unclear.



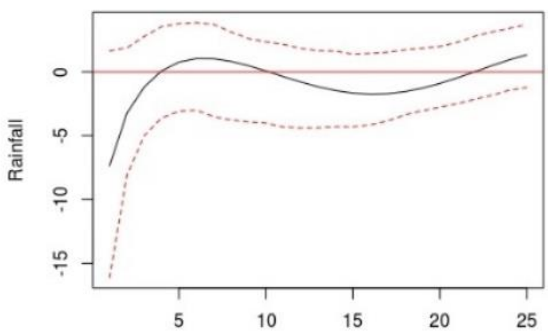
MaxT $_{w1}$



MaxT $_{w2}$



MaxT $_{w3}$



MaxT $_{w4}$

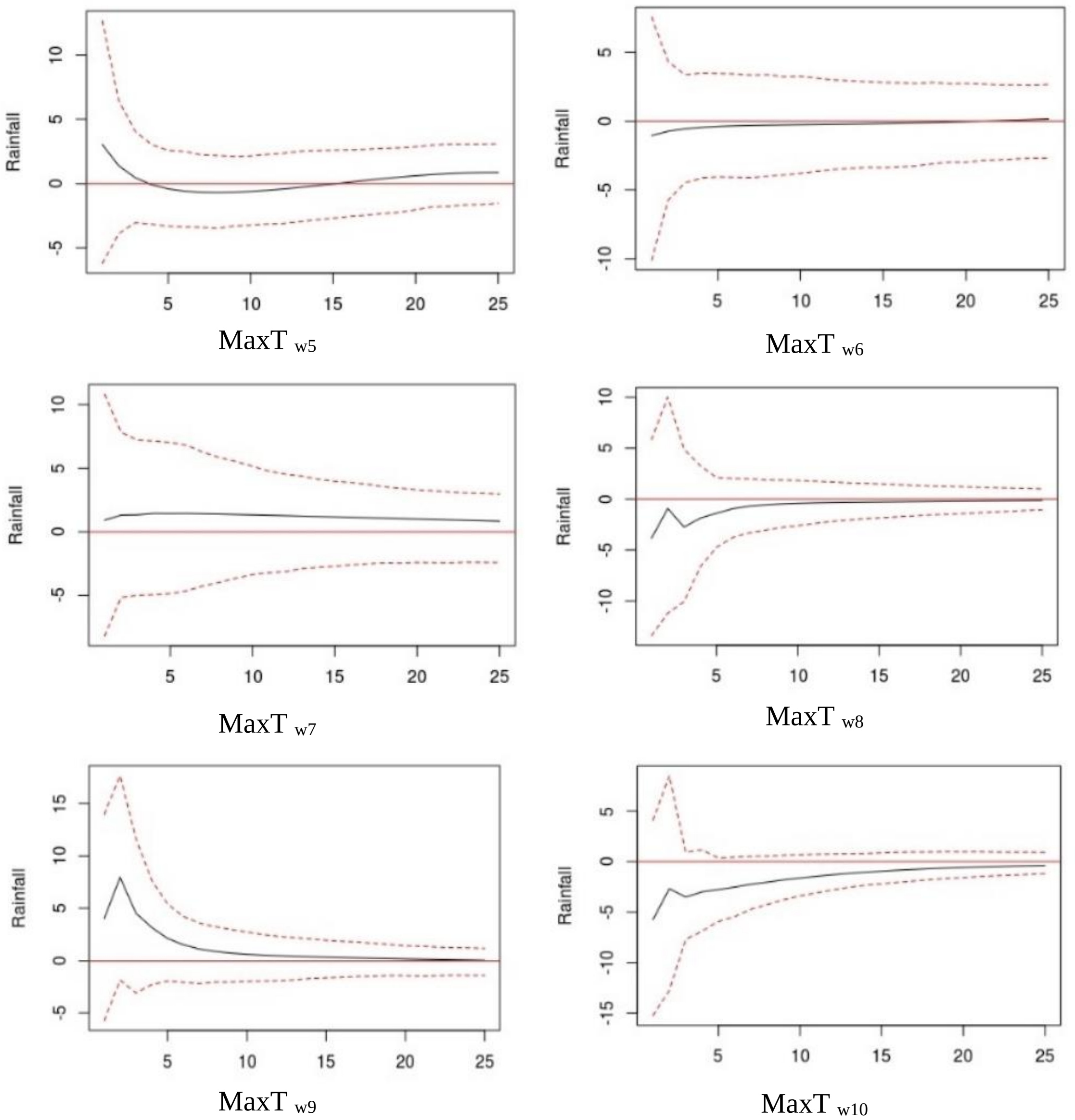
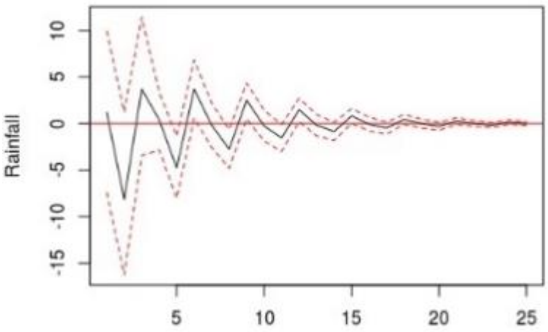
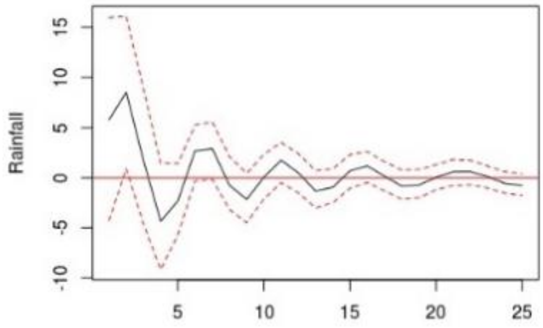


Figure 7. 12 Response of rainfall towards MaxT (w1-w10) at GHY

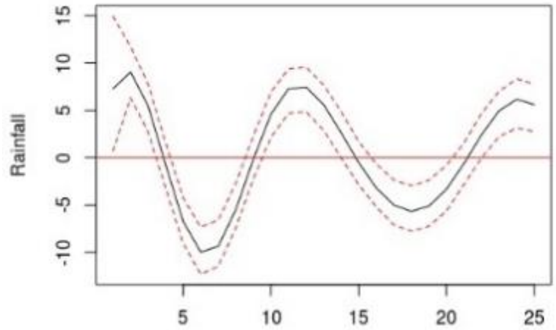
In case of w5 of MinT, the rainfall response was initially negative, but it became zero after 3 months (Figure 7.13). The response pattern seemed to be the inverse pattern of the response that was observed in case of MaxT; only the moment of delay in the response was different. The initial response in rainfall was negative also while shock was applied to w6, but the strength of this response declined and became zero after 6-10 months. In case of w7 the no clear response by rainfall could be identified.



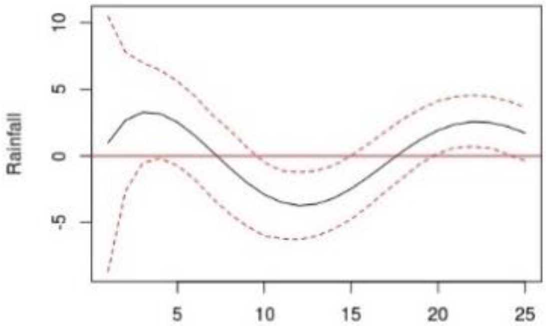
MinT $_{w1}$



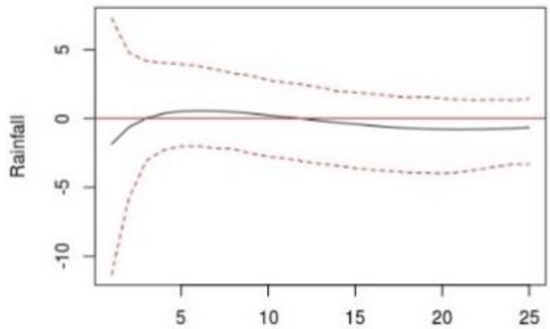
MinT $_{w2}$



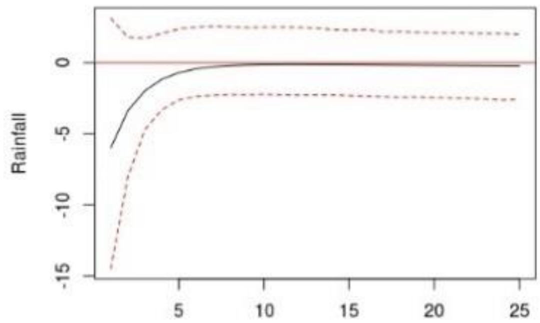
MinT $_{w3}$



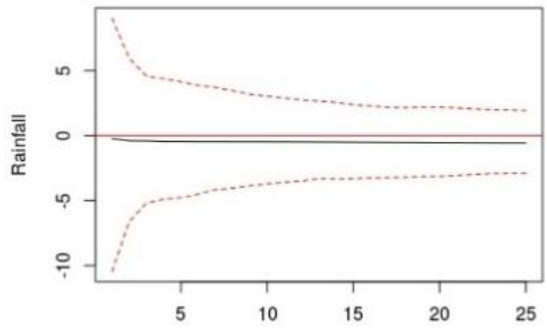
MinT $_{w4}$



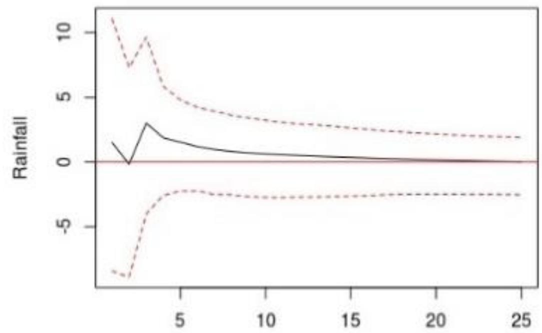
MinT $_{w5}$



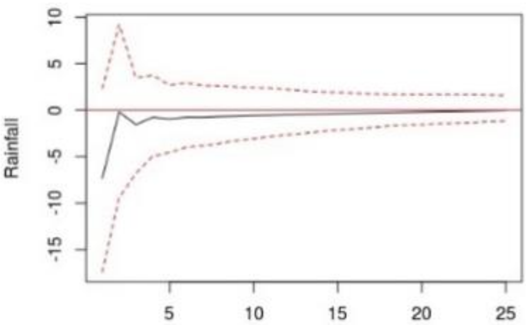
MinT $_{w6}$



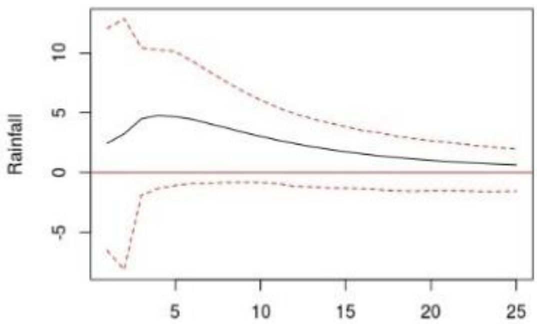
MinT $_{w7}$



MinT $_{w8}$



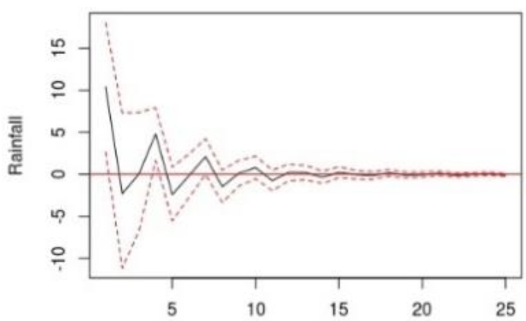
MinT $_{w9}$



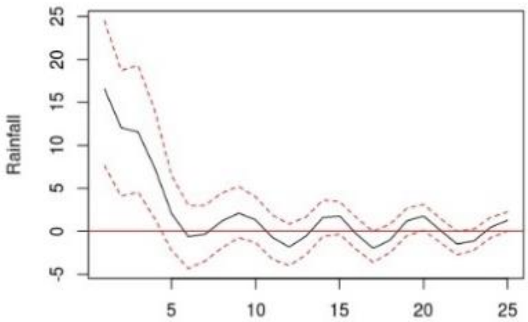
MinT $_{w10}$

Figure 7. 13 Response of rainfall towards MinT (w1-w10) at GHY

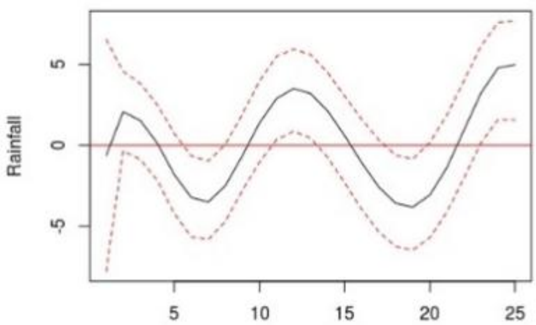
In case of w5 of RH, no clear response was observed in rainfall towards impulse in w5 (Figure 7.14). The initial response of rainfall to shock in w6 was negative, which died off after 6 months. On the other hand, the initial response by rainfall was positive to shock in w7, but the response became stable after 6 months.



