Chapter-1

A BRIEF OVERVIEW OF FLUIDS EXISTENT ON DIVERSE SCALES

Abstract: *A brief overview of diverse fluids spanning a broad spectrum of naturalistic existence is systematically presented. The fundamental processes governing the various structure formation mechanisms in diversified circumstances ranging from neutral to non-neutral configurations are briefed. The significance of laboratory plasma sheath existing in different astrophysical, stellar, and space environments is illustrated. At the last, the equilibrium structure of the Sun and its circumambient atmosphere is summarily described in the fabric of existing diverse plasma fluid models alongside extensive scope.*

1.1 INTRODUCTION

A basic postulate behind understanding a system of free particles with the help of fluid model formalism is to consider the model system as a continuum. This is because of the fact that the basic fluid equations concern relevant physical quantities, such as fluid velocity, density, pressure, temperature, etc., which are assumed to vary continuously from point to point in every part throughout the fluid. We suppose that the macroscopic properties can be associated with any volume of the fluid, however small it is, which are also associated with the fluid in bulk. It enables us to consider that, at each point in the fluid model volume; there is a fluid element, such that the fluid as a whole consists of a continuous sum of such elements. Each of such elements has a particular value of the associated physical quantities. But this assumption is not true at the molecular level (micro-scale) of the fluid due to the high fluctuations of the macroscopic properties. However, we can formulate the basic governing equations based on the continuum fluid hypothesis in such model circumstances. It is methodically achieved by defining a fluid element, which is large enough to include a large number of molecules (constitutive particles), thereby validating the entire macroscopic (bulk) fluid properties at any point in the fluid region (macro-scale). It is possible because various macroscopic properties are defined by averaging them over a large number of fluid constitutive molecules [1].

In figure 1.1, we display the variation of the fluid temperature due to the Brownian motion of its constituent particles with the logarithmic length scale specifying the fluid extension in the spatial region of concern [1]. On the molecular length scale (~ L_1), the temperature fluctuates very highly. Here, L_1 is on the order of the inter-particle distance. The temperature change due to an increase in the length scale depends particularly on the random speeds of the incoming molecules. As the length scale increases (~ $L_2 >> L_1$), a large number of molecules are accommodated in the volume and the temperature fluctuation becomes negligible, towards a steady-state temperature. Again, on a wider length scale (~ $L_3 >> L_2$), the volume becomes large enough to extend to regions of different temperature. The definition of a fluid element in the fabric of continuum hypothesis is applicable only in the intermediate region (~ L_2), where the macroscopic properties can be distinctly defined. One may consider the fluid element within the order of the length scale L_2, regarding it to be infinitesimal to the extent that macroscopic effects are concerned and formulate the fluid dynamical evolution equations, ignoring the fluctuations of the fluid properties on microscopic length scales.

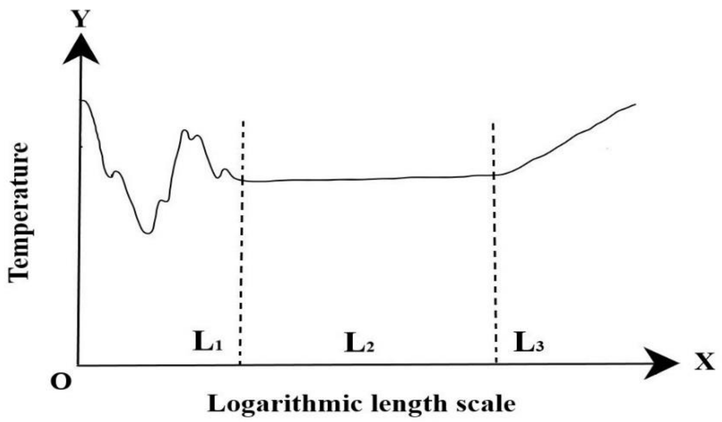

Figure 1.1: *Schematic variation of the fluid temperature (average Brownian molecular energy) with the system scale length (logarithmic scale).*

According to the continuum hypothesis, a fluid element at one point at one time is physically the same as that at another point at another time, although the constitutive molecules are incessantly colliding with each other [1, 2]. Thus, if λ is the molecular mean free path, we must have $\lambda << L_2$ such that a molecule can undergo many collisions with the neighbours without causing any effect on the dynamics of the fluid element.

