Chapter 4

Self-diffusion of complex plasma in the presence of magnetized wake potential

In this chapter, the diffusion of a three-dimensional dust ensemble embedded in a flowing plasma in the presence of a moderate magnetic field is investigated using Langevin dynamics simulation. It is found that the asymmetric wake potential created due to streaming ions drives the dusty plasma from a sub-diffusive to a super-diffusive regime. A novel wake dominant regime is identified that shows super-diffusive behavior for suitable adjustment of the magnetic field. The dependence of cross-field diffusion coefficient on magnetic field exhibits three distinct behaviors characterized by (a) B^{2-3} for ultra-low magnetic field and strongly correlated state, (b) $B^{\simeq 0.1}$ for moderate magnetic field, and (c) classical B^{-2} for relatively large magnetic field. Our analysis shows that the magnetic field is mediated via the streaming ions in regimes (a) and (b) of a relatively low magnetic field, where the dependence of diffusion coefficient on the magnetic field is faster than in the usual classical regime.

4.1 Introduction:

Diffusion is a fundamental transport process that involves the movement of particles or molecules from regions of high concentration to regions of low concentration [168]. It is a result of random thermal motion at the microscopic level, which leads to a net flow of particles. Diffusion occurs in various physical systems, such as gases, liquids, and solids, and plays a crucial role in numerous phenomena and processes [168; 169; 170; 171].

Particle diffusion in plasma is the study of how the particles, such as electrons and ions, move within plasma. These charged particles exhibit random thermal motion, primarily driven by their kinetic energy, with collisions between particles playing a significant role in determining their movement. Importantly, magnetic fields have a profound influence on particle diffusion in plasmas, a feature particularly relevant in fields like nuclear fusion research, where magnetic confinement is employed to stabilize and control the plasma. Understanding particle diffusion in plasma has far-reaching implications in both scientific and technological domains. In astrophysics, it plays a pivotal role in comprehending phenomena such as the solar wind's behavior, stellar formation, and the dynamics of supernova remnants. Furthermore, in technological applications, such as plasma-based material synthesis, semiconductor manufacturing processes, and plasma thrusters for spacecraft propulsion, a deep understanding of particle diffusion is essential for optimizing performance and efficiency.

The study of solid particle diffusion in plasma is important for various reasons, including its applications in plasma processing and technology, its significance in fusion research, etc. [172; 173]. For example, plasma-wall interactions are a major concern in the design and operation of fusion reactors [172]. Dust particles present in the plasma can damage the reactor walls and disrupt the plasma, leading to a decrease in performance or even reactor shutdown. Understanding the diffusion of these dust particles is crucial for predicting their behavior and developing mitigation strategies. In industries that utilize plasma processes, such as semiconductor manufacturing, dust can significantly affect the quality of the final product [174]. Dust particles can cause defects in the delicate structures of a semiconductor wafer, leading to device failure. It's crucial to understand how dust particles move and interact with plasma in order to develop effective dust mitigation strategies. This could involve creating clean-room environments, implementing filters, or using plasma parameters that minimize dust creation and adherence [174]. Overall, the study of dust particle diffusion in these different environments allows us to better understand and manipulate these systems for improved performance and safety.

4.1.1 Diffusion in complex plasma:

The diffusive behavior of dusty plasmas is governed by various factors, including self-diffusion resulting from particle-particle interactions and collisions with background neutral particles. This property plays a crucial role in understanding the thermodynamics of physical systems, providing insights into inter-particle potential, structural characteristics, and criteria for melting or freezing [175; 176; 177; 178]. The interaction potential between dust particles plays a critical role in determining their diffusion behavior. In the presence of streaming ions and an external magnetic field, the potential around a dust grain may deviate from the Yukawa potential and significantly affect the transport property of the dust ensemble.