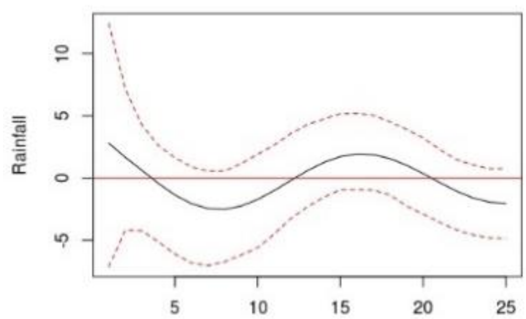
RH $_{w1}$



RH $_{w2}$



RH $_{w3}$



RH $_{w4}$

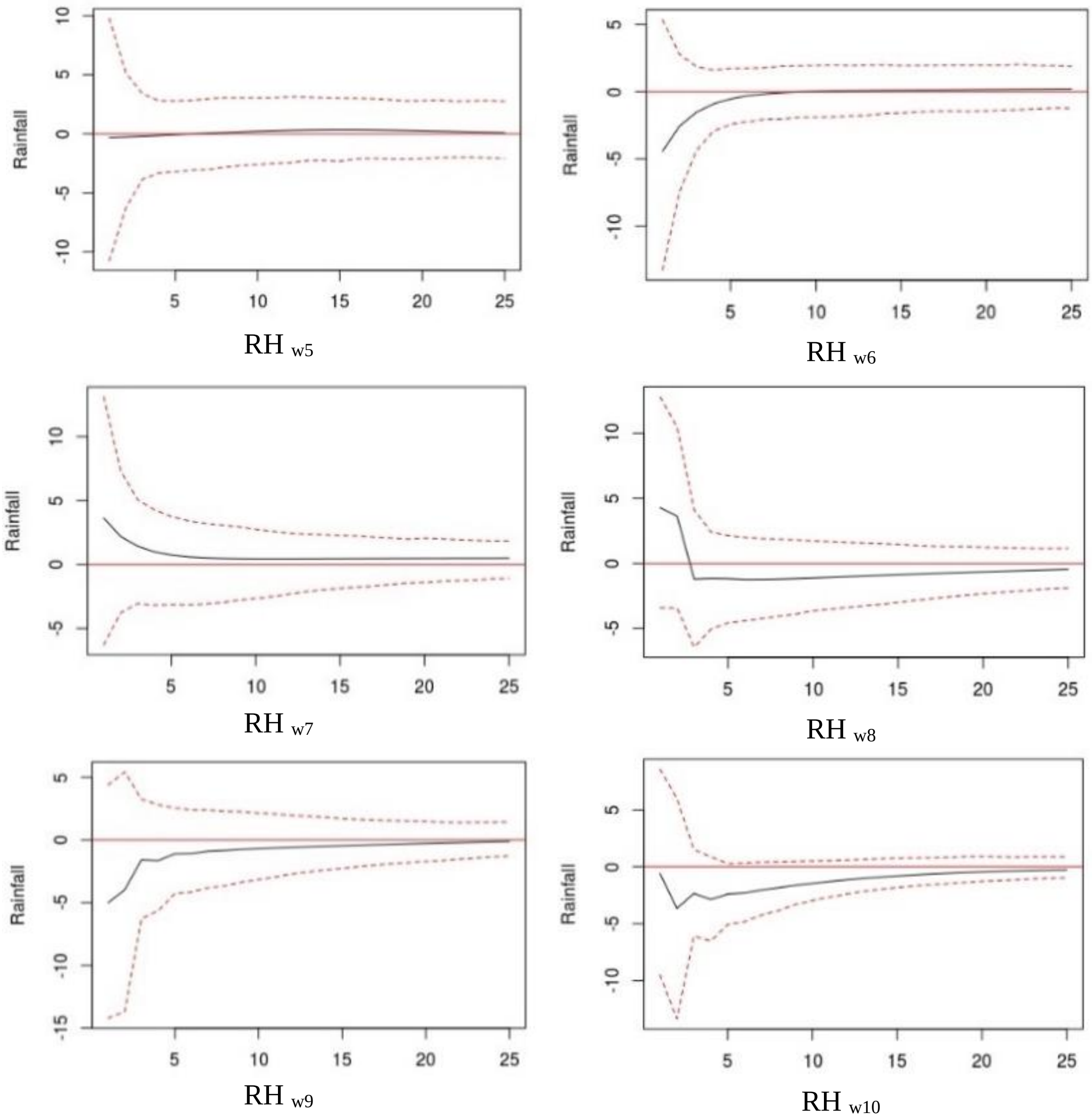
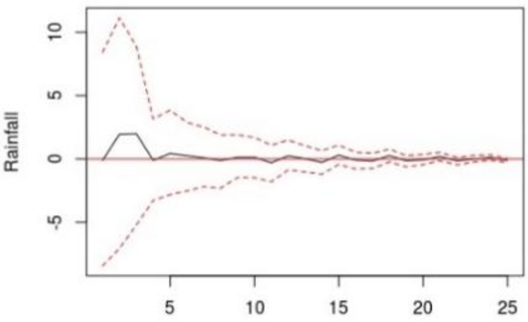
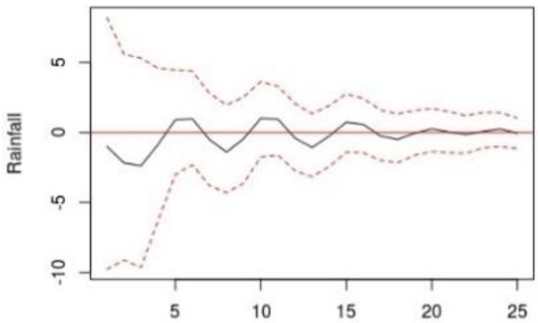


Figure 7. 14 Response of rainfall towards RH (w1-w10) at GHY

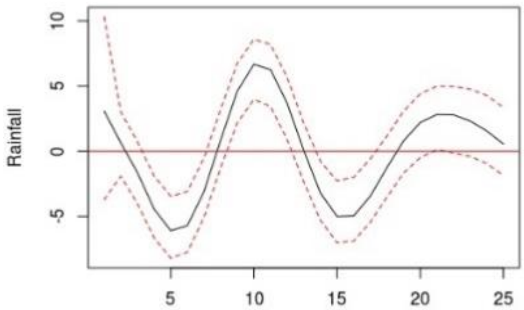
The response pattern of rainfall toward shock in w5 of SLP was similar to that in w5 of MaxT at GHY. In case of w6, an initially positive response was observed in rainfall, but it lasted for less than five months. In case of w7, no clear response was observed by rainfall (Figure 7.15).



