Chapter-2

JEANS INSTABILITY IN ASTROPHYSICAL VISCOELASTIC FLUIDS WITH GEOMETRICAL CURVATURE EFFECTS

Abstract: *The semi-analytic study presented herein primarily focuses on analyzing the Jeans (gravitational) instability dynamics excited in strongly correlated viscoelastic astrophysical fluid media in the presence of geometrical curvature effects* [†]*. The basic governing equations are accordingly formulated in the generalized hydrodynamic framework in spherical geometry on the astrophysical spatiotemporal scales. A quadratic dispersion relation of an atypical analytical construct is obtained by a spherically symmetric normal mode analysis over the perturbed spherical medium. A numerical illustrative platform based on judicious multiparametric inputs is provided to see the various stabilizing and destabilizing factors of the fluid volume. It is specifically demonstrated that the effect of geometrical curvature introduces a compound viscous influence onto the dispersion relation. It is in contrast to the earlier reports analytically based on a planar geometry. The fact that larger clouds, but with constant density, are gravitationally more stabilized and vice-versa is an intriguing non-trivial result herein. Astronomical implications and applications of our study are summarily highlighted.*

2.1 INTRODUCTION

The first step towards structure formation that occurs in interstellar gas clouds is the gravitational instability [1-8]. Interstellar gas clouds naturalistically have inhomogeneous mass distribution [1-8]. As such, the denser regions become more dense, thereby undergoing contraction under the influence of their own gravity. Thus, the gas cloud becomes unstable in certain regions where the radiation pressure of the gas is insufficient to balance the inward force of gravity. This kind of instability that arises due to the gravity is known as the gravitational instability. Gravitational instability leads to the collapse of the interstellar gas clouds, leading to the formation of smaller pre-stellar structures or protostars. In the formation and evolution of stellar objects, protostar forms the earliest phase [9-11]. The protostars lead to the formation of stellar structures over a typical time-scale of million years or so. In stellar physics, the

[†]Dasgupta, S. and Karmakar, P. K. The Jeans instability in viscoelastic spherical astrophysical fluid media. *Astrophysics and Space Science*, 364:213, 2019

problem of gravitational instability was first addressed and analyzed by Sir James Jeans [12]. Hence, in stellar physics, gravitational

instability is known as the Jeans instability [12]. However, the stellar model considered by Jeans was rather a simplified one as compared to real astronomic circumstances.

In plasma physics, one of the most important parameter that is used to classify the plasma is the coupling parameter (Γ) or the coupling constant. The coupling constant is defined as the ratio of the Coulomb interaction energy to that of the average kinetic energy. The fluid is weakly coupled if $\Gamma << 1$ [13-15]. The plasma fluid is assumed to be strongly coupled in the limit $1 << \Gamma < \Gamma_C$ [13-15]; Γ_C is the critical value of the coupling parameter beyond which crystallization occurs. It is in this regime that the plasma exhibits viscoelastic behaviour [13]. This behaviour is actually found to exist in stellar nuclear matter [16]. In addition, signatures of viscoelasticity have also been found in the laboratory investigations of nuclear dynamics [16]. There exists naturalistically a good number of promising evidences in support of the fact that the large-scale astrocosmic fluids are indeed viscoelastic in nature [16, 17]. As far as seen lately, a large number of theoretic studies have been conducted to analyze the onset of the Jeans instability and on the different ways to arrest it [18-24]. However, analysis of the Jeans instability in presence of curvature and viscoelastic influence has been unaddressed and unattended, to the best of our knowledge.

In the present chapter, we analyze the Jeans instability dynamics in a massive viscoelastic molecular cloud by using the generalized hydrodynamic formalism. The key assumption employed herein is that in the transition phase between elastic solid, and viscous liquid, the stellar matter displays the characteristics of both viscosity and elasticity. The focal aim of the semi-analytic investigation presented here is to model the excitation of the instability dynamics in self-gravitating complex viscoelastic fluids. We include the geometric curvature effects in the adopted spherically symmetric geometric model configuration. A standard technique of spherically symmetric normal mode analysis yields a quadratic dispersion relation having atypical multiparametric coefficients. Analytical reliability of our results is validated by a perfect matching with that obtained earlier by using the plane-wave analysis in planar geometry [22]. Various key factors that contribute to the stability or instability of the composite viscoelastic cloud are numerically highlighted and broadly interpreted. The non-trivial implications and applications of our semi-analytic investigation are lastly contextualised in the real astronomic circumstances.