Molecular dynamics (MD) simulation is an incredibly useful tool for studying self-diffusion in a Yukawa system and other complex systems. Ohta and Hamaguchi utilized MD simulation to determine the self-diffusion coefficients of Yukawa systems in the fluid phase, covering a wide range of thermodynamic parameters [179]. The effects of a normalized electric field (E^*) on parallel and perpendicular diffusion coefficients ($D_\perp$) are investigated through equilibrium molecular dynamics (EMD) simulations in three-dimensional strongly coupled dusty plasmas [180]. During the last decade, a number of experiments on diffusion in quasi-two-dimensional dusty plasmas have been performed. In many of these, diffusion was found to behave anomalously [181; 182; 179]. In dusty plasma experiments, the magnetic field is often used as a tool to control and manipulate the conditions of the plasma. Ott and Bonitz [183] studied the diffusion of dust grains in strongly coupled plasma (SCP) in the presence of a magnetic field. They have shown the dependence of parallel ($D_\parallel$) and perpendicular ($D_\perp$) diffusion coefficients on the magnetic field. In the presence of a strong magnetic field, they found Bohm diffusion in a strongly coupled regime. With the help of Brownian dynamics simulation, Hou *et al.* [184] analyzed self-diffusion in a two-dimensional dusty plasma system. From the simulation results, they have inferred that the occurrence of super diffusion is transient and the system exhibits normal diffusion when the Coulomb coupling parameter Γ approaches the melting value. Recently Bezbaruah *et al.* [185] have discussed the effect of screened interaction and ion-neutral collisions on anomalous diffusion of dust grains. In the theoretical work of Ott *et al.* [186], the system dimension is found to play an important role in super-diffusive behavior. In the experimental work by Liu *et al.* [187], super-diffusion was found in a two-dimensional dusty plasma liquid that experiences damping due to the background neutrals. In the work, diffusion coefficients are obtained from mean-squared displacement (MSD), and the corresponding scaling with respect to the temperature of the system has been analyzed. In a recent work, self-diffusion in two-dimensional dusty plasma under an extremely strong magnetic field was carried out both experimentally and through molecular dynamics simulations [188]. The experimental setup involved a rotating particle cloud, which allowed for effective magnetic fields up to 3000 T. The results validated the simulation predictions of super-diffusive behavior at intermediate timescales and confirmed its dependence on the magnetic field.

In all the above studies, it was assumed that the dust particles interact with each other via Yukawa interaction potential, thus ignoring the effect due to ion flow. Here, we emphasize that simulation with only Yukawa potential does not fully explain the behavior of strongly coupled dusty plasma in the presence of ion flow. Due to the complex behavior of plasma particles in the presence of ion flow, an attractive wake potential may arise (as described in Chapter 3) that gets further modulated in the presence of a magnetic field [104; 162; 163; 105; 114]. In the present work, diffusion of a three-dimensional dust ensemble embedded in a flowing plasma in the presence of a moderate magnetic field is investigated using Langevin dynamics simulation by considering the effective inter-particle potential as the superposition of isotropic Yukawa and anisotropic attractive wake potential. The sinusoidal dependence of the potential on the magnetic field and streaming velocity brings novel effects to the behavior of the diffusion of dust grains. It is also interesting to investigate the dependence of perpendicular and parallel diffusion coefficients on the magnetic field in the presence of a Yukawa potential superimposed with an anisotropic attractive wake potential. We also try to establish the diffusion coefficient as an indicator of the phase transition from an ordered to a disordered state.