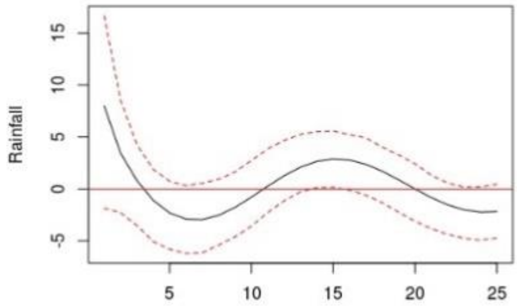
SLP $_{w1}$



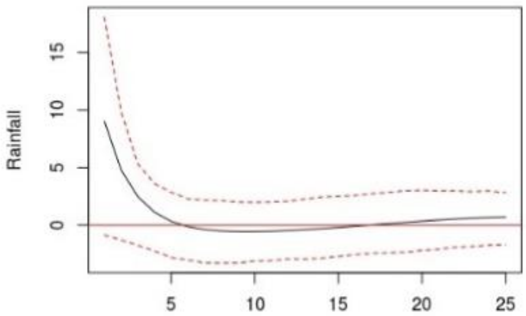
SLP $_{w2}$



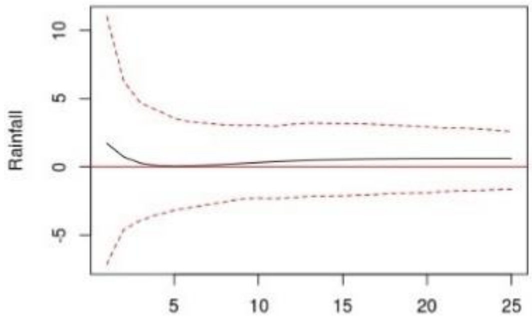
SLP $_{w3}$



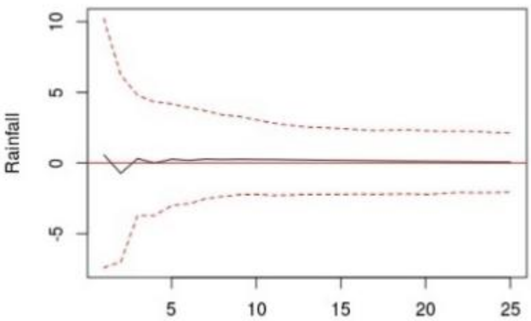
SLP $_{w4}$



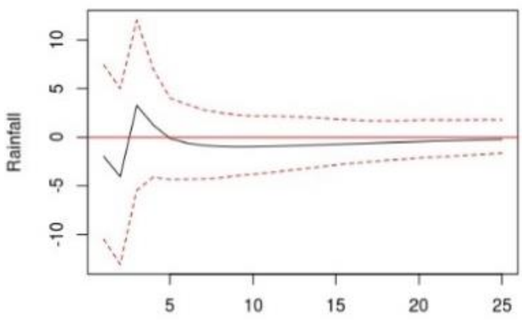
SLP $_{w5}$



SLP $_{w6}$



SLP $_{w7}$



SLP $_{w8}$

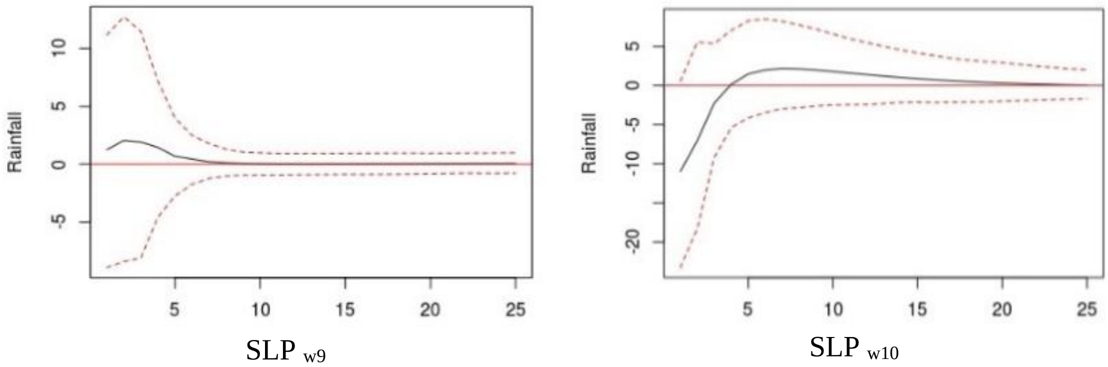
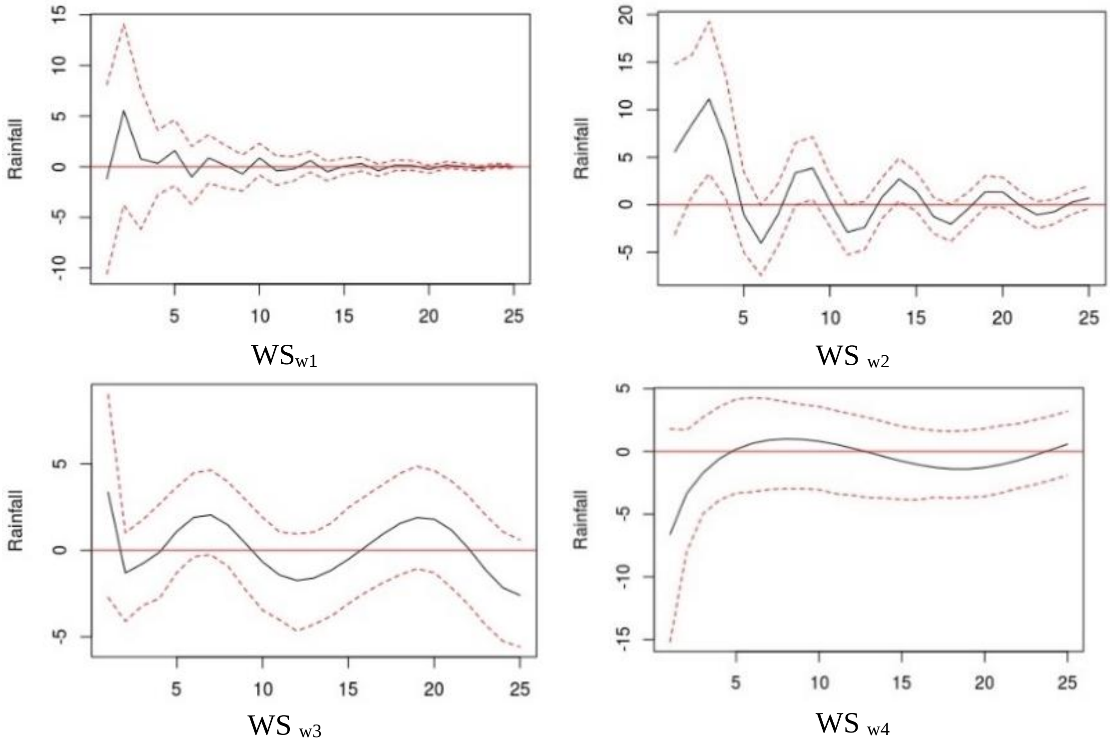


Figure 7. 15 Response of rainfall towards SLP (w1-w10) at GHY

As evident from the IRFs in Figure 7.16, an initial positive response by rainfall was detected towards shock in w5, while the opposite was observed in case of w6 and w7 of WS. In case of w5 the response declined gradually and after crossing zero near 9th lag, the response became negative and stable after 12 months, while the response by rainfall became nil and stable after 6 and 4 months respectively in case of w6 and w7.



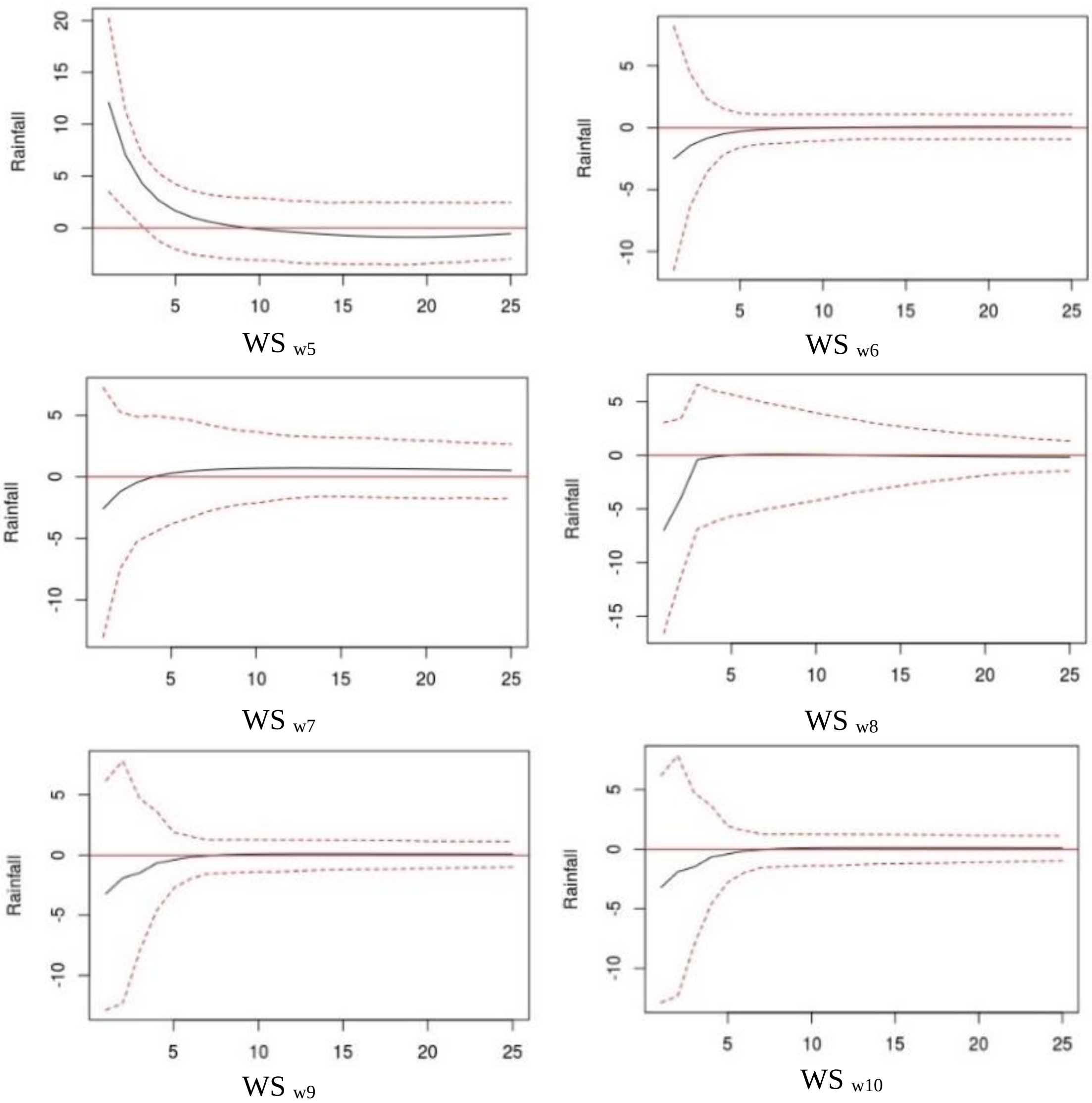
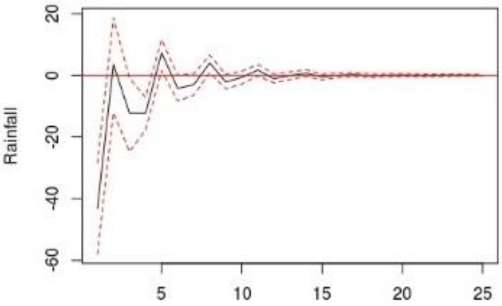


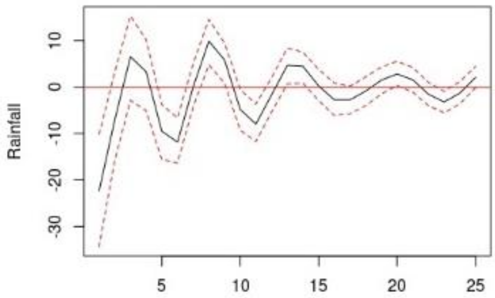
Figure 7. 16 Response of rainfall towards WS (w1-w10) at GHY

At KSH

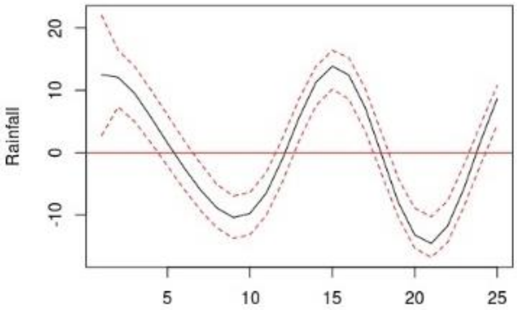
The response of rainfall as impulse towards shock in different resolutions of meteorological variables for KSH is presented in Figure 7.16-7.21. In case of MaxT, initial positive response was detected in w5-w7, which gradually declined and became zero after 5-6 months in case of w5, the response revered to become negative in case of w6 after 2-3 months for an indefinite period and in case of w7 the response kept declining for an indefinite period.