2.2 PHYSICAL MODEL AND FORMALISM

We consider a strongly correlated viscoelastic fluid media confined in a spherically symmetric geometric construct. The justification behind the consideration of spherical geometry lies in the fact that most of the astrophysical structures are usually spherical. The system is modelled with the help of generalized hydrodynamic formalism in the presence of geometric curvature effects. A deviation from the planar geometry approximation ($R \to \infty$) towards the nonplanar spherical geometric consideration is known to play a key role in moderating the supported instabilities in diversified astrophysical situations [25]. The fluid system is strongly coupled so that both the viscosity and elasticity act on the same footing [22]. As such, the fluid system considered herein is effectively viscoelastic in nature. The basic governing equations are the equation of continuity, force-balancing momentum equation, and the gravitational Poisson equation, respectively cast as

$$\partial_t \rho + \left(r^{-2}\right)\partial_r\left(r^2 \rho v\right) = 0\ , \tag{2.1}$$

$$\begin{aligned}&\left[1+\tau_m\{\partial_t + (v)\partial_r\}\ \right]\left[\rho\ \{\partial_t + (v)\partial_r\}v + \rho\partial_r\psi + c_s^2\partial_r\rho\right] \\ &= \eta(r)^{-2}\partial_r\left\{(r)^2\partial_r v\right\} + \left\{\zeta + (3)^{-1}\eta\right\}\partial_r\left\{(r)^{-2}\partial_r\left(r^2 v\right)\right\},\end{aligned} \tag{2.2}$$

$$(r)^{-2}\partial_r\left\{(r)^2\partial_r\psi\right\} = 4\pi G(\rho - \rho_0)\,. \tag{2.3}$$

Here, r and t denote the radial and temporal coordinates, respectively. ρ denotes the mass density. Velocity of the fluid is denoted by v. τ_m is the viscoelastic relaxation time. ψ stands for the unnormalized gravitational potential. $c_s = \sqrt{P/\rho}$ denotes the phase speed of the fluid sound mode. As in the customary notations [21, 24], ζ stands for the bulk viscosity coefficient (related to the vibrational energy of the constituent molecules, denoting the effects of the fluid compressibility) and η is the shear viscosity coefficient (measuring the resistance to the fluid flow). The universal gravitational constant is denoted as $G = 6.67\times10^{-11}$ N m^2 kg^{-2}. Here, ρ_0 stands for the hydrostatic equilibrium material density accounting for the Jeans swindle in order to relax the zeroth-order gravitational force field effects as in an assumed homogeneous equilibrium.

It is noteworthy to add that the appearance of $1/r$- terms is due to the geometrical curvature effects, which are normally, otherwise, absent in the case of the planar geometric approximation ($r \to \infty$). Equation (2.1) is the equation of continuity. The net force balancing amid spatiotemporally fixed viscoelasticity coefficients on our observational spatiotemporal scales of current interest is modelled by equation (2.2). The closure of the constructed model is obtained with the help of the gravitational Poisson equation thereby relating the gravitational potential evolution with the material density perturbation as evident in equation (2.3). A standard astronormalization technique in the customary notations in the Jeansian scales of space and time [21, 24], is herewith employed in our relevant model equations. The normalized set of equations are respectively cast as

$$\partial_T D + M\,\partial_R D + D\,\partial_R M + 2(R)^{-1} DM = 0, \tag{2.4}$$

$$\begin{aligned}&\left[1+\tau_m(\tau_J)^{-1}\{\partial_T + (M)\partial_R\}\right]\left[D\{\partial_T + (M)\partial_R\}M + D\partial_R \Psi + \partial_R D\right]\\ &= \left[\chi^*\partial_R^2 M + 2(R)^{-1}\chi^*\partial_R M - 2(R)^{-2}\beta^* M\right],\end{aligned} \tag{2.5}$$