The fluid dynamical equations can be formulated in two sensible approaches: Eulerian and Lagrangian model formalisms. In the first one, the frame of reference is considered to be fixed at a particular point in space. The time evolution of a physical quantity is studied at that fixed position in space. Thus, the Eulerian description explains the fluid dynamics as seen from a particular coordinate in real space. The Eulerian approach is useful if the motion of individual fluid elements is not important [1, 3]. In the second approach, the reference frame is considered to be fixed with a particular fluid element, which is in motion with the bulk fluidic flow. The time evolution of a physical quantity characterizing the fluid is studied at that particular fluid element. Thus, the Lagrangian description explains the fluid dynamics as seen by an observer riding with the particular element (commoving). The Lagrangian fluid model description is extensively useful if the fluid properties uniquely distinguish a particular constitutive fluid element from the remaining ones in the same fluidic setup [1, 3].

1.2 THE FLUID WITH IONIZED COMPONENTS: PLASMA

If a fluid (gas) is ionized (fully or partially), then the state called plasma if it is quasi-neutral and shows collective behaviour [4]. The quasi-neutrality condition is satisfied when the macroscopic fluidic state is neutral enough so that one can accept $n_e \approx n_i = n$, where n_e is electron number density, n_i is ion number density, and n is a common density called the quasi-neutral plasma density. However, it is not so much neutral that the long-range electro-dynamic effects vanish. Here, the "collective behaviour" means that the motion of the constitutive plasma elements under consideration depends not only on the local conditions, but also on the macroscopic state of that plasma in the remote regions as well, unlike neutral gaseous matter dominated by short-range interactions [4, 5].

1.3 FLUIDS IN THE UNIVERSE

The most common states of matter composed of protons and neutrons in the Universe can be well studied with the help of fluid dynamical approach [3]. In the astrophysical scenarios, the liquid state is not as frequently encountered as the gaseous state is. The former state is widely relevant in the high-pressure environments of planetary interiors, exteriors, and circumvents [3, 6]. In contrast, the latter, i.e., the fluid dynamical approach finds extensive scope in successfully modelling the stellar interiors and exteriors, stellar winds, jets, accretion disks, interstellar medium (ISM), intergalactic medium (IGM), intracluster medium (ICM), and so forth [3, 6].

For a system of neutral atoms or molecules, in which the interspecies interaction is only through the short-range van der Waals force, the collisional mean free path is expressed as

$$\lambda = \frac{1}{n\sigma} \quad . \tag{1.1}$$

Here, n and σ are the number density of the collisional partners and the corresponding scattering cross-section, respectively. In the molecular level, one finds $\sigma \sim 10^{-19}$ m^2. Near the sea level, the air has $n \sim 10^{25}$ m^{-3}. Then, $\lambda \sim 10^{-6}$ m facilitates the fluid dynamic approach [2]. On the other hand, the interstellar space is much more rarefied. For an atomic hydrogen gas cloud (H I region), one gets $n \sim 10^7$ m^{-3}. Then the molecular mean free path becomes $\lambda \sim 10^{12}$ m, which is about the distance from the Sun to the Jupiter. This distance is very large as compared to everyday encountered human standards. But typically, the H I clouds span length scale of $L \sim 10^{17}$ m or more. Thus, the condition, $\lambda << L$, here justifies the validation of the fluid dynamical treatment to be applied. A similar mathematical justification is extensively valid for the most of the constitutive thermal gases distributed widely in the entire interstellar space and medium [2].

1.4 EXISTENCE OF THE PLASMA FLUID

It is well known that matter in the known universe is often classified as solid, liquid, gas, and plasma. The first three states are distinguished by the difference in the inter-particle bond strength of the constituent particles. These inter-particle binding forces are the strong in solids, weak in liquids and almost negligible in case of gases. The equilibrium between the particle thermal kinetic energy and the inter-particle binding potential energy determines the state among the first three. If sufficient energy is provided to a molecular gas, it will gradually dissociate into an atomic gas by overcoming the molecular binding energy as a result of collision among the molecules. As the temperature is raised sufficiently, an increasing fraction of atoms gain sufficient energy due to the collisions, to get ionized by releasing the outermost orbital electrons. Hence, a plasma state is reached. It is to be noted here that the transition from a gas to a plasma state is not a phase transition thermodynamically, because ionization of the gas occurs gradually with an increase in the temperature of the gas in a continuous manner [5].