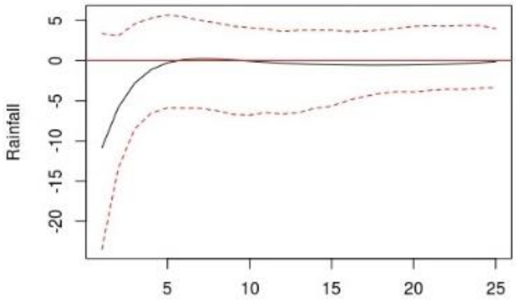
$MaxT_{w1}$



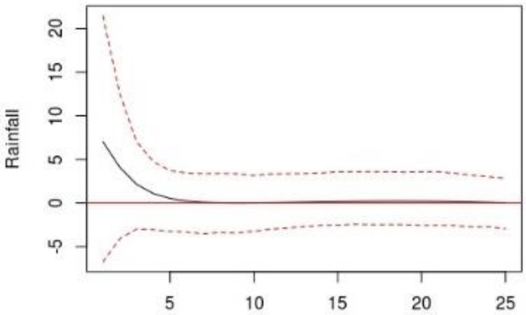
$MaxT_{w2}$



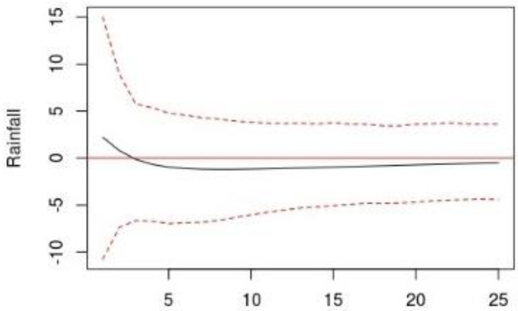
$MaxT_{w3}$



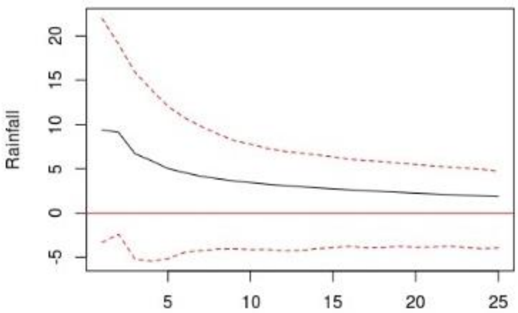
$MaxT_{w4}$



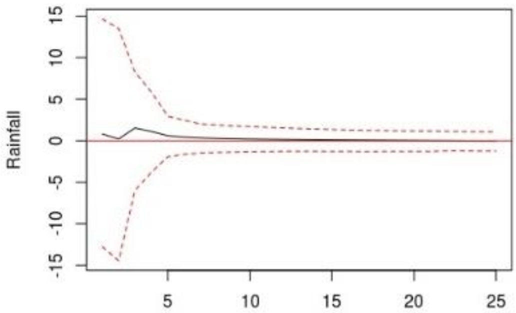
$MaxT_{w5}$



$MaxT_{w6}$



$MaxT_{w7}$



$MaxT_{w8}$

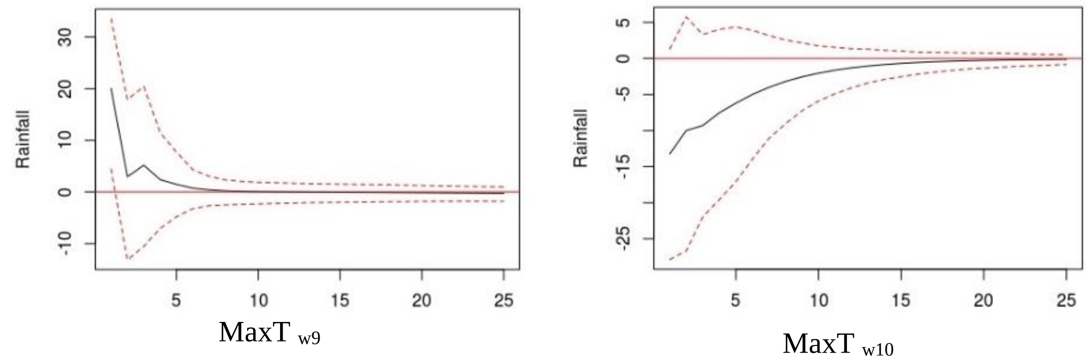
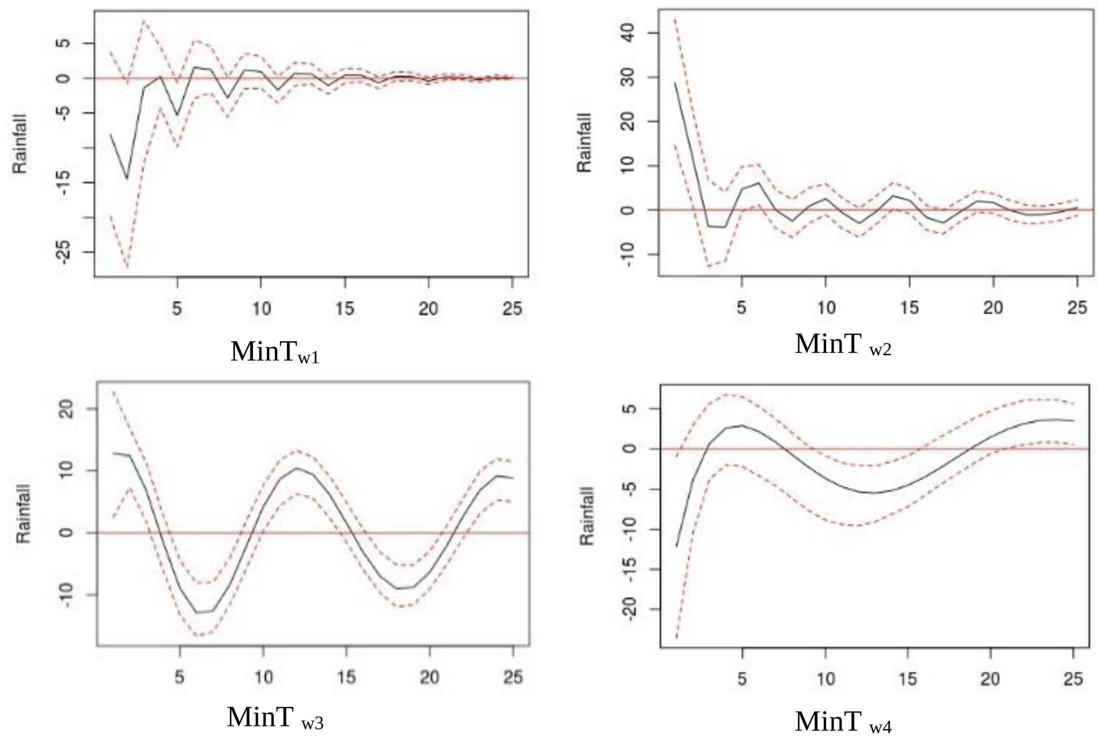


Figure 7. 17 Response of rainfall towards MaxT (w1-w10) at KSH

In case of w5 of MinT, same pattern of response as that in w5 of MaxT was observed, only the effect of the initial shock lasted for a little long (~8 months). On the other hand, in case of both w6 and w7 of MinT, the initial response shown in rainfall towards the shock in MinT was negative and it became zero after 6 and 4 months respectively for w6 and w7 (Figure 7.18).



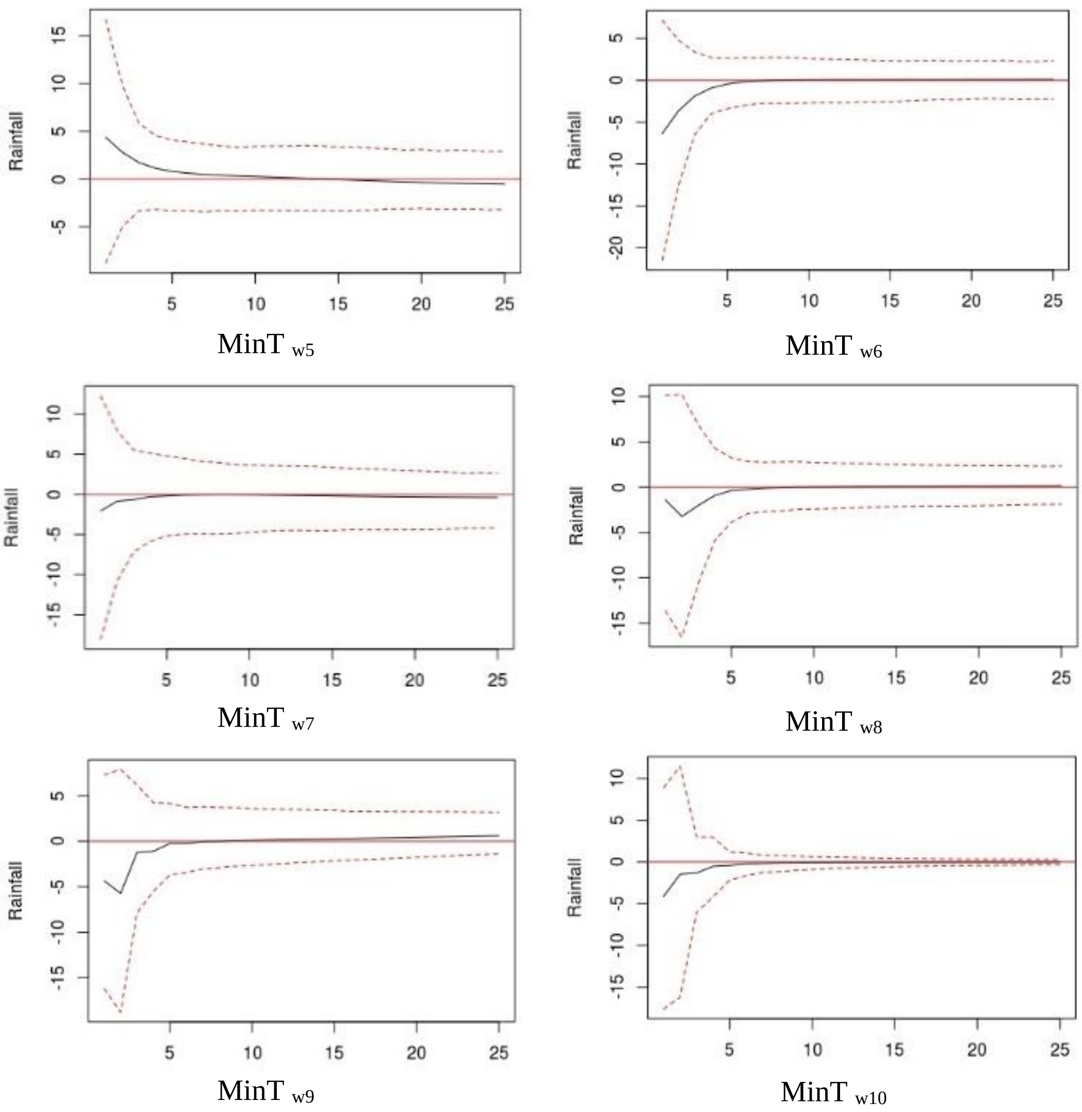
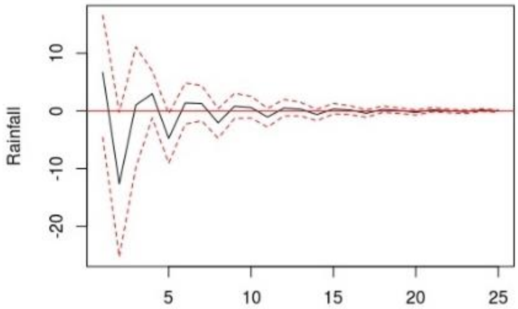
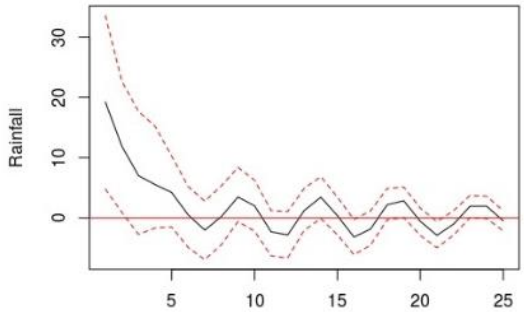


Figure 7. 18 Response of rainfall towards MinT (w1-w10) at KSH

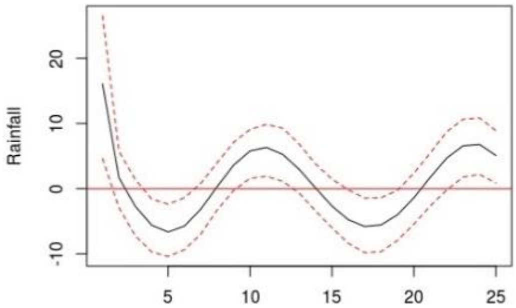
Initial negative response was observed in rainfall (Figure 7.19) towards shock in w5 of RH, which died off after 8-9 months. No response was observed in rainfall towards shock in w6 of RH. In case of w7, a low negative response in rainfall could be seen in case of w7, but the response reversed after less than 3 months and then slowly became zero.



