

Table of Contents

| | |
|--|------|
| Abstract | iii |
| Acknowledgements | vii |
| Table of Contents | viii |
| 1 Introduction | 1 |
| 1.1 q -products | 1 |
| 1.2 Ramanujan's theta functions | 2 |
| 1.3 Partial Bell polynomials | 3 |
| 1.4 Polygonal numbers | 3 |
| 1.5 Partitions and the partition generating function | 3 |
| 1.6 Ramanujan's partition congruences | 4 |
| 1.7 The Rogers-Ramanujan continued fraction | 5 |
| 1.8 m -dissection | 5 |
| 1.9 The n -color partitions | 6 |
| 1.10 Overpartitions | 7 |
| 1.11 t -Core Partitions | 8 |
| 1.12 Objective of the thesis | 9 |
| 1.13 Overview of the chapters | 9 |
| 2 Number of representations of n as a sum of generalized polygonal numbers | 11 |
| 2.1 Introduction | 11 |

| | | |
|----------|---|-----------|
| 2.2 | Lemmas and Theorem 2.2.3 | 11 |
| 2.3 | Corollaries | 14 |
| 3 | The n-color partition function and some counting theorems | 20 |
| 3.1 | Introduction | 20 |
| 3.2 | Proof of Lemma 3.1.1 | 23 |
| 3.3 | Proof of Theorem 3.1.2 | 24 |
| 3.4 | Corollaries and an example | 25 |
| 3.5 | A special identity involving Euler's totient ϕ | 30 |
| 3.6 | A generalization of Theorem 3.1.3 and some counting theorems for n -color partitions | 31 |
| 4 | Some restricted overpartition functions | 35 |
| 4.1 | Introduction | 35 |
| 4.2 | Proof of Theorem 4.1.2 | 44 |
| 4.3 | Proof of Theorem 4.1.5 | 46 |
| 4.4 | Proof of Theorem 4.1.8 | 49 |
| 5 | Arithmetic identities for 10-cores and self-conjugate 10-cores | 56 |
| 5.1 | Introduction | 56 |
| 5.2 | Preliminaries and useful lemmas | 61 |
| 5.3 | Proof of Theorem 5.1.1 | 65 |
| 5.4 | Proof of Theorem 5.1.3 | 66 |
| 5.5 | Proof of Theorem 5.1.4 | 68 |
| 5.6 | Proof of Theorem 5.1.6 | 71 |
| 6 | Arithmetic identities for some analogues of 5-core partition func- tion | 72 |
| 6.1 | Introduction | 72 |
| 6.2 | Preliminary lemmas | 76 |
| 6.3 | Proof of Theorem 6.1.1 | 77 |

| | | |
|-----|----------------------------------|-----------|
| 6.4 | Proof of Theorem 6.1.3 | 80 |
| 6.5 | Proof of Theorem 6.1.4 | 82 |
| 6.6 | Concluding observation | 84 |
| | Bibliography | 86 |