$$\partial_R^2 \Psi + 2(R)^{-1}\partial_R \Psi = (D-1). \tag{2.6}$$

Here, $R = r/\lambda_J$ denotes the Jeans normalized radial coordinate. $\lambda_J = c_s\tau_J$ is the Jeans scale length. $T = t/\tau_J$ denotes the Jeans-normalized temporal coordinate with $\tau_J = \omega_J^{-1} = (4\pi G\rho_0)^{-1/2}$ as the Jeans time scale. $D = \rho/\rho_0$ is the normalized fluid material density. $M = v/c_s$ is the normalized fluid flow speed or Mach number. $\chi^* = \chi/c_s\rho_0\lambda_J$ is the normalized effective generalized viscosity. $\beta^* = \beta/c_s\rho_0\lambda_J$ is the normalized compound viscosity. $\Psi = \psi/c_s^2$ is the normalized gravitational potential. Here, the Jeans parameters, namely, λ_J, τ_J, and the other relevant Jeans parameters such as $k_J = 2\pi/\lambda_J$, $\omega_J = (4\pi\rho_0 G)^{1/2} = \tau_J^{-1}$ have been used to normalize the parameters in astrophysical scales. Further, the expression for the Jeans mass reads as $M_J = 4\pi\rho\lambda_J^3/3$.

2.3 LINEAR STABILITY ANALYSIS

We linearly perturb the relevant physical fluid parameters F as F_1 about their respective homogeneous equilibrium values F_0 [26] given as

$$F(R,T) = F_0 + F_1 = F_0 + F_{10}(R)^{-1}\exp\left[-i\left(\Omega T - k^* R\right)\right], \tag{2.7}$$

$$F = \left[D \quad M \quad \Psi\right]^T, \tag{2.8}$$

$$F_0 = \left[1 \quad 0 \quad 0\right]^T, \tag{2.9}$$

$$F_1 = \left[D_1 \quad M_1 \quad \Psi_1\right]^T. \tag{2.10}$$

We assume a spherically symmetric geometry with perturbations along radial direction only. Perturbations along the direction given by spherical harmonics are kept for future course of investigative studies. Another important point that is noteworthy herein is that the assumption of spherically symmetric geometry simplifies the complicated three-dimensional problem into a simple one-dimensional problem, by omitting the azimuthal and polar dependence. Here, Ω $\left(\sim \omega/\omega_J\right)$ and k^* $\left(\sim k/k_J\right)$ appear in equation (2.7) in a self-consistent auto-normalized Fourier form. In the Fourier transformed wavespace, the spatial and temporal operators get transformed as $\partial/\partial R \rightarrow \left(ik^* - R^{-1}\right)$ and $\partial/\partial T \rightarrow \left(-i\ \Omega\right)$, respectively. Equation (2.7) is methodically applied to equations (2.4)-(2.6). The linearly perturbed relevant physical parameters are given in an algebraic form as

$$M_1 = i\,\Omega\left(i\,k^* + R^{-1}\right)^{-1} D_1, \tag{2.11}$$

$$\left[1 - i\ \Omega \tau_m \left(\tau_J^{-1}\right)\right]\left[-i\ \Omega\ M_1 + \left(i\ k^* - R^{-1}\right)\Psi_1 + \left(i\ k^* - R^{-1}\right) D_1\right]$$
$$= \left[\chi^* \left(2R^{-2} - 2ik^* R^{-1} - k^{*^2}\right) M_1 + 2R^{-1}\chi^* \left(ik^* - R^{-1}\right) M_1 - 2R^{-2}\beta^* M_1\right], \tag{2.12}$$

$$\Psi_1 = -D_1 \left(k^*\right)^{-2}. \tag{2.13}$$

A procedural algebraic substitution of equations (2.11) and (2.13) in equation (2.12) yields a polynomial equation in Ω and k^*, cast as

$$\left[1 - i\ \Omega \tau_m \tau_J^{-1}\right]\left[1 + \Omega^2 - R^{-2}\left\{1 - \left(k^*\right)^{-2}\right\} - k^{*2}\right] = -i\,\Omega\ \left[\chi^* k^{*2} + 2R^{-2}\beta^*\right]. \tag{2.14}$$