4.2 Theoretical Model:

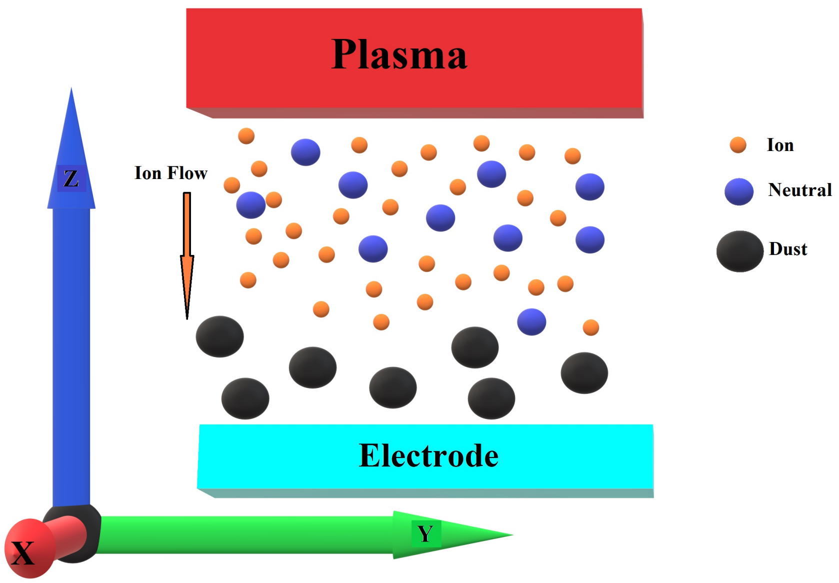

Figure 4.1: A schematic diagram illustrating a dusty plasma setup with streaming ions and an externally applied magnetic field.

We consider a 3D dusty plasma consisting of electrons, ions, neutral particles, and micron-sized grains. The ions are streaming in the vertical Z - direction perpendicular to an external magnetic field applied along the X - direction. In the absence of streaming, the charged dust particles interact via repulsive screened Coulomb or Debye – Hückel potential defined as:

$$\phi_Y = \frac{Q_d}{4\pi\epsilon_0 r} exp\left(-\frac{r}{\lambda_D}\right) \tag{4.1}$$

Here, $\lambda_D = \frac{\lambda_{De}\lambda_{Di}}{\sqrt{\lambda_{De}^2+\lambda_{Di}^2}}$ is the Debye screening length, where $\lambda_{De} = (\epsilon_0 K_B T_e/n_{e0}e^2)^{1/2}$ is electron Debye length and $\lambda_{Di} = (\epsilon_0 K_B T_i/n_{i0}e^2)^{1/2}$ is ion Debye length. n_{e0}, n_{i0}, T_e, T_i, Q_d are the electron density, ion density, electron temperature, ion temperature, and dust charge respectively. We are mainly interested in the wake potential that arises due to the interaction between dust grains and electrostatic dust ion cyclotron mode propagating in the system.

In the present theory, the electrons are considered to obey the Boltzmann distribution. The deviation from Boltzmann equilibrium is observed in the presence of a strong magnetic field in the bulk plasma region. However, the current model is focused on dust grains levitating in the near sheath region, where the electrons still follow the Boltzmann distribution even in the presence of a moderate magnetic field. The electron temperature being sufficiently high (10^4 K), the species get thermalized and behave as a classical gas in the ion timescale. Although the effect of ion-neutral collision is found to be insignificant on the formation of the magnetized wake for the parameter regime considered here, the influence of the presence of neutral particles can still play a role when we look at the Langevin equation for describing the dust particles. The effective interaction potential among the dust particles in such flowing magnetized plasma may be expressed as a superposition of spherically symmetric Debye H*ü*ckel and anisotropic wake potential [162].

$$\phi = \phi_Y + \phi_W \tag{4.2}$$

Where, ϕ_Y is the Yukawa potential (equation 4.1) and ϕ_W is the wake potential defined as [162]

$$\phi_W = \frac{-Q\rho}{\lambda_{De}\epsilon_0}\frac{sin(\sqrt{\alpha\rho}z)}{2(\rho+1)(\rho\alpha+1)} \tag{4.3}$$

Here, $\alpha = \frac{P}{2M^2}$, ρ = -1 + $\sqrt{1+\left(\frac{R}{P}\right)^2}$, $P = M^2 - f_i^2 - 1$ and $R = 2Mf_i$ are normalized parameters, $M = \frac{u_{io}}{\omega_{pi}\lambda_{De}}$ is the Mach number, $f_i = \frac{\omega_{ci}}{\omega_{pi}}$ is the normalized ion gyro frequency, u_{io} is the ion drift velocity, ω_{pi} is the ion plasma frequency and ω_{ci} is the ion cyclotron frequency. The nature and behavior of the potential have been analyzed in detail in that work. The amplitude and phase of this asymmetric wake potential are controlled by two important parameters i.e., Mach number (M) and magnetic field (B). Using this potential, the combined role of wake and Yukawa potentials on the dust diffusion process as a characteristic of different ion flow speeds and magnetic fields have been investigated using Langevin dynamics simulation.

