Abstract

In this thesis, we find new arithmetic identities involving sums of polygonal numbers, n-color partitions, certain restricted overpartitions, the 10-core partitions and some analogues of the 5-core partition function. The approaches utilized to discover the mentioned arithmetic identities encompass various techniques such as combinatorial reasoning, dissections of q-products, Ramanujan's theta function identities, and certain identities related to the Rogers-Ramanujan continued fraction.

Recently, Jha (arXiv:2011.11038, 2020; *Rocky Mountain Journal of Mathematics*, 51:581–583, 2021) has found identities that connect certain sums over the divisors of n to the number of representations of n as a sum of squares and triangular numbers. In Chapter 2, we prove a generalized result that gives such relations for s-gonal numbers for any integer $s \geq 3$. Jha's results follow as corollaries.

Merca and Schmidt (*The American Mathematical Monthly*, 125:929–933, 2018; *The Ramanujan Journal*, 49:87–96, 2019) found some decompositions for the partition function $p(n)$ in terms of the classical Möbius function as well as Euler's totient. In Chapter 3, we define a counting function $T_k^r(m)$ on the set of n-color partitions of m for given positive integers k, r and relate the function with the n-color partition function and other well-known arithmetic functions like the Möbius function, Liouville function, etc. and their divisor sums. Furthermore, we use a counting method of Erdös to obtain some counting theorems for n-color partitions that are analogous to those found by Andrews and Deutsch (*Integers*, 16: Article A24, 2016) for the partition function.

In Chapter 4, we prove arithmetic properties of some restricted overpartition

functions in which the parts are from certain residue classes of 8. For example, if $\overline{p}_{3,4,5}(n)$ and $\overline{p}_{1,4,7}(n)$ denote the number of overpartitions of a positive integer n into parts congruent to $3, 4$, or 5 modulo 8 and congruent to $1, 4$, or 7 modulo 8, respectively, then $\overline{p}_{3,4,5}(16n + 1) \equiv 0 \pmod{16}$ and $\overline{p}_{1,4,7}(16n + 9) \equiv 0 \pmod{16}$ for all nonnegative integers n.

Chapter 5 is devoted to find new linear recurrence relations satisfied by the 10-core and self-conjugate 10-core partition functions. In the process, we also find several exact generating function representations and congruences satisfied by those functions.

Gireesh, Ray and Shivashankar (*Acta Arithmetica*, 199:33-53, 2021) have considered an analogue $\overline{a}_t(n)$ of the t-core partition function $c_t(n)$ and proved new identities involving $\overline{a}_t(n)$. In Chapter 6, we revisit the function $\overline{a}_5(n)$ in conjunction with $c_5(n)$ as well as another analogous function $\overline{b}_5(n)$. We find new recurrence relations for $\overline{a}_5$, $\overline{b}_5$ and relations among $c_5(n)$, $\overline{a}_5(n)$ and $\overline{b}_5(n)$. In the process, we prove a stronger version of one of their congruences for $\overline{a}_5(n)$.