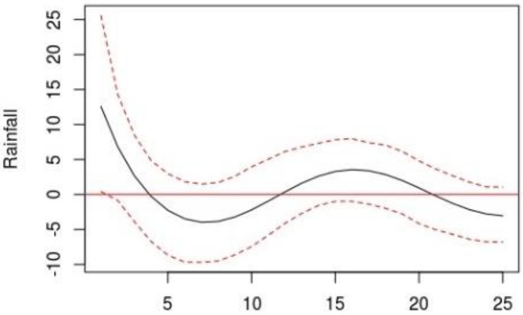
RH_{w1}



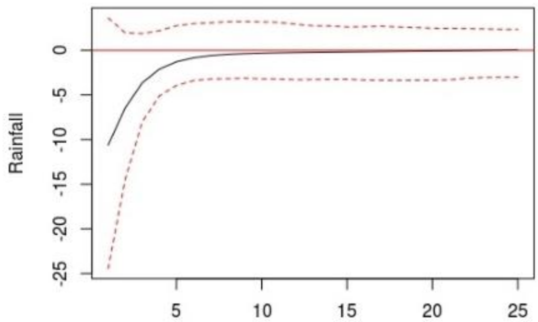
RH_{w2}



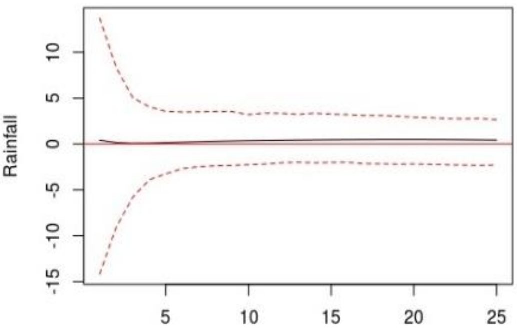
RH_{w3}



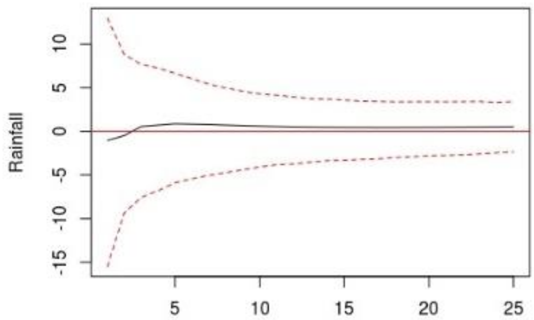
RH_{w4}



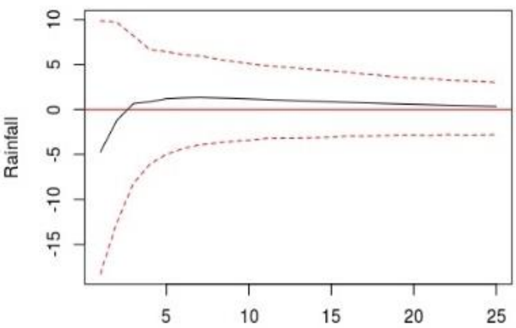
RH_{w5}



RH_{w6}



RH_{w7}



RH_{w8}

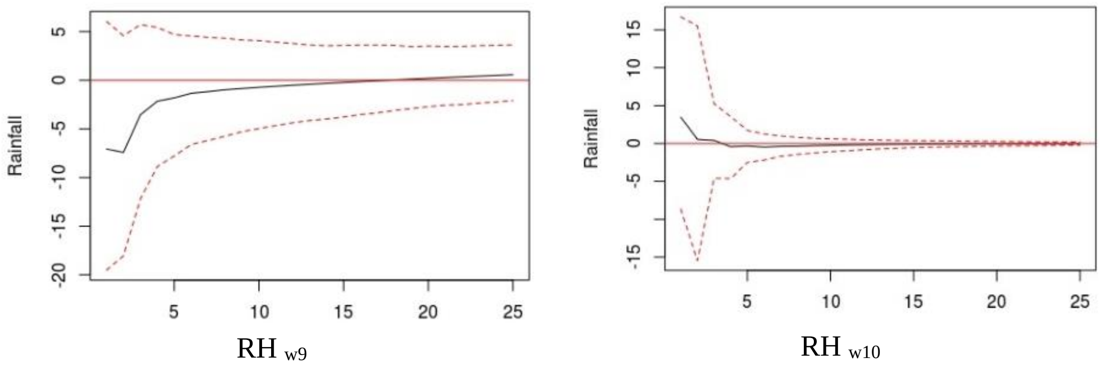
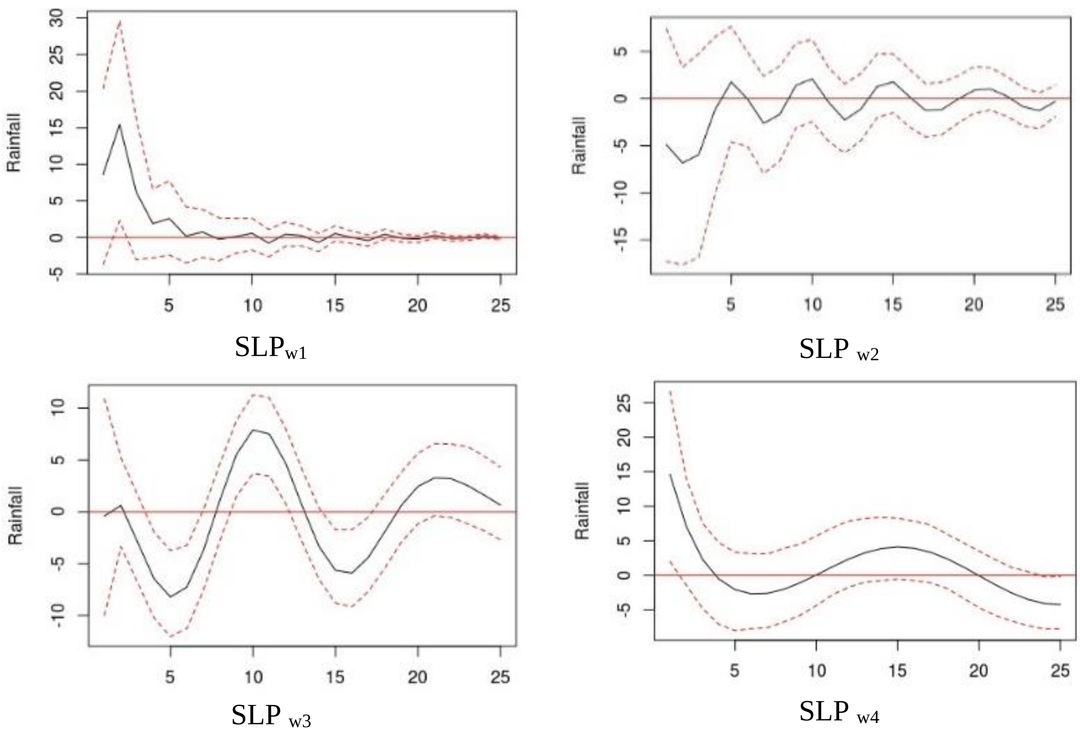


Figure 7. 19 Response of rainfall towards RH (w1-w10) at KSH

Rainfall showed positive response at the beginning to the shocks in w5-w7 of SLP (Figure 7.20. The response declined became stable after 8 and 5 months in case of w5 and w6, while case of w7 the response became negative quickly, and afterwards stayed as such for an indefinite period.



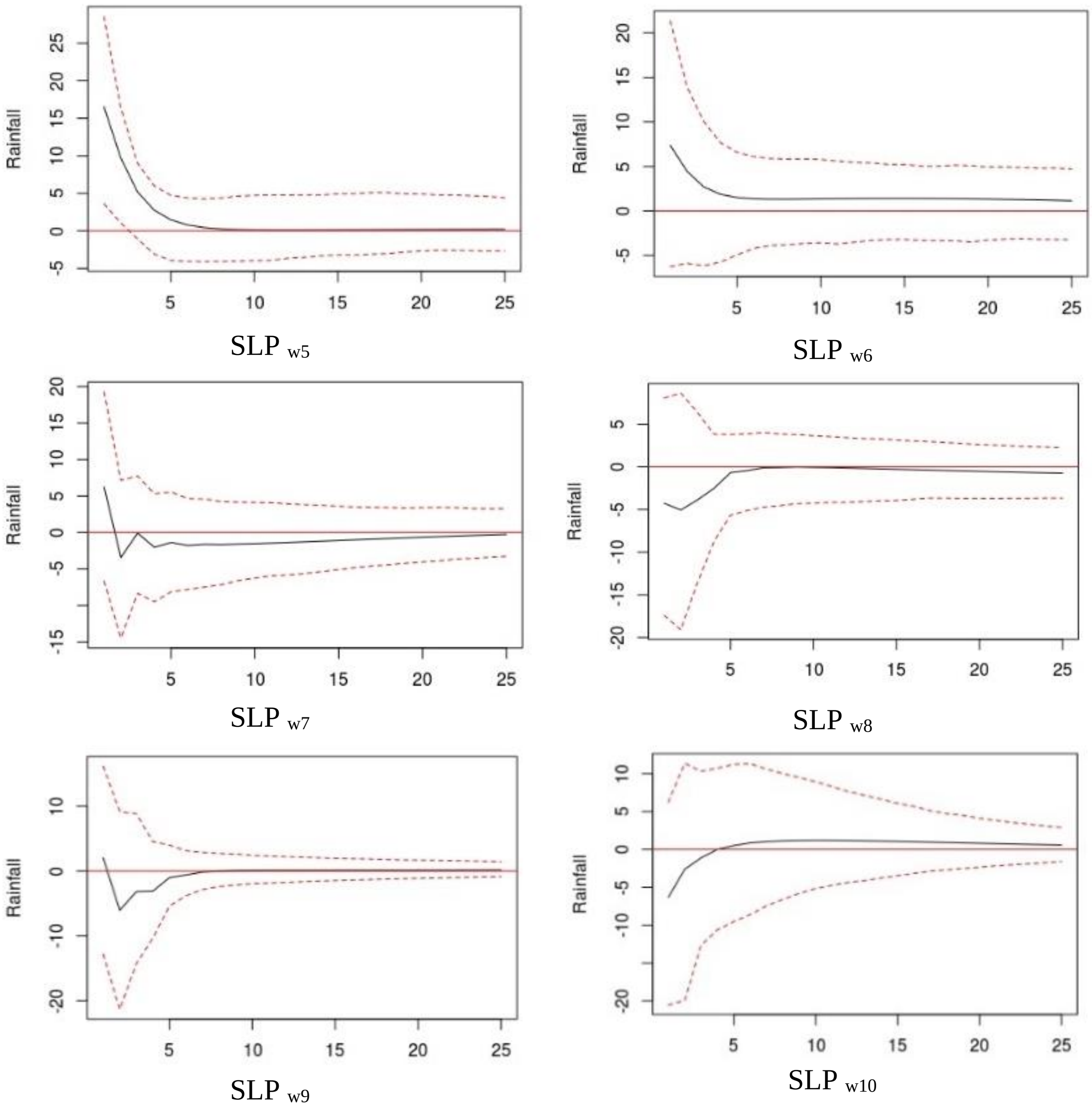
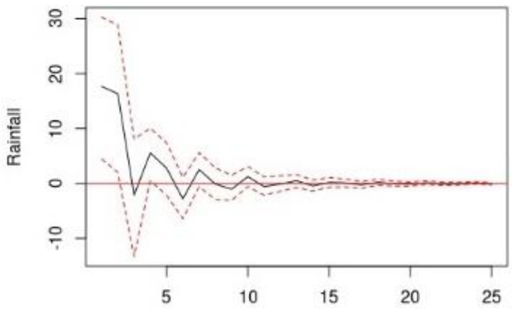
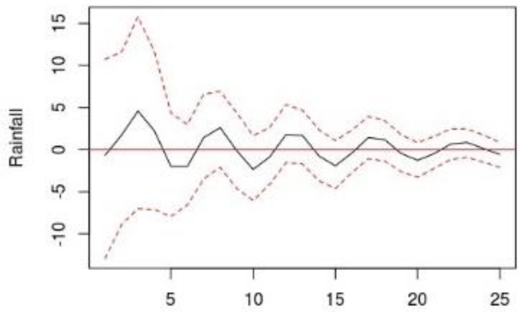


Figure 7. 20 Response of rainfall towards SLP (w1-w10) at KSH

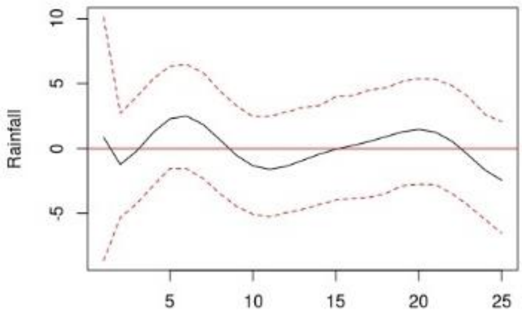
In case of WS, initial negative response was observed by rainfall towards shock in w5 and w7, while the opposite was detected in case of w6. The response became stable after 10 and 18 months in case of w5 and w6. In case of w7, the persistence period of the initial negative response was less than 5 months, which afterwards became positive and stable for an indefinite period (Figure 7.21).