4.3 Langevin dynamics simulation:

The calculation of transport coefficients is one of the most powerful applications of Molecular Dynamics simulations. The equilibrium methods, make use of the Helfand-Einstein or the Green–Kubo relations to obtain information about the transport process from systems in thermodynamic equilibrium. Langevin Dynamics Simulation has been performed for 864 dust particles, with an additional Lorentz force acting on dust particles due to the external magnetic field. In this simulation scheme, for each particle i, the Langevin equation has been integrated.

$$\ddot{r_i}(t) = \frac{Q_d\left(\dot{r_i}(t) \times B\right)}{m_d} - (1/m_d)\nabla \sum_{i \neq j} \phi_{i,j} - \nu \dot{r_i}(t) + \frac{1}{m_d}\zeta(t) \tag{4.4}$$

Here, $Q_d\left(\dot{r_i}(t) \times B\right)$ is the Lorentz force, $\nu \dot{r_i}(t)$ is the frictional drag, which appears due to the dust particle motion in surrounding buffer plasma. ν is a friction coefficient, which is associated with the frequency of the dust particle collisions with the buffer plasma particles, and $\zeta(t)$ is the random force which is assumed to have a Gaussian distribution [125]. $\phi_{i,j}$ is the inter-particle potential energy of the dust particles. $\phi_{i,j}$ takes account of all the possible interactions between dust grains. The behavior of the system is controlled by the dimensionless parameters which are $\Gamma = \frac{Q_d^2}{4\pi\epsilon r_{av} K_B T_d}$, $\kappa = \frac{r_{av}}{\lambda_D}$, $M = \frac{u_{io}}{\omega_{pi}\lambda_{De}}$ and $f_i = \frac{\omega_{ci}}{\omega_{pi}}$. The particles are assumed to interact via isotropic Yukawa potential, defined as $\phi_Y = \Gamma\kappa\frac{\exp(-r)}{r}$, and anisotropic asymmetric ion flow induced wake potential which is given by equation (4.3). This asymmetric wake potential comes into play along the direction of ion flow (z-direction). In our simulation time and length are normalized by $\sqrt{\frac{m_d\lambda_D^2}{K_B T_d}}$ and λ_D respectively. In the simulation, parameters related to dust grains are dust mass $m_d = 10^{-15}$ Kg, dust particle radius $r_d = 10^{-6}$ m, dust grain density $n_d = 10^{11} m^{-3}$. We have implemented a Periodic Boundary Condition (PBC) so that the particle number in the system is conserved during the time of the simulation. The structural behavior of the dust grains is investigated on the basis of radial distribution function g(r), defined as

$$g(r) = \frac{V}{N}\frac{N(r,dr)}{4\pi r^2 dr} \tag{4.5}$$

Where, V is the volume of the simulation box, $N(r,dr)$ is the number of particles in the shell with infinitesimal thickness dr. The nature of diffusion in the system of particles is described using Einstein's formula with the help of Mean-squared displacement (MSD), defined as -

$$\mathrm{MSD(t)} = \frac{1}{\mathrm{N}}\sum_{\mathrm{i=1}}^{\mathrm{N}} (\mathrm{r_i(t)} - \mathrm{r_i(t_0)})^2 \tag{4.6}$$

Where, $r_i(t)$ is the position of the i^{th} particle at time t. In general, $MSD(t) \propto t^{\alpha}$ where $\alpha = 1$ for normal diffusion, $\alpha < 1$ is for sub-diffusion and $\alpha > 1$ is for super-diffusion. The components of the Diffusion coefficient, parallel and perpendicular to the magnetic field are defined as -

$$D_{\parallel} = \lim_{t\to\infty} \frac{\left\langle |x(t) - x(t_0)|^2 \right\rangle}{2t} \tag{4.7}$$

$$D_{\perp} = \lim_{t\to\infty} \frac{\left\langle |y(t) - y(t_0)|^2 \right\rangle + \left\langle |z(t) - z(t_0)|^2 \right\rangle}{4t} \tag{4.8}$$

Here, the angular brackets represent the averaging of displacement squares over the entire ensemble of particles. The phase behavior and structural properties have been analyzed by the radial distribution function g(r) [125], Lindemann parameter (defined as $\frac{\sqrt{MSD}}{r_{av}}$) [189] and lattice correlation factor (which is the Fourier transform of the radial distribution function) [125].