It is well known that the plasma state mostly exists in a vacuum-like physical system configuration. If not so, the presence and impact of ambient air molecules cool it down. Consequently, the process of recombination of the plasma constituent electrons and ions may transform the plasma back into a neutral gaseous state. Therefore, in a laboratory, plasma is generated in an evacuated chamber (Pressure ~ 0.1 Pa, Temperature ~ 300 K [4]). In such circumstances, plasmas are produced by various ionization processes, such as photoionization, electric field discharge, thermal ionization, etc. Such processes significantly increase the degree of ionization compared to the negligible charged-to-neutral particle number density ratio of a normal gas in a normal environment [4, 5]. In contrast to the above, most of the matter in the interstellar environments of the cosmos naturally exists in the plasma state because of such ionization processes [4].

A fundamental property of the plasma is its ability to shield any external electric potential applied to it by forming a cloud of opposite polarity charges around the applied source of charge. In hypothetical cold plasmas, without any particle-thermal motion, this shielding will be perfect and there will not be any electric field in the plasma body outside the shielding cloud. However, for real plasmas with a finite temperature (T), the particles at the edge can escape by overcoming the weak long-range electrostatic potential well (ϕ). This cloud-edge occurs at the radius where the particle thermal energy ($k_B T$) is approximately equal to the electrostatic potential energy ($e\phi$). Due to the leakage of the charged particles, a finite magnitude of electric field exists in that region. The shielding effect is spatially measured by a screening scale length, termed as the Debye length (λ_D). The Debye length is the distance relative to the source point, where the externally applied potential reduces by a factor of e ≈ 2.72 (i.e., $\phi_{\lambda_D} = \phi_0 / \mathrm{e}$, where ϕ_0 is the maximum effective potential at the location of the external charge source). The Debye length in generic notations is mathematically expressed as

$$\lambda_D = \left(\frac{\varepsilon_0 k_B T_e}{n_0 e^2} \right)^{1/2} . \tag{1.2}$$

Here, ε_0 is the permittivity of free space, k_B is the Boltzmann constant, T_e is the electron temperature (in Kelvin scale), n_0 is the equilibrium electron density in the quasi-neutral plasma where the electric potential is almost zero and e is the electronic charge [4, 5].

To maintain quasi-neutrality, the scale length L of the global plasma must be very large compared to the Debye length. At this point, the density of an ionized gas defines whether the gas qualifies to be studied as plasmas. Let the number of particles in the sphere with radius of the order of the Debye length be N_D. The value of N_D must be very large to cause well shielding of the external potential. Again, the long-range electrostatic effect to be pronounced, the collision of the charged particles with the neutrals in the plasma has to be restricted. The plasma oscillation frequency (ω) and the mean time between collisions with the neutrals (τ) must be coupled via the relation $\omega\tau > 1$ [4].

To summarize, a medium to be in the plasma state, it should fulfil the following three fundamental conditions:

1. $L >> \lambda_D$ (Characteristic plasma space dimension),
2. $N_D >> 1$ (Supercritical Debye number), (1.3)
3. $\omega\tau > 1$ (Plasma frequency condition).

1.4.1 PLASMA SHEATH FORMATION MECHANISM

The plasma sheath is a non-neutral space-charge layer composed of less mobile species (inertial), found near the wall of a vacuum chamber in which the plasma is produced. In a general plasma (hydrogenic) environment, when electrons and ions hit the wall, they get lost by recombination. The light and hot electrons compared to the heavy and cold ions can leave the bulk plasma body very easily making the plasma potential positive with respect to the confining wall. Due to development of a negative wall potential, the positive ions tend to accumulate in that region. The negative potential of the wall gets shielded by the Debye screening mechanism on a scale length comparable to a few Debye lengths. The plasma sheath acts as an electrostatic potential barrier for the more mobile species (electrons) by confining them in the plasma system electrostatically [4].