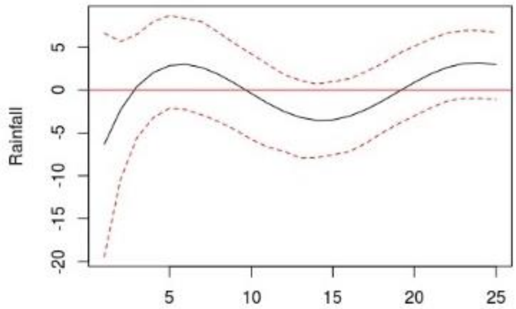
WS_{w1}



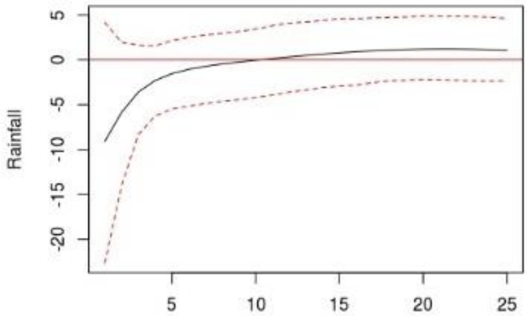
WS_{w2}



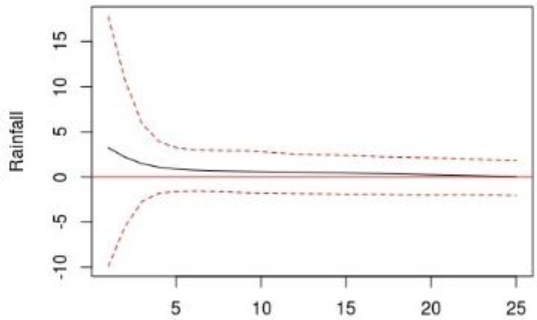
WS_{w3}



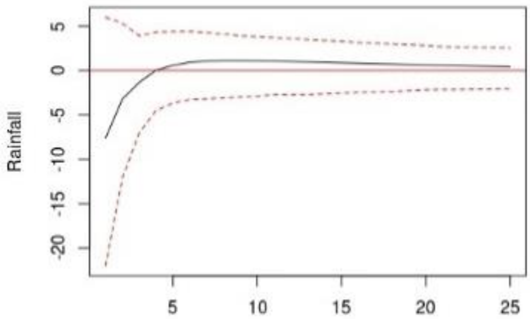
WS_{w4}



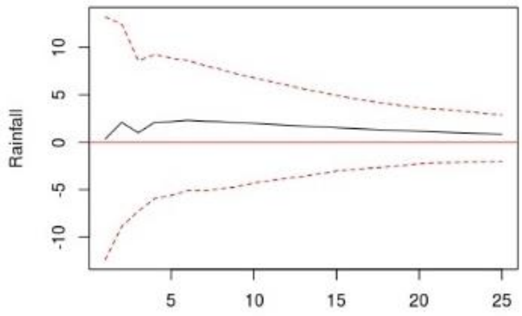
WS_{w5}



WS_{w6}



WS_{w7}



WS_{w8}

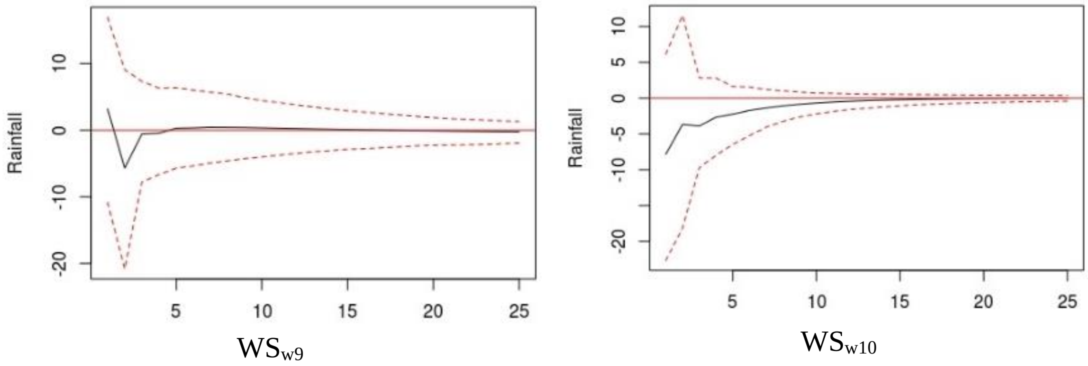
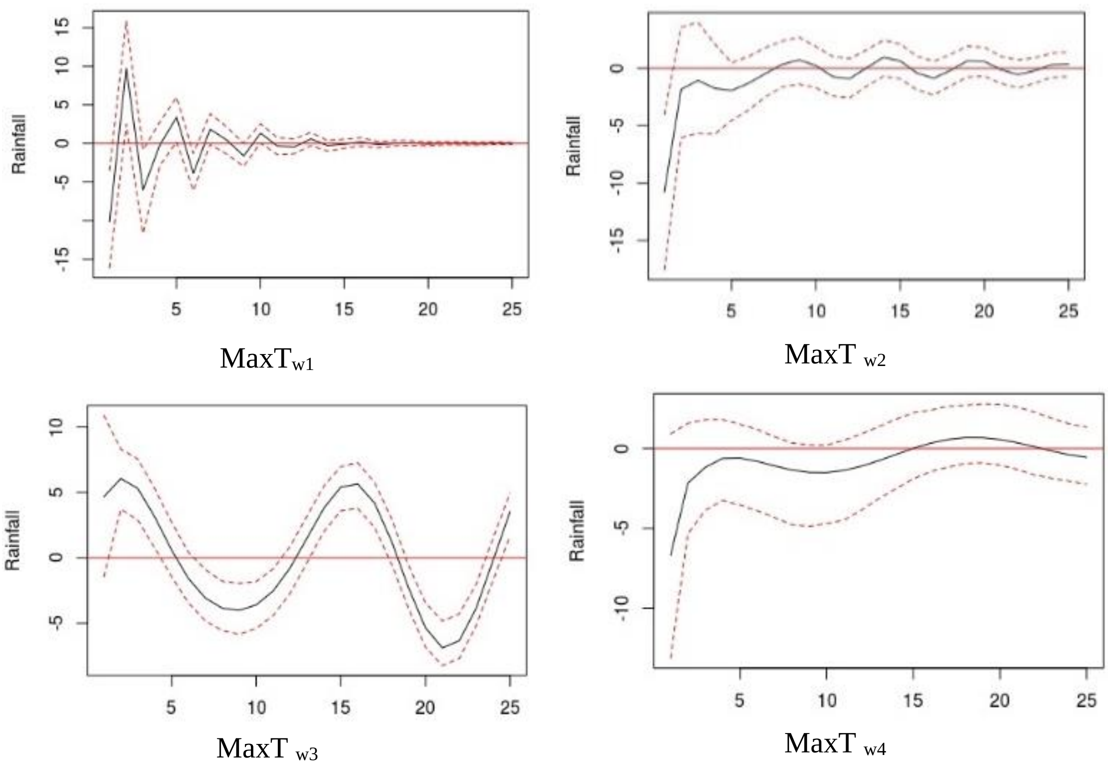


Figure 7. 21 Response of rainfall towards WS (w1-w10) at KSH

At TUL

Response of RF as impulse towards shock of different *meteorological variables* (for wavelet decomposed series w_1-w_{10}) for TUL is presented in Figure 7.22-7.26.

It can be seen that rainfall initially responded positively after the shock was applied on w5-w7. However, the response declined quickly and became negative after 5, 2 and ~10 months in case of w5, w6 and w7 respectively. It was observed that the response became stable at negative state for an indefinite time except w5. In case of w5 the negative response reversed after nearly 20 months (Figure 7.22).



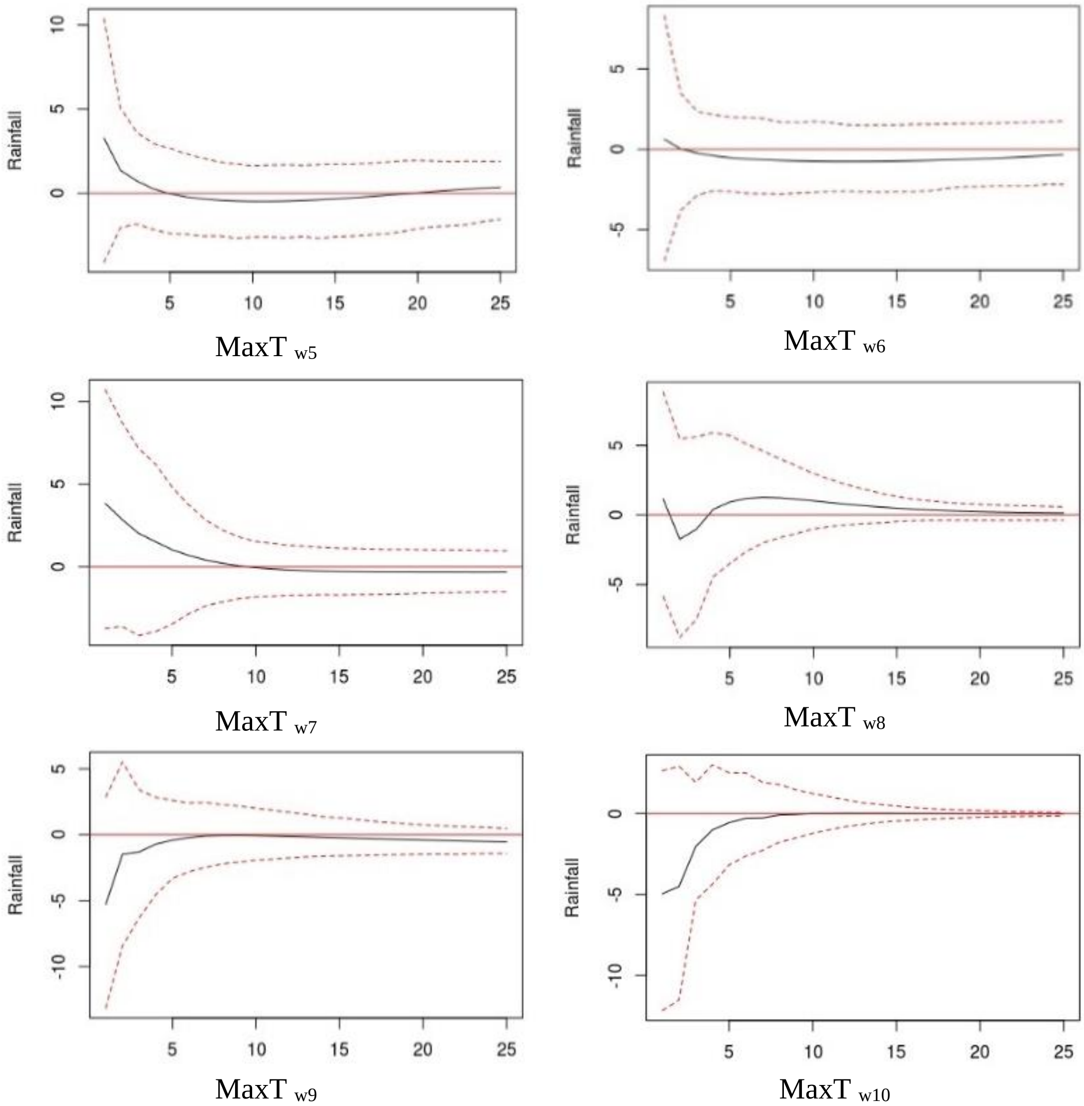
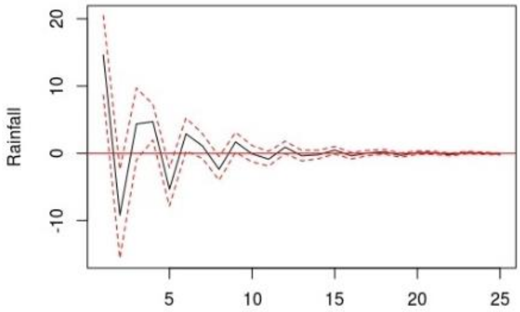
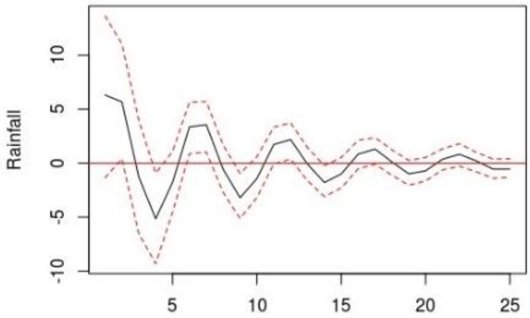


Figure 7. 22 Response of rainfall towards MaxT (w1-w10) at TUL

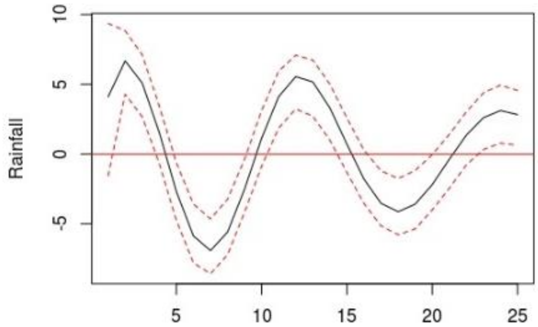
In case of MinT (Figure 7.23), the initial response of rainfall was found to be slightly negative when shock was applied on w5 and w6, and the response became stable soon (before 5 months) for an indefinite time. To the contrary, the initial response of rainfall was positive at the application of shock to w7, and it also became stable for an indefinite period after 5 months.