4.4 Results and Discussions:

Here, we report some results on the self-diffusion of charged dust particles in the presence of an external magnetic field and ion flow. The diffusion property of dust grains in the present work is mainly controlled by the Coulomb coupling parameter Γ, screening parameter κ, normalized ion flow velocity i.e. Mach number M, magnetic field B, and dissipation arising from neutral pressure.

In order to understand the nature of transport of a driven, dissipative dusty plasma system as discussed above, we perform several sets of Langevin dynamics simulation for a wide range of values of Coulomb coupling parameter Γ, screening parameter κ, normalized ion flow velocity i.e, Mach number (M) and magnetic field (B). MSD is plotted against time in Figs. 4.2(a) and 4.2(b) in linear and logarithmic scales respectively for values of Γ ranging from 45 to 370 keeping

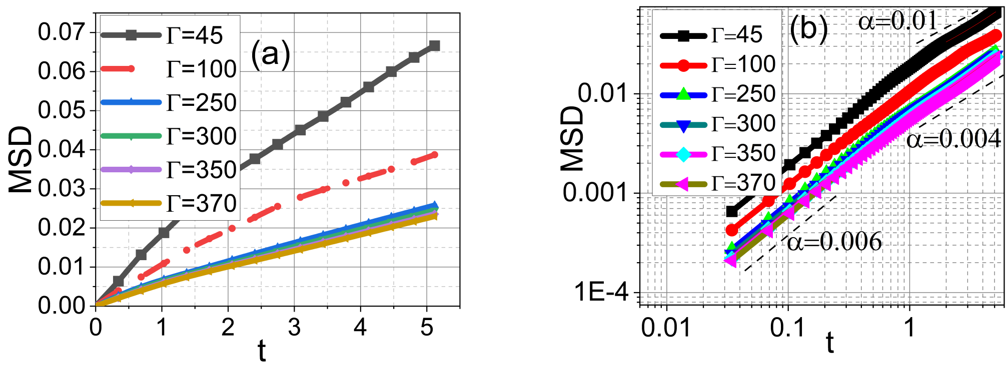

Figure 4.2: (a) Mean-squared displacement (MSD) for a range of Γ values when B = 0.001 T, κ = 2, M = 1.9 and $n_n = 10^{21} m^{-3}$. In our simulation, both MSD and time are normalized. Time and length are normalized by $\sqrt{\frac{m_d \lambda_D^2}{K_B T_d}}$ and λ_D respectively. Fig. 4.2(b) is the result of fitting the MSD time series of Fig. 4.2(a) in a log-log plot (logarithmic scale) to find the value of diffusion exponent α for different values of coupling parameter.

neutral density n_n at $10^{21} m^{-3}$, B = 0.001 T, κ = 2 and M = 1.9. MSD values decrease with an increase in the Coulomb coupling parameter Γ as expected. In order to understand the nature of diffusion of the dusty plasma system, the diffusion exponent α is obtained in Fig. 4.2(b) from the slope of MSD vs. time graph on the log-log scale ($MSD(t) \propto t^{\alpha}$). For normal diffusion, α is equal to unity, whereas for anomalous diffusion, $\alpha > 1$ (superdiffusion) or $\alpha < 1$ (subdiffusion). The system undergoes subdiffusion in the entire time of observation, since $\alpha < 1$.