1.4.2 THE BOHM SHEATH CRITERION

The dynamics of plasma sheath formation is usually found to require a fundamental condition to be well fulfilled. Accordingly, David Bohm derived an analytic condition on the threshold ion flow speed in the sheath entrance region for the formation of the plasma sheath. This local condition on the threshold ionic flow in the sheath-entrance point is known as the Bohm sheath criterion. In the steady-state, the ion flux reaching the wall of the plasma container by overcoming the potential barrier balances the electron flux (floating condition) to nullify the electron deficit in the plasma. This ion speed, according to the Bohm sheath criterion must be at least as high as ion sound phase speed in the bulk (interior) plasma medium [4]. This sheath formation criterion is mathematically expressed as $M \geq 1$, where M is the Mach number at the sheath-edge, normalized to the acoustic phase speed in the bulk plasma medium. This condition is obtained following a standard procedure of functional monotonicity in association with the matter and energy conservation laws followed by the plasma constitutive species [4].

1.5 STRUCTURE FORMATION IN ASTROPHYSICAL PLASMA FLUIDS

As encountered in **Sections 1.3** and **1.4**, plasma-fluids are extensively found in the interstellar media, circumstellar environments, molecular clouds, etc. The origin of gravitational condensation in the uniformly distributed fluids leading to the formation of the bounded structures can be traced back to Newton's initial conjectures. Nonetheless, the initial mathematical analysis is attributed to James Jeans, analysing the stability of nebular gaseous media on the astrophysical spatiotemporal scales, reported in 1902 [7].

1.5.1 JEANS INSTABILITY

The initiation of bounded structures in the cosmos is the mechanical instability in the interstellar gas clouds resulting in their fragmentation and collapse. This instability is known as the Jeans instability, named after British physicist Sir James Jeans. Bounded structures occur when the internal gas pressure (outward) in the cloud is not strong enough to counteract self-gravitational pressure (inward) of the cloud. The instability gets triggered when the cloud mass exceeds a critical mass limit (Jeans mass); otherwise, the cloud remains stable and this instability results in no bounded structure formation [7].

It is evident from above that star formation in molecular clouds is primarily governed by the Jeans instability. The critical conditions, known as the Jeans criteria, indicate that a cloud becomes less stable against gravitational collapse with increase in mass, reduction in size, and decrease in gas temperature of the cloud [6, 7]. The figure below depicts a schematic diagram of a typical gravito-thermally bounded spherical molecular cloud alongside its diverse constitutive species and operative forces.

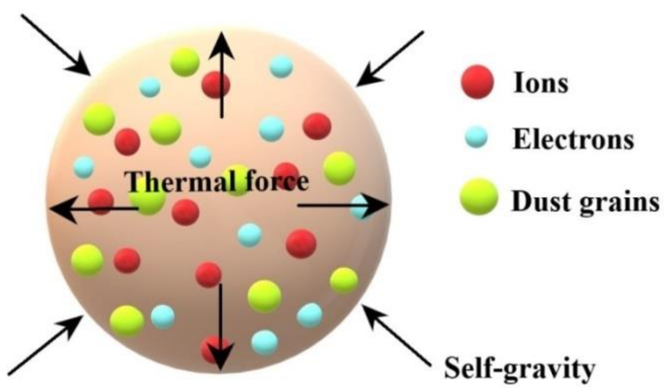

Figure 1.2: *Cartoonist sketch of a gravito-thermally bounded spherical molecular cloud.*

1.5.2 SHEATH FORMATION IN ASTROPHYSICAL PLASMAS

Like in the laboratory scale, the plasma-wall interaction and various surface mechanisms have been speculated in the various astrophysical and space plasma environments. Two of such astro-events are γ-ray and X-ray bursts, which can be explained as exploding double layers [8]. It has also been pointed out that the relativistic double layers in the interstellar space may be the source of acceleration of the ions to cosmic ray energies [8]. The plasma sheath structures have recently been well reported on the sunlit regions of the Moon. Such lunar plasma sheath environments are created by the interaction of highly energetic photons as well as the solar wind particles with the surface of the moon [9]. Though there happens to be no physical wall, the self-gravity of the system acts as a non-rigid physical potential barrier to well-structurize the heavier plasma species by restraining their flow than the lighter ones. Such an idea has already been applied to formulate a new model of the Sun, known as the gravito-electrostatic sheath (GES) model, to confront some fundamental solar plasma issues [10]. The GES model enables us to see the Sun as a bi-scaled plasma system. The solar self-gravity-bounded plasma is known as the solar interior plasma (SIP) and the unbounded plasma medium with infinite radial extent is known as the solar wind plasma (SWP). The two plasma layers are coupled via the non-rigid interfacial solar surface boundary (SSB). This model is successful enough in explaining the solar surface origin of the SWP by the dynamic conversion of the SIP. This GES links the Sun with its atmosphere from a new plasma-wall interaction viewpoint as previously encountered on the laboratory scales only [10].