The plasma fluid becomes strongly coupled (significantly viscoelastic) for the coupling parameter $1 \le \Gamma \le \Gamma_c$, for which only the high frequency modes are excitable, relative to the usual viscoelastic relaxation mode $\omega \tau_m >> 1$. On the other hand, the system becomes weakly coupled for which only the low frequency modes are excited subject to the fulfillment of the threshold condition $\omega \tau_m << 1$. In the strongly coupling limit, $\omega \tau_m >> 1$, which is $\Omega \tau_m / \tau_J >> 1$ in the normalized form; we obtain the generalized quadratic dispersion relation from equation (2.14) in a new normalized form cast as

$$\Omega\ =\ \left[\left(\chi^* k^{*2} + 2\beta^* R^{-2}\right) - \left\{\left(1 - k^{*2}\right)\left(1 + \left(k^*\right)^{-2} R^{-2}\right)\right\}\right]^{\frac{1}{2}}. \tag{2.15}$$

To clearly indicate the effect of the geometrical curvature considered herein, we express equation (2.15) as

$$\Omega\ =\ \left[\left\{k^{*^2}\chi^* - 1 + k^{*^2}\right\} + \left\{2\beta^* R^{-2} + R^{-2} - k^{*^{-1^2}} R^{-2}\right\}\right]^{\frac{1}{2}}. \tag{2.16}$$

The first bracketed group containing no *R*-terms reveals exactly the same as those obtained by using planar geometry and the second one containing *R*-factors corresponds to the outcome due to the presence of geometric curvature effects. The analytic results so far explored herein are well bolstered with the help of a perfect matching of equation

(2.16), within the no-curvature limit defined by $R \to \infty$, with the previously reported dispersion relation in the planar reduced construct (equation (11) in [19]) presented as

$$\omega = \left[(\rho_0 \tau_m)^{-1} \left(\zeta + 4(3)^{-1}\eta\right) k^2 (2\pi)^{-2} - \omega_J^2 + c_s^2 \; k^2 (2\pi)^{-2}\right]^{\frac{1}{2}}. \tag{2.17}$$

The presence of the numerical constant here appears due to the unnormalized form of the dispersion relation (equation (2.17)). From equation (2.16), it is clearly evident that the perturbations would increase if

$$\left[\left(\chi^* k^{*2} + 2\beta R^{-2}\right)\right] < \left[\left(1 - k^{*2}\right)\left(1 + k^{*-2} R^{-2}\right)\right]. \tag{2.18}$$

Thus, the growth rate of the gravitational instability in a spherically symmetric self-gravitating viscoelastic plasma medium is cast as

$$\Omega_i = \left[\left(1 - k^{*2}\right)\left(1 + k^{*-2} R^{-2}\right) - \left(\chi^* \; k^{*2} + 2\beta^* R^{-2}\right)\right]^{\frac{1}{2}}. \tag{2.19}$$

Thus, we see that growth rate of the instability is influenced by the curvature effects, effective generalized viscosity, and compound viscosity.

2.4 RESULTS AND DISCUSSIONS

The dynamics of the gravitational instability is investigated in a spherically symmetric self-gravitating viscoelastic fluid medium. The system is modelled using the generalized hydrodynamic framework. A standard normal spherical mode analysis yields a unique generalized form of quadratic dispersion relation (equation (2.15)). A unique combination of bulk and shear viscosities is found to arise due to the curvature effect, in addition to the effective generalized viscosity. This new viscous influence is termed as the compound viscosity. A numerical illustrative platform is provided to clearly analyze the influence of different parameters in stabilizing/destabilizing the considered system, that is, to analyze the growth rate of the considered instability. The different input values employed herein have been taken from different trustworthy literary sources [14, 21, 24].

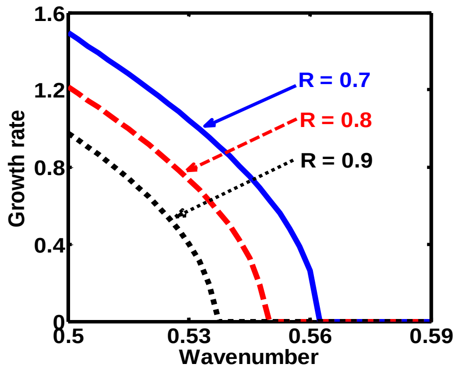

Figure 2.1: *Profiles of the Jeans-normalized growth rate* (Ω_i) *with variation in the Jeans-normalized wavenumber* k^* *for different values of the Jeans-normalized cloud size* (R). *The different lines link to* $R = 0.7$ *(blue solid line),* $R = 0.8$ *(red dashed line), and* $R = 0.9$ *(black dotted line), respectively.*