Figs. 4.2($c_1 - c_4$) are the radial distribution functions (g(r)) depicting the variation in grain ordering for different values of Γ. A transition from fluid to solid crystalline state is admitted on increasing the value from Γ = 45 to Γ = 370 (Figs. 4.2($c_1 - c_4$)). The appearance of multiple peaks in g(r) plots indicates a transition from a disordered to an ordered solid-like state with a reduction in MSD values. An increase in coupling parameters is responsible for a strong correlation among particles by counteracting the anisotropy arising due to ion flow. Our primary interest is to see whether the diffusion property of complex plasma

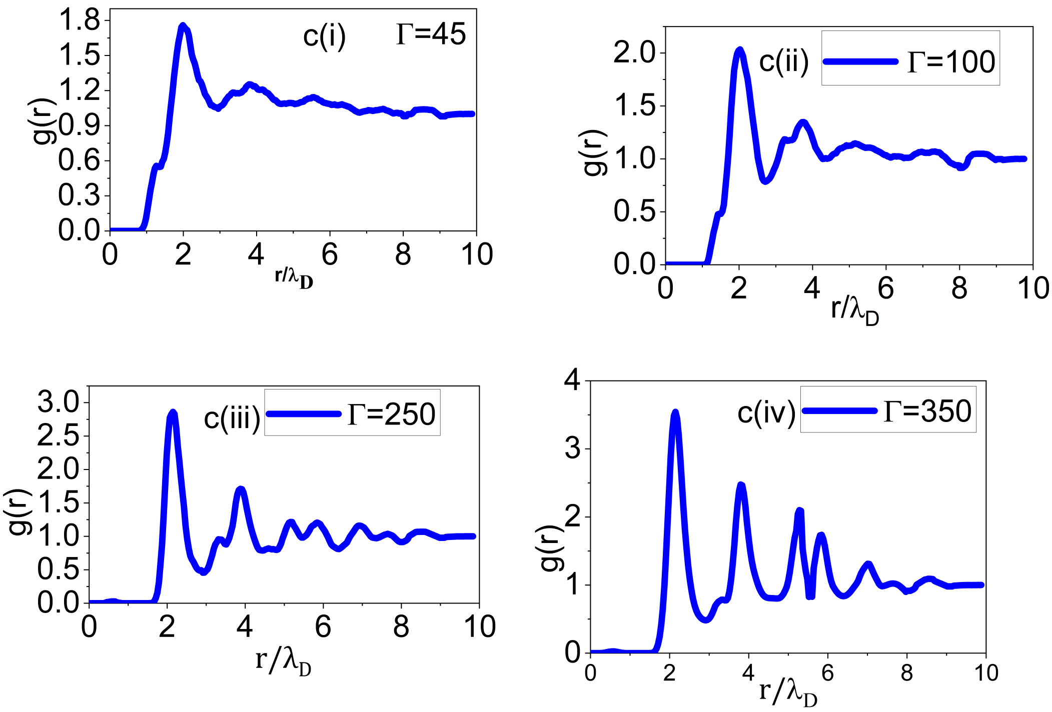

Figure 4.2: $(c(i), c(ii), c(iii), c(iv))$ are the plot of radial distribution functions (g(r)) depicting the variation in grain ordering for different values of Γ. The parameters are the same as in Figure 4.2(a).

can be controlled by tuning the amplitude and frequency of the magnetized wake.