1.6 SOLAR (STELLAR) PLASMA STRUCTURE

The Sun, its surrounding atmosphere, and the intricate interplay between them remain a fascinating realm yet to be well explored, which offer insights into diverse cosmic plasma phenomena. The Sun emerged about 4.6 billion years ago, from a condensed interstellar cloud through the Jeans collapse mechanism. As a luminous sphere of hot plasma in steady-state, it sustains incandescence through nuclear fusion, transforming hydrogen into helium at its core. This steady-state equilibrium structure results from the ceaseless balancing of the gravitational, thermal, and electromagnetic forces [6]. The atmospheric evolution of the Sun significantly impacts the Earth's space weather and climate. As our closest star, it plays a central role in exploring other similar stars (G-class) and various astrophysical phenomena, such as the solar dynamos, coronal mass ejections, sunspots, magnetic turbulence, solar wind dynamics, and so forth [11, 12].

The Sun is an ordinary G-type main sequence star categorized in the spectral class G2V. It has an absolute magnitude of 4.83 [6]. It is mostly composed of Hydrogen (~73%) and Helium (~25%). The rest of the mass comes from the much smaller quantities of the heavier elements like Oxygen, Carbon, Iron, Neon, Nitrogen, etc. [11, 12]. The fundamental physical properties of the Sun, according to the SSM, are listed in Table 1.1 as widely found in the literature [11].

Table 1.1: Physical properties of the Sun according to the SSM

S. No.	Physical parameter	Characteristic value
1	Age	4.6×10^{9} years
2	Mass (M_{Θ})	2×10^{30} kg
3	Radius (R_{Θ})	6.96×10^{8} m
4	Average density	1.4×10^{3} kg m^{-3}
5	Average distance from Earth	1.49×10^{11} m
6	Surface gravitational acceleration	2.74×10^{2} m s^{-2}
7	Luminosity (L_{Θ})	3.84×10^{26} J s^{-1}
8	Core temperature	1.5×10^{7} K
9	Surface temperature	5.775×10^{3} K
10	Coronal temperature	10^{6} K
11	Average magnetic field	1 G

1.6.1 STANDARD SOLAR MODEL (SSM)

According to the widely accepted SSM, the Sun lacks an actual surface. However, what can be perceived is a zone where the solar plasma medium becomes optically less dense, allowing photons from that area to move through space without much obstruction. That is, a distinct and well-defined solar boundary is visible due to the alteration in the optical density. The transition from optically thin to optically thick region in the solar atmosphere occurs within a mere radial span of approximately 600 km. This relatively minute distance (approximately 0.09% of R_{Θ}) imparts the sharpness to the Sun's edge [6].

The solar interior, in accordance with the SSM can be classified into three distinct regions based on their energy transport mechanisms as the core, the radiative zone and the convective zone in the radially outward direction. Energy is generated in the core from the thermonuclear fusion reaction. Nearing 99% of the solar energy is produced in the core

[12]. Next to the core, there lies the comparatively dense radiative zone, where the core-generated energy is transported via the thermal radiation processes. After the radiative zone, there exists the convective zone. Here, the material density is suitable for energy transport via the convective circulation dynamics. The transition region between the dense uniformly rotating radiative interior and the differentially rotating outer convective zone is known as the tachocline [11-13].

The solar atmosphere, in the same way can be classified into various spherically symmetric layers with varying physical properties as the photosphere, chromosphere, transition region and corona radially outwards [11, 12]. A schematic diagram of the solar constitutive interior and exterior layers as discussed above is shown in figure 1.3.