In figure 2.1, we depict the profile for the variation of normalized growth rate (Ω_i) of the system with variation in normalized wavenumber (k^*) for different values of the normalized radial size of the cloud (R). The different coloured lines correspond to $R = 0.7$ (blue solid line), $R = 0.8$ (red dashed line), and $R = 0.9$ (black dotted line), respectively. The different multiparametric input values used are $\rho_0 = 1.67 \times 10^{-5}$ kg m^{-3} ($\rho_0 = m \times n$, $n = 10^{22}$ m^{-3} is the particle concentration and $m = 1.67 \times 10^{-27}$ kg is the hydrogenic particle mass), $\tau_m = 10^{-2}$ s, $\chi = 10^{-1}$ kg m^{-1} s^{-1}, $\beta = 10^{-2}$ kg m^{-1} s^{-1}. It is seen that, as the cloud size increases with its mass and material density kept constant, the instability growth rate decreases gradually, exhibiting simultaneous shifting towards the lower k^*-regime; and vice-versa. Thus, an interesting unusual inference which could be drawn herefrom is that larger clouds for a given mass and density are more stable; and vice-versa. The physical reason for this unusual inference of the cloud stability enhancement with its size in a given specified fluidic configuration can be traced back to the unique unipolar virtue of the long-range Newtonian inverse-square law of gravitational interaction. That is, larger the interparticle distance, smaller is the gravitational interaction for a given system of particles; and vice-versa. We, herein, in other words, increase the cloud size; but, the cloud mass and particle concentration remain the same. As a result, the self-gravitational attraction (inward, organizing) gets reduced in its strength with the constituent rarefaction (dilution in concentration) and falls weaker in comparison with the radiative pressure force (outward, randomizing). The dominancy of the latter over the former is indeed actively responsible for the cloud stability enhancement with its increasing geometrical size against the canonical Jeans collapse dynamics. It is again stressed that this contrast will occur only if the radius of the spherical cloud is increased without increasing its mass or number density. In other words, the anti-gravitational cloud stability is responsible for a relatively reduced rate of structure formation in the galaxies in the open cosmos. Thus, spherical geometry introduces a curvature term which reduces the growth rate of the instability.

In figure 2.2, we depict the same as figure 2.1, but for different values of the combined viscosity (β). It is seen that, for a fixed β, Ω_i decreases towards the higher k^*-regime, and vice-versa. In fact, such growth behaviours become visibly more significant as we gradually increase the value of β from 10^{-4} kg m^{-1} s^{-1} (blue solid line) to $\beta = 10^{-3}$ kg m^{-1} s^{-1} (red dashed line), and finally, to $\beta = 10^{-2}$ kg m^{-1} s^{-1} (black dotted line). We see the three distinct Ω_i-patterns for the different β-values (figure 2.2), which enable us to conclude that Ω_i decreases with enhancement in β; and vice-versa. Thus, the geometric curvature-induced compound viscosity, β, acts as a stabilizing agency to the gravitating fluid instability dynamics. It is physically attributable to the constitutional molecular resistive influences sourced in the interlayer microscopic frictional effects.

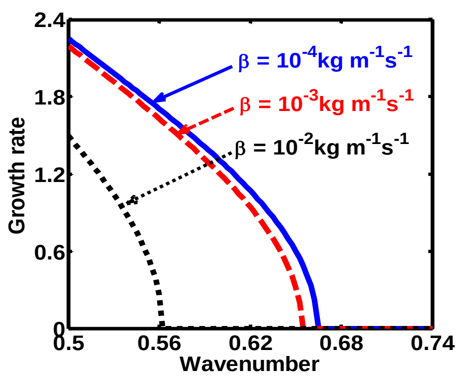

Figure 2.2: *Same as figure 2.1, but for the different viscosity* (β) *values. The varied coloured lines link to* $\beta = 10^{-4}$ *kg m*$^{-1}$ *s*$^{-1}$ *(blue solid line),* $\beta = 10^{-3}$ *kg m*$^{-1}$*s*$^{-1}$ *(red dashed line), and* $\beta = 10^{-2}$ *kg m*$^{-1}$ *s*$^{-1}$ *(black dotted line), respectively.*