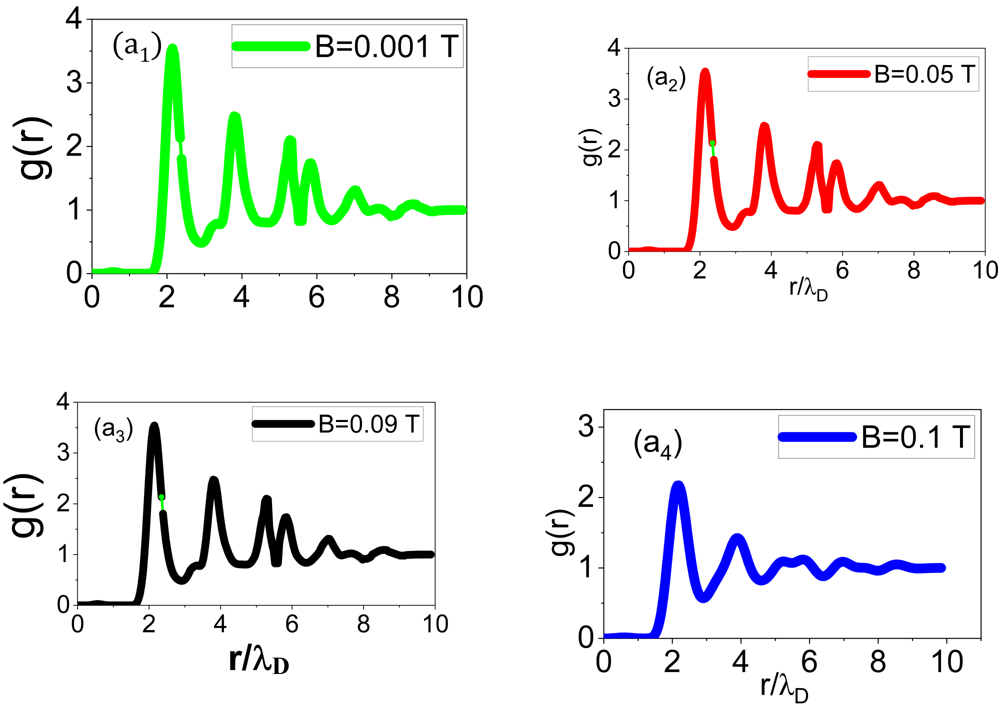

Figure 4.3: (a_1, a_2, a_3, a_4) are the plots of g(r) depicting the variation in grain ordering for different values of B when $\Gamma = 350$, $\kappa = 2$, M $=$ 1.9 and $n_n = 10^{21} m^{-3}$; the simulation was performed with Yukawa potential only along with the Lorentz force term.

The Lorentz force $Q_d\left(\dot{r_i}(t) \times B\right)$ acting on dust grains is negligibly small due to the small charge-to-mass ratio of dust grains and the magnetic field manifests itself, mainly through the wake potential for the weak to moderate values of the magnetic field considered here. This is evident from the comparison of g(r) plots of Figs. 4.3$(a_1 - a_4)$ and 4.3$(b_1 - b_4)$ respectively. Figs. 4.3$(a_1 - a_4)$ depicts the results when the simulation was performed with Yukawa potential only along with the Lorentz force term. The system remains in the crystalline state when B is varied from 0.001 T to 0.09 T in this case. On the other hand, when wake potential is included, one observes the transition from a solid crystalline to a gaseous state (Figs. 4.3$(b_1 - b_4)$).

The effect of the magnetized wake on MSD when B is varied from 0.05 T to 0.1 T, is shown in Fig. 4.4(a). The values of neutral density n_n, coupling parameter