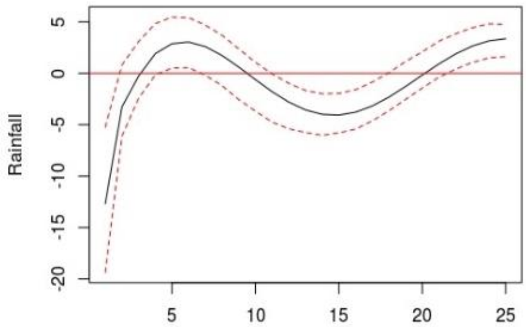
$MinT_{w1}$



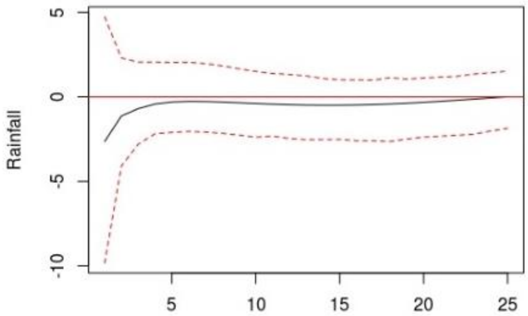
$MinT_{w2}$



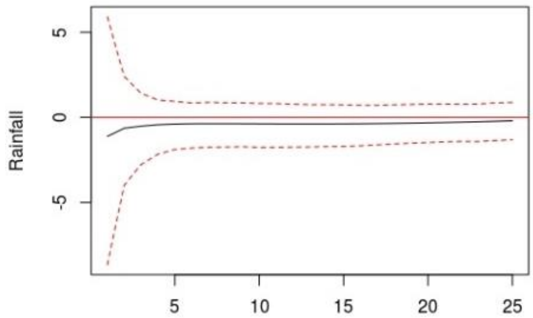
$MinT_{w3}$



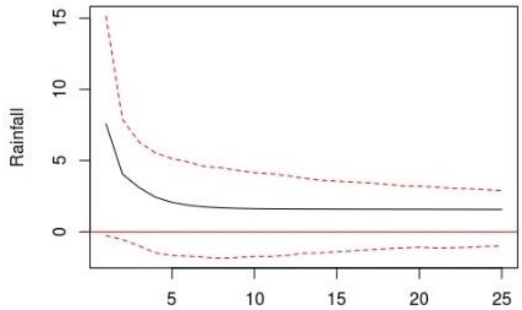
$MinT_{w4}$



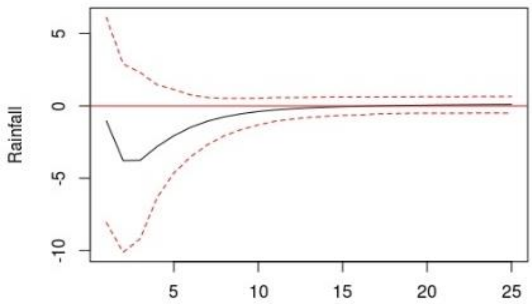
$MinT_{w5}$



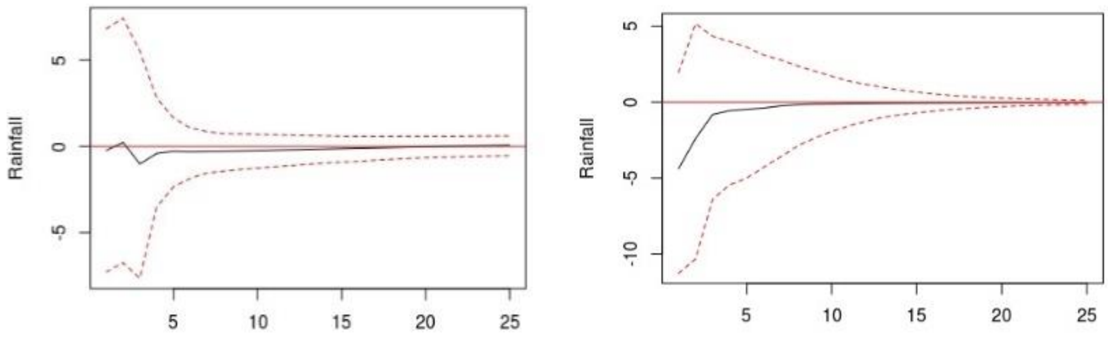
$MinT_{w6}$



$MinT_{w7}$



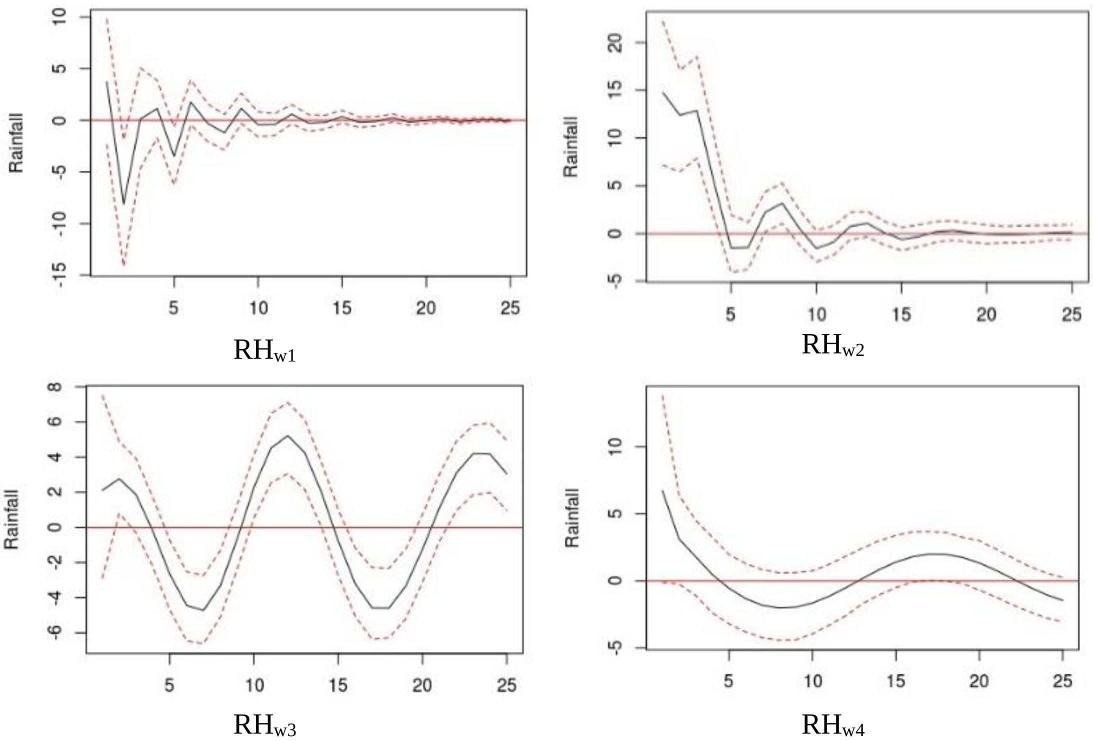
$MinT_{w8}$



MinT w9 MinT w10

Figure 7. 23 Response of rainfall towards MinT (w1-w10) at TUL

In case of RH also, rainfall initially responded negatively to w5 and w6, but either very soon the response became zero (w6, after <5 months) or reversed and remained positive for indefinite period (w5). In case of w7 the initial response was the opposite and it became stable after 5 months (Figure 7.24).



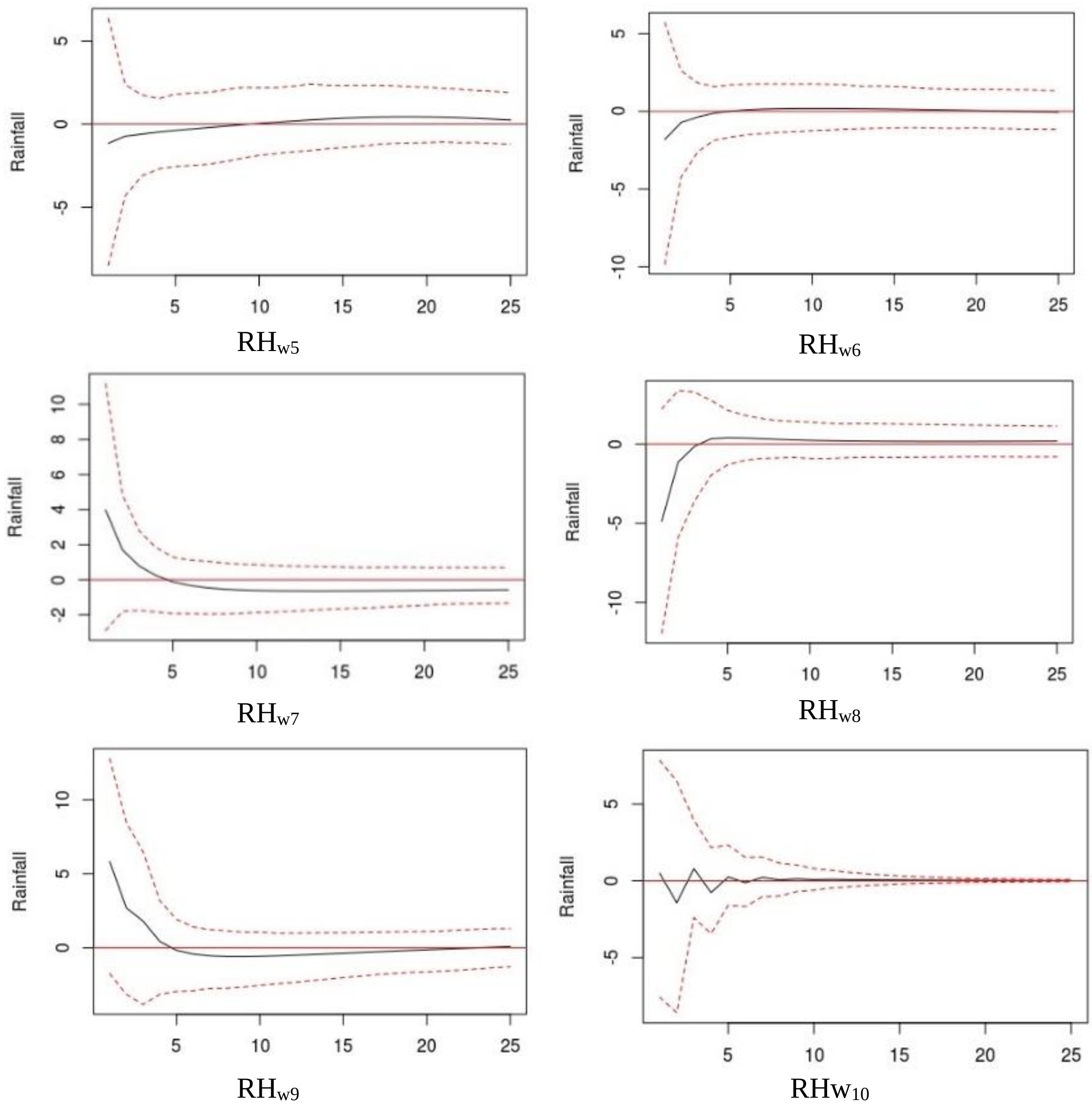
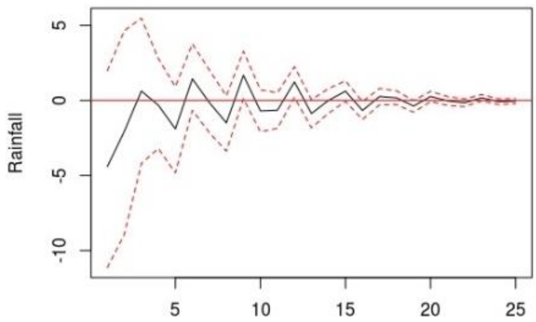
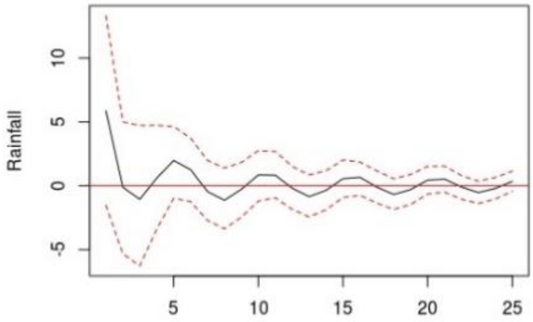


Figure 7. 24 Response of rainfall towards RH (w1-w10) at TUL

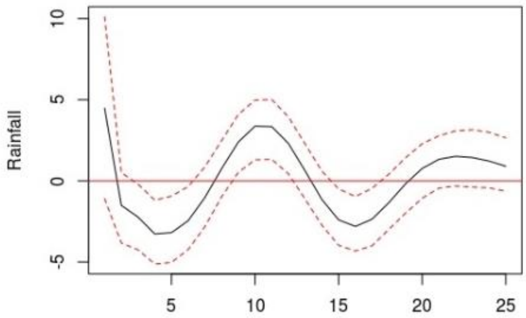
As evident in Figure 7.25, Rainfall responded positively, as shock was applied on w5 of SLP, while the opposite initial response was observed in case of w6 and w7. However, like the previous cases, the response became zero quickly (on or before 6 months) in case of w5 and w7, while in case of w6 the response fluctuated weakly between positive and negative Y axis at certain lags and no clear conclusion could be drawn.