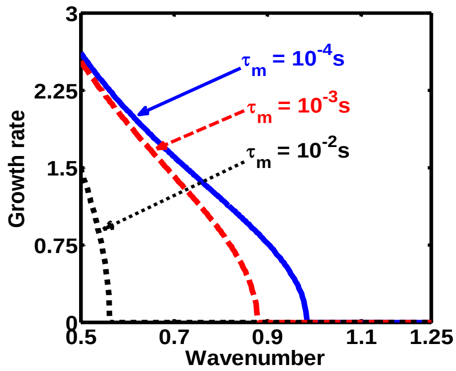

Figure 2.3: *Same as figure 2.1, but for the different values of the viscoelastic relaxation time* (τ_m) *with variation in the Jeans-normalized wavenumber* (k^*)*. The three distinct coloured lines link to different values of* $\tau_m = 10^{-4}$ *s (blue solid line),* $\tau_m = 10^{-3}$ *s (red dashed line), and* $\tau_m = 10^{-2}$ *s (black dotted line), respectively.*

In figure 2.3, we display the same as figure 2.1, but for the different values of the viscoelastic relaxation time (τ_m). The different coloured lines show the trend of the growth variation for $\tau_m = 10^{-4}$ s (blue solid line), $\tau_m = 10^{-3}$ s (red dashed line), and $\tau_m = 10^{-2}$ s (black dotted line). As τ_m increases, Ω_i decreases; and vice-versa. Thus, larger the τ_m-value, the more stable is the fluid; and vice-versa. So, τ_m plays a key role in the dynamics as a stabilizing factor. It, hereby, physically indicates a parametric index of how memory effects modify the shear wave propagation in the fluid. Thus, it is evident that τ_m is a crucial factor determining the effect of shear viscosity in the fluids. As such, it plays a key role in determining the viscoelastic behaviour. As a consequence, the viscoelastic relaxation time plays a key role in determining the stability of extremely dense astrocosmic objects and their neighbouring atmospheres collectively made up of correlative fluids.

2.5 CONCLUSIONS

The Jeans gravitational instability dynamics excited in self-gravitating spherical complex fluids is analyzed in the generalized hydrodynamic fabric on the astrophysical spatiotemporal scales. The effect of geometrical curvature moderated with viscoelasticity is specifically included in the basic model setup. A spherical normal mode analysis over the perturbed complex fluid results in a quadratic dispersion relation with atypical multiparametric coefficients. A numerical illustrative platform is constructed to identify and characterize different stability factors. It is seen that the compound viscosity (β) introduces a stabilizing influence to the instability. It is interestingly seen that the fluid system moves towards higher stability with an enhancement in the cloud size (R) and vice-versa. The viscoelastic relaxation time (τ_m) is found to play a stabilizing role on the fluid.

The investigated results in the proposed Chapter can be extensively employed in the stability analysis of degenerate molecular clouds, dense nebula, compact astrophysical structures and their dense correlated atmospheres well known to be composed of viscoelastic fluids. The derived results, despite some facts and faults, could be fairly applicable to see the instability evolutionary processes in the compact astroobjects, such as white dwarf stars, neutron stars, and so forth. Also, the theoretic analysis in the spherical geometry might come handy in rigorous application to several special astrosystems, where the planar geometry-approximation is both improper and inappropriate. It would, of course, require a proper inclusion of the gravitational effects with the judicious replacement of the non-relativistic Newtonian gravity by the relativistic Einstein gravitational formalism due to the fact that compact astrofluids are widely relativistic in nature. It is finally admitted that the presented theoretic investigation is a simplified one. It is based on the spatiotemporally fixed viscoelasticity coefficients. It hereby opens a new research scope for more refined ameliorations with all the zeroth-order inhomogeneity effects inclusively.

REFERENCES

[1] Camenzind, M. *Compact Objects in Astrophysics: White Dwarfs, Neutron Stars and Black Holes*. Springer, 2007.

[2] Bonnar, W. B. Boyle's Law and Gravitational Instability. *Monthly Notices of the Royal Astronomical Society*, 116(3): 351-359, 1956.

[3] Boss, A. P. Giant Planet Formation by Gravitational Instability. *Science*, 276(5320): 1836-1839, 1997.