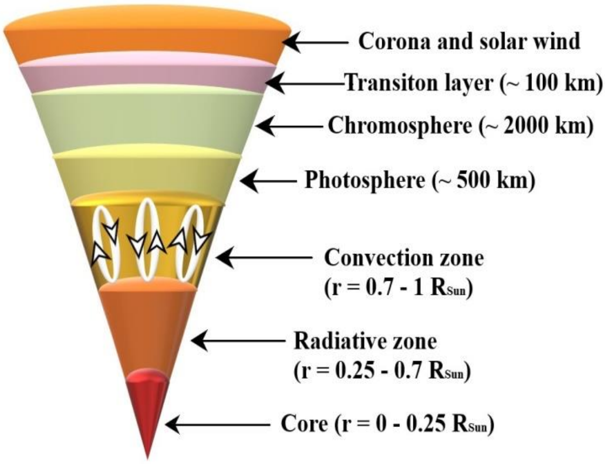

Figure 1.3: *Cartoonist sketch of the structure of the Sun according to the SSM.*

1.6.2 GRAVITO-ELECTROSTATIC SHEATH (GES) MODEL OF THE SUN

The conventional solar model based on hydrostatic equilibrium as described above is successful enough in explicating various physical properties of the Sun, such as its mass, radius, temperature, and density in different layers, surface gravity, etc. [11, 12]. To investigate the solar plasma flow dynamics, E. N. Parker in 1958 developed the first hydrodynamic model of the solar atmosphere which could explain clearly the continuous hot solar plasma mass expansion into the space extending up to infinity. According to this model, the mechanism behind the acceleration of the solar wind plasma from the subsonic to the supersonic/hypersonic flow is the physics of the de Laval nozzle effect over differing cross-section [14, 15]. However, the Parker model breaks down in coupling the solar interior plasma medium with the solar exterior/wind plasma system. Besides, several other crucial drawbacks of the Parker model have been the addressed in the literature, introducing the basic GES model of the bi-scaled solar plasma system composed of the bounded Sun and its unbounded surrounding atmosphere [10].

The GES solar model is based on the application of the fundamental principles of the laboratory scale plasma-wall interaction mechanisms to the astrophysical scales of space and time. This model successfully explains the origin of the supersonic SWP flow from the dynamic conversion of the subsonic SIP via a non-rigid interfacial SSB. The SSB is uniquely defined by the gravito-electrostatic force balancing. This model explains the Sun as a bi-scaled plasma system of the SIP and the SWP coupled via the SSB. All the physics behind the genesis of the GES model have been elucidated in the available literature [10, 16]. A schematic diagram of the Sun and its ambient atmosphere, and the distinct particle flow processes in the GES model perspective are depicted in figure 1.4.

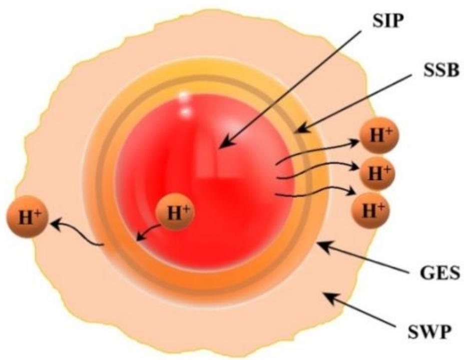

Figure 1.4: *Schematic diagram of the Sun with its surrounding as per the GES model.*

1.6.3 POLYTROPIC SOLAR PLASMA FLUID

The main sequence stars like the Sun are self-gravitating bodies at hydrostatic equilibrium in which the forces (inward gravity and outward thermal forces) on any fluid element almost counter balance each other [6]. A polytropic star is a generalized model developed to study the dynamics and structurization of the stellar fluid. In this model, a simple relation between pressure P and material density ρ is assumed as

$$P(\rho) = k\rho^{\gamma} = k\rho^{(n+1)/n}, \tag{1.4}$$

where k and $\gamma > 0$, and are constants. Here, k, γ, and n are known as the polytropic constant, polytropic exponent, and polytropic index, respectively. Equation (1.4) is known as the polytropic equation of state [6, 13]. From here we see that the polytropic index is related to the polytropic exponent by the relation

$$n = \frac{1}{\gamma - 1}. \tag{1.5}$$

A polytropic index n = 3 is usually used to model the main-sequence stars like the Sun, particularly in the radiative layer [13]. Moreover, a polytropic model may also be developed to study the evolution of the stellar atmospheres and structurizations [16, 17].

