

DECLARATION OF THE CANDIDATE

I, **Kallol Ray**, hereby declare that the subject matter in this thesis entitled “**Discontinuous Galerkin Finite Element Methods for the Incompressible Fluid Flows**” is the record of work done by me, that the contents of this thesis did not form the basis of the award of any previous degree to me or to the best of my knowledge to anybody else, and that the thesis has not been submitted by me for any research degree in any other university/ institute. This thesis is being submitted by me to Tezpur University for the degree of Doctor of Philosophy in the department of Mathematical Sciences.

Place: Tezpur

Date:

Signature of the Candidate

DECLARATION OF THE CANDIDATE

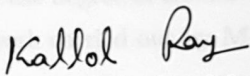
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CERTIFICATE OF THE SUPERVISOR


The undersigned hereby certifies that the thesis entitled "**Discontinuous Galerkin Finite Element Methods for the Incompressible Fluid Flows**" submitted to the School of Science, Tezpur University is a record of research work done by the candidate for the award of the degree of Doctor of Philosophy in the department of Mathematical Sciences. The contents of the thesis have been duly acknowledged and approved by the undersigned.

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This is to certify that the thesis entitled “**Discontinuous Galerkin Finite Element Methods for the Incompressible Fluid Flows**” submitted to the School of Sciences, Tezpur University in partial fulfillment for the award of the degree of Doctor of Philosophy in Mathematical Sciences is a record of research work carried out by **Mr. Kallol Ray** under my supervision and guidance.

All help received by him from various sources have been duly acknowledged.

No part of this thesis has been submitted elsewhere for award of any other degree.

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Place: Tezpur

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Dr. Deepjyoti Goswami

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Place: Tezpur

Date:

Kallol Ray

Dedicated to
my Mother
for her love, sacrifice and devotion

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