[4] Nipoti, C. Local gravitational instability of stratified rotating fluids: three-dimensional criteria for gaseous discs. *Monthly Notices of the Royal Astronomical Society*, 518(4): 5154-5162, 2023

[5] Krumholz, M., McKee, C., and Klein, R. The formation of stars by gravitational collapse rather than competitive accretion. *Nature*, 438:332–334, 2005.

[6] Schonberg, M. and Chandrasekhar, S. On the evolution of main-sequence stars. *The Astrophysical Journal*, 96(2):161-172, 1942.

[7] Kippenhahn, R., Weigert, A., and Weiss, A. *Stellar Structure and Evolution*. Springer, Heidelberg, 2nd edition, 2012.

[8] *Life cycles of stars: How Supernovae are formed*. Retrieved on 18 Dec. 2022 from https://imagine.gsfc.nasa.gov/educators/lessons/xray_spectra/background-lifecycles.html

[9] Binney, J. and Tremaine, S. *Galactic Dynamics*. Princeton University Press, Princeton, 1987.

[10] Schneider, S. and Arny, T. *Pathways to Astronomy*. McGraw-Hill Education, 4th edition, 2014.

[11] Spitzer, L. *Physical Processes in the Interstellar Medium*. Wiley-Interscience Publication, New York, 1978

[12] Jeans, J. H. The Stability of a Spherical Nebula. *Philosophical Transactions of the Royal Society of London*, 199:1-53**,** 1902.

[13] Kaw, P. K. and Sen, A. Low frequency modes in strongly coupled dusty plasmas. *Physics of Plasmas*, 5(10):3552, 1998.

[14] Borah, B., Haloi, A., and Karmakar, P. K. A generalized hydrodynamic model for acoustic mode stability in viscoelastic plasma fluid. *Astrophysics and Space Science*, 361, 165 (2016)

[15] Ichimaru, S. Strongly coupled plasmas: high-density classical plasmas and degenerate electron liquids. *Reviews of Modern Physics*, 54(4):1017-1059, 1982.

[16] Bastrukov, S. I., Weber, F., and Podgainy, D. V. On the stability of global non-radial pulsations of neutron stars. *Journal of Physics G: Nuclear and Particle Physics*, 25:105-127, 1999.

[17] Brevik, I. Temperature variation in the dark cosmic fluid in the late universe. *Modern Physics Letters A*, 31(8):1650050(1)-1650050(12), 2016.

[18] Dhiman, J. S. and Sharma, R. Gravitational Instability of Cylindrical Viscoelastic Medium Permeated with Non Uniform Magnetic Field and Rotation. *Journal of Astrophysics and Astronomy*, 37:5(1)-5(15), 2016.

[19] Dhiman, J. S. and Sharma, R. Effect of rotation on the growth rate of magnetogravitational instability of a viscoelastic medium. *Physica Scripta*, 89:125001, 2014.

[20] Sharma, P. K., Tiwari, A., Khan, N., and Argal, S. Gravitational instability of an anisotropic and viscoelastic plasma. *Journal of Physics: Conference Series*, 836:012020, 2017.

[21] Kalita, D. and Karmakar, P. K. Nonlinear dynamics of gravitational instability in complex viscoelastic astrofluids. *AIP Advances*, 8(8): 085207, 2018.

[22] Janaki, M. S., Chakrabarti, N., and Banerjee, D. Jeans instability in a viscoelastic fluid. *Physics of Plasmas*, 18(1):012901, 2011.

[23] Prajapati, R. P., Sharma, P. K., Sanghvi, R. K., and Chhajlani, R. K. Jeans instability of self-gravitating magnetized strongly coupled plasma. *Journal of Physics: Conference Series*, 365, 012040, 2012.

[24] Karmakar, P. K. and Kalita, D. Dynamics of gravitational instability excitation in viscoelastic polytropic fluids. *Astrophysics and Space Science*, 363: 239, 2018.

[25] Tomisaka, K. and Ikeuchi, S. Gravitational instability of isothermal gas layers - Effect of curvature and magnetic field. *Publications of Astronomical Society of Japan*, 35(2):187-208, 1983.

[26] Karmakar, P. K. and Das, P. Nucleus-acoustic waves: Excitation, propagation, and stability. *Physics of Plasmas*, 25:082902, 2018.