1.6.4 NON-THERMAL SPACE PLASMA

For a particle system in thermal equilibrium, for example, the air in the Earth's atmosphere, the particles are distributed such that there are many particles with intermediate velocity range and very few with the extremely small or large velocity edges. The mathematical equation describing how many particles are observed at each velocity is given by the "Maxwellian distribution." However, it is commonly seen in the space plasma system that there are more high-velocity particles than there should be, if the space plasma were in thermal equilibrium. The mathematical equation used to describe such non-thermal space plasma particle velocity distribution is known as the "kappa (κ) distribution." The stronger the inter-particle correlation, the further the plasma medium resides from thermal equilibrium that is higher degree of non-thermality. The existence of strong inter-particle correlations warrants for a collision-less plasma; as otherwise, collisions destroy the correlation among plasma constituent species [18, 19].

Space plasmas in diverse realms of existence, such as the solar wind, planetary magnetospheres, and the outer heliosphere are the systems well known to be out of thermal equilibrium. These plasma systems are better modelled by the generalized structure of a κ-distribution rather than a Maxwellian one, where the particles with higher velocities are well incorporated when κ has some finite small value. Generally, the larger the κ-parameter, the closer the plasma medium goes towards the thermal equilibrium. When the κ reaches infinity, the plasma is exactly at thermal equilibrium and the distribution reduces to a normal Maxwellian one [18, 19].

REFERENCES

[1] Tritton, D. J. *Physical Fluid Dynamics*. Oxford University Press, Inc., New York, 2nd edition, 1988.

[2] Shu, F. *The Physics of Astrophysics Vol. II: Gas Dynamics*. University Science Books, California, 1992.

[3] Clarke, C. and Carswell, R. *Principles of Astrophysical Fluid Dynamics*. Cambridge University Press, Cambridge, UK, 2007.

[4] Chen, F. F. *Introduction to Plasma Physics and Controlled Fusion*. Springer, New York, 1984.

[5] Bittencourt, J. A. *Fundamentals of Plasma Physics*. Springer, New York, 2004.

[6] Carroll, W. and Ostlie, D. A. *An Introduction to Modern Astrophysics*. Cambridge, UK, 2017.

[7] Jeans, J. H. The stability of a spherical nebula. *Philosophical Transaction of the Royal Society*, 199:1-53, 1902.

[8] Alfven, H. Double layers and circuits in astrophysics. *IEEE Transactions on Plasma Science*, PS-14 (6):779-793, 1986.

[9] Sana, T. and Mishra, S. K. Plasma sheath around sunlit moon: monotonic and non-monotonic structures. *Monthly Notices of the Royal Astronomical Society*, 520 (1):233–246, 2023.

[10] Dwivedi, C. B., Karmakar, P. K., and Tripathy, S. C. A gravito-electrostatic sheath model for surface origin of subsonic solar wind plasma. *Astrophysical Journal*, 663:1340-1353, 2007.

[11] Stix, M. *The Sun: An Introduction*. Springer, Berlin, 1991.

[12] Priest, E. *Magnetohydrodynamics of the Sun*. Cambridge University Press, Cambridge, New York, 2014.

[13] Kippenhahn, R., Weigert, A., and Weiss, A. *Stellar Structure and Evolution*. Springer, Berlin, 2012.

[14] Parker, E. N. Dynamics of the interplanetary gas and magnetic fields. *Astrophysical Journal*, 128:664-676, 1958.

[15] Ockendon, H. and Ockendon, J. R. *Waves and Compressible Flow*. Springer, Berlin, 2004.

[16] Sarma, P. and Karmakar, P. K. Solar plasma characterization in Kappa (κ)-modified polytropic turbomagnetic GES-model perspective. *Monthly Notices of the Royal Astronomical Society*, 519(2):2879-2916, 2023.

[17] Sarma, P. and Karmakar, P. K. Analyzing non-thermal steady solar plasmas in the κ-modified polytropic GES model framework. *Journal of Astrophysics and Astronomy*, 43:46(1)-46(7), 2022.

[18] Livadiotis, G. and McComas, D. J. Understanding kappa distributions: a toolbox for space science and astrophysics. *Space Science Reviews*, 175:183–214, 2013.

[19] Livadiotis, G. *Kappa Distributions: Theory and Applications in Plasmas*. Elsevier, Amsterdam, 2017.