ABSTRACT

This thesis analyzes discontinuous Galerkin (DG) finite element methods for three incompressible fluid flows, namely, the Navier-Stokes equations, that describes viscous Newtonian flows, and the Kelvin-Voigt model and the Oldroyd model of order one, that represent linear viscoelastic flows, and can be treated as smooth perturbations of the Navier-Stokes equations. A primal DG method to solve nonlinear problems comes with a high computational costs, and as a result, a study of a combined approach involving a cost-effective two-grid method alongside the DG method is conducted for these three fluid flows, which are first of its kind in these directions.

We study a DG finite element method for the transient Navier-Stokes equations. Optimal velocity and pressure error estimates in $L^\infty(\mathbf{L}^2)$ and L^∞ (L^2)-norms, respectively, for the semi-discrete DG case are derived. In order to attain these results, we employ the newly introduced L^2-projection and modified Stokes operator on appropriate broken Sobolev spaces, and then standard duality arguments. Estimates are shown to be uniform in time under smallness condition on the data. Then, a completely discrete scheme is analyzed where the time discretization is based on the first-order backward Euler method and fully discrete error estimates are derived.

We next propose and analyze a DG formulation for the Kelvin-Voigt model, a first of its kind. Based on new *a priori* and regularity bounds for the semi-discrete discontinuous solutions, well-posedness of the DG scheme is discussed. Optimal semi-discrete error estimates of the velocity and pressure in $L^\infty(\mathbf{L}^2)$ and $L^\infty(L^2)$-norms, respectively, are established. The standard elliptic duality argument and a newly developed modified Sobolev-Stokes operator defined on a suitable DG finite element space are the foundations of the analysis. The error estimates are uniform in time for sufficiently small data. Then, based on the backward Euler method, a completely discrete scheme is analyzed. *A priori* bounds for the fully discrete solution, and optimal fully discrete error estimates for the velocity and pressure are derived.

Then, we employ a DG finite element method for the Oldroyd model of order one, for the first time. Based on the new *a priori* results for the semi-discrete solution, well-posedness and consistency of the DG scheme are discussed. A fully discrete scheme with the backward Euler method for time discretization is studied, and error estimates for the velocity and pressure in energy and $L^2(L^2)$-norms, respectively, are established. In addition, optimal fully discrete $L^\infty(\mathbf{L}^2)$ and $L^\infty(L^2)$-norms error estimates for the

velocity and pressure, respectively, are derived with the help of a modified Stokes-Volterra projection.

Next, a two-step two-grid scheme combined with DG approximations is utilized for the transient Navier-Stokes equations. In the first step, we solve the nonlinear problem on a coarse mesh, and then linearize the nonlinear problem with one Newton iteration around the coarse grid solution and solve it over a fine mesh in the second step. Optimal semi-discrete error estimates of the two-grid DG approximations for the velocity and pressure in energy and L^2-norms, respectively, are derived for an appropriate choice of coarse and fine mesh parameters. Under smallness condition on data, these estimates are shown uniformly with time. A full discretization of the semi-discrete two-grid model is achieved by applying the backward Euler method in the time direction, and fully discrete error estimates are derived.

We then analyze a three-step two-grid method based on DG approximations for the Kelvin-Voigt model. The first two steps are similar to what we have attempted for the Navier-Stokes equations, as stated above. We introduce here a third step, which is a correction step for the solutions of the second step on the fine mesh. With the help of newly derived interpolated Sobolev and trace inequalities, optimal semi-discrete error estimates for the velocity in $\mathbf{L}^2$ and energy norms and for the pressure in L^2-norm are derived, for an suitable choice of coarse and fine mesh parameters. Also, for sufficiently small data, uniform in time error estimates are presented. We discretize the semi-discrete two-grid DG model in time, using the backward Euler method and derive the fully discrete error estimates.

Finally, a three-step two-grid algorithm combined with the DG method for the Oldroyd model of order one is studied. We employ the same algorithm which is applied to the Kelvin-Voigt model. In this case, the resulting scheme is a fully discrete scheme where the time discretization is achieved using the backward Euler method. Optimal error estimates for the velocity in $\mathbf{L}^2$ and energy norms and for the pressure in L^2-norm are established under certain relations between the coarse and fine mesh sizes.

Numerical experiments are carried out in support of all our theoretical findings.

We conclude the thesis with a brief summary of our results and a brief discussion of our future problems.