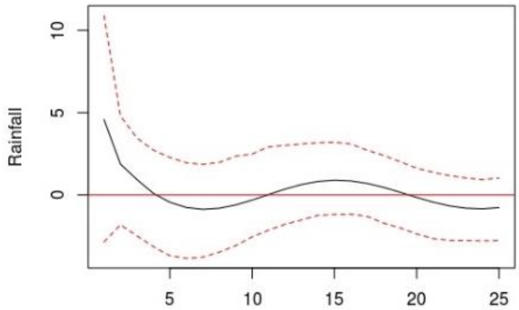
SLP_{w1}



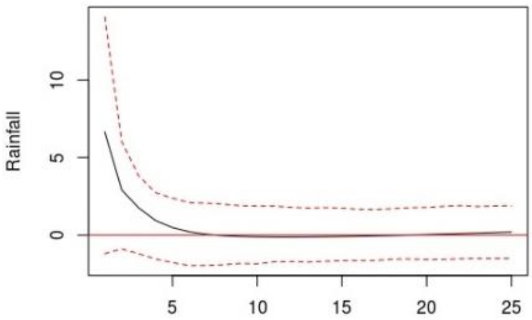
SLP_{w2}



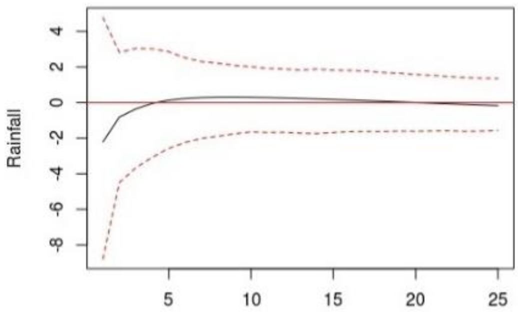
SLP_{w3}



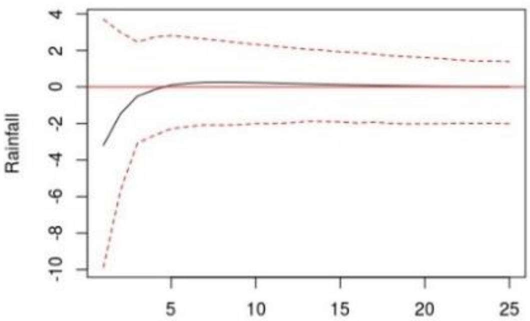
SLP_{w4}



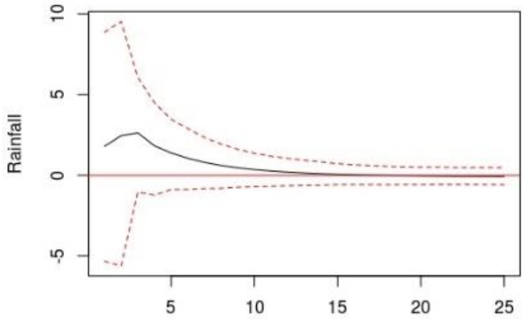
SLP_{w5}



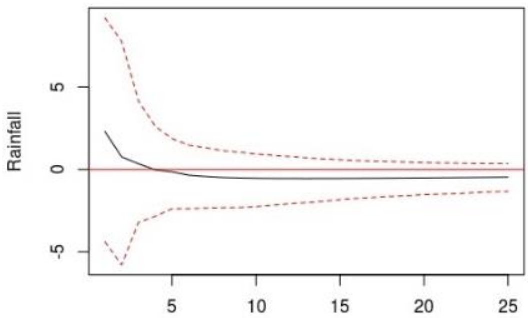
SLP_{w6}



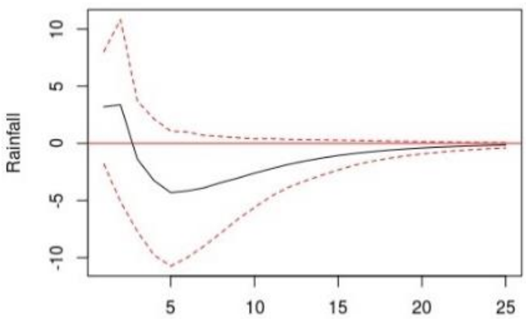
SLP_{w7}



SLP_{w8}



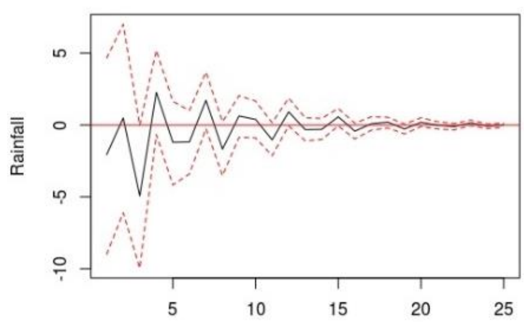
SLP_{w9}



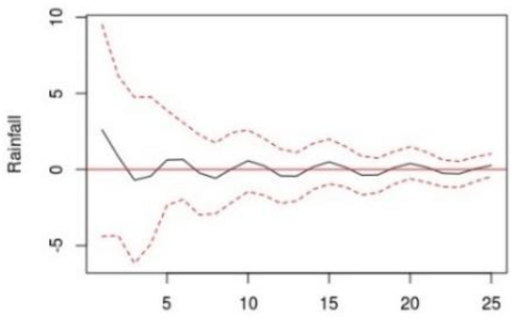
SLP_{w10}

Figure 7. 25 Response of rainfall towards SLP (w1-w10) at TUL

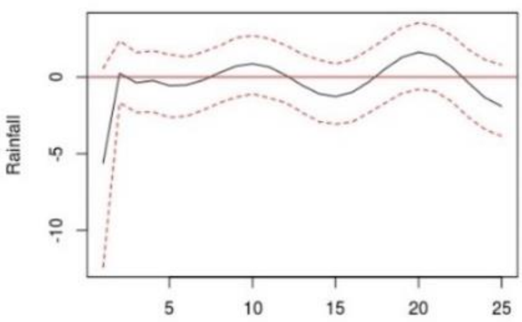
The initial response of rainfall was positive towards w5-w7 of WS at TUL (Figure 7.26). In case of w5 of WS, response of rainfall became negative from the initial positive response after 15 months and on the other hand the response of rainfall became zero after 6 months in case of w6. In case of w7, the response of rainfall became stable as a negative response after 6 months.



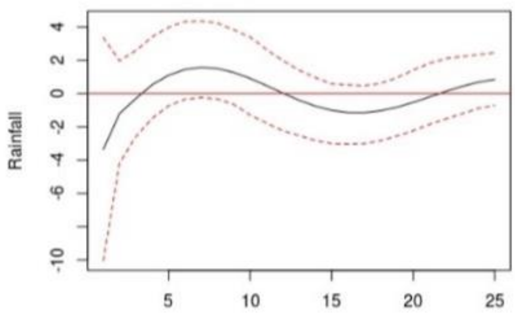
WS_{w1}



WS_{w2}



WS_{w3}



WS_{w4}

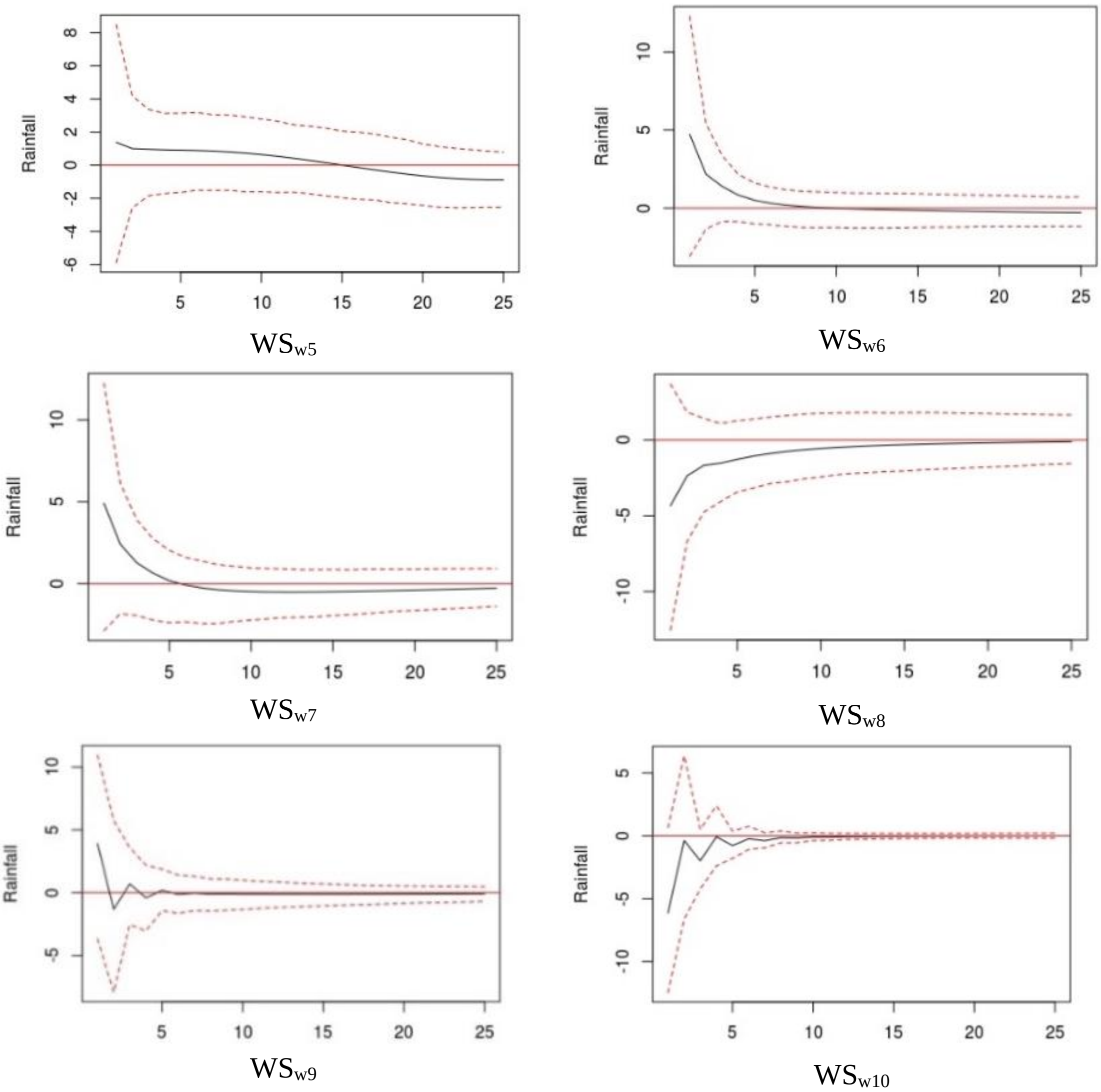


Figure 7. 26 Response of rainfall towards WS (w1-w10) at TUL

7.4 Summary

The determination of inter-association of meteorological parameters is crucial for understanding the sensitivity and sustenance of a region in a changing climatic scenario. The VAR method along with IRF approach is such a technique that explores the inter-relationships among different meteorological variables with change in time. In this study, an attempt was made to study the inter-relational sensitivity of rainfall with different meteorological variables such as temperature (MaxT and MinT), RH, SLP and WS using VAR-IRF modelling technique, coupled with wavelet. The sensitivity of rainfall was studied on different resolutions of the selected meteorological variables. The time series of meteorological variables of different resolutions were obtained from the wavelet decomposition. Depending upon the data length, ten such series were obtained for each variable for the selected locations of NER and 1σ shock was applied on each resolution to see the behaviour of monthly rainfall total as an impulse. It could be seen that the response of rainfall in the high and low frequency series (w1-w4 as the high frequency series and w8-w10 as the low frequency series) was cyclical, as induced by noise. The observations from the climatic series (w5-w7) revealed that the response of rainfall upon changes to each of these different resolution's series per variable could last for 5-12 months of initial application of shocks. It could be seen that the initial response by rainfall in most of the cases was sharp in either negative or positive way, but the response died off soon. Also, in some of the cases the response of rainfall was found to be persisting for an indefinite period.

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