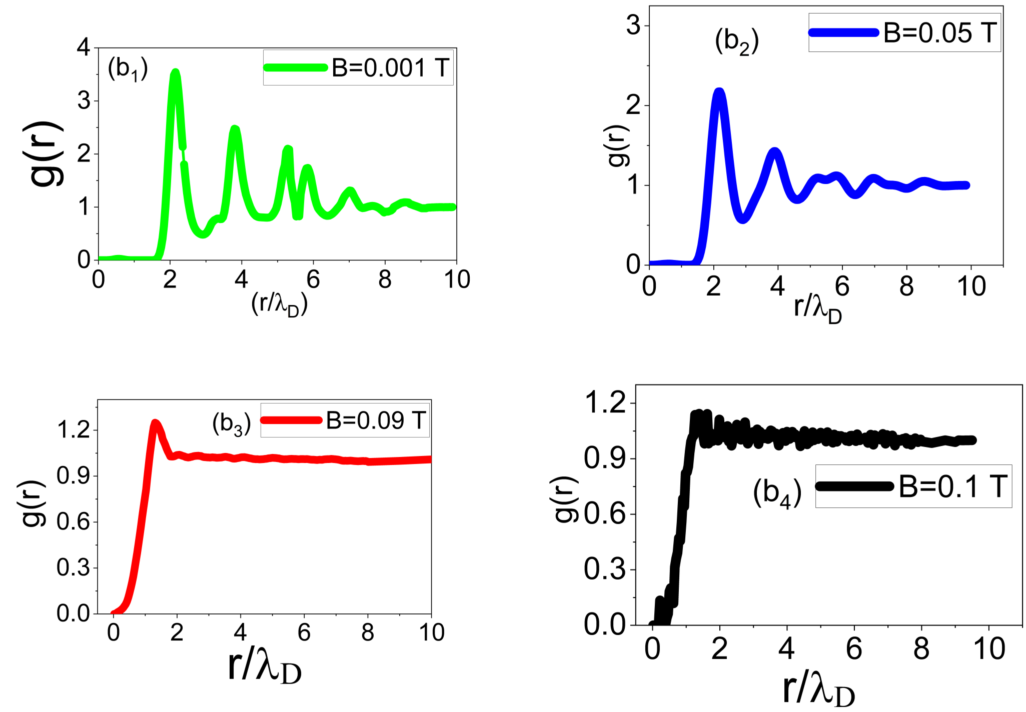

Figure 4.3: (b_1, b_2, b_3, b_4) - plots of g(r) for different values of B considering both Yukawa and magnetized wake potential.

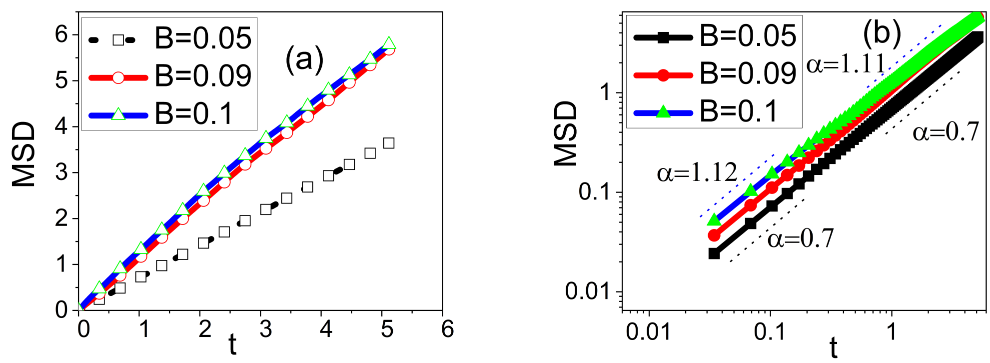

Figure 4.4: (a) The figure depicts the evolution of MSD for a range of B values when $\Gamma = 350$, $\kappa = 2$, M $= 1.9$ and $n_n = 10^{21} m^{-3}$. Fig. (b) - MSD in the log-log plot as a function of time.

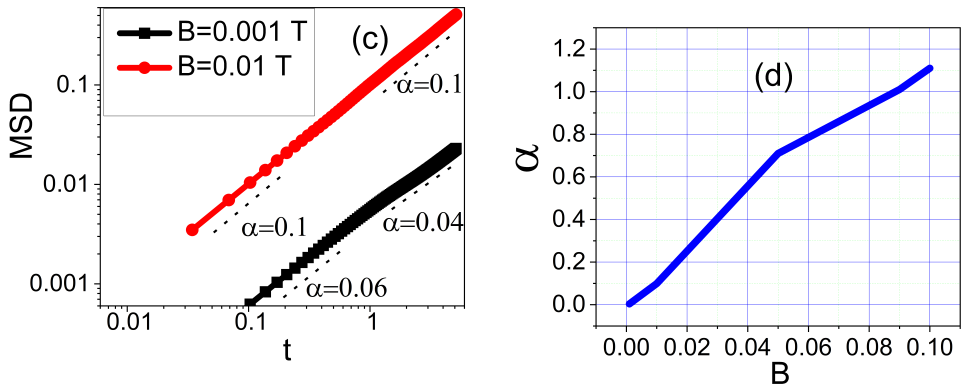

Figure 4.4: (c) The figure depicts the MSD in the log-log plot as a function of time. (d) The diffusion exponent (α) for different values of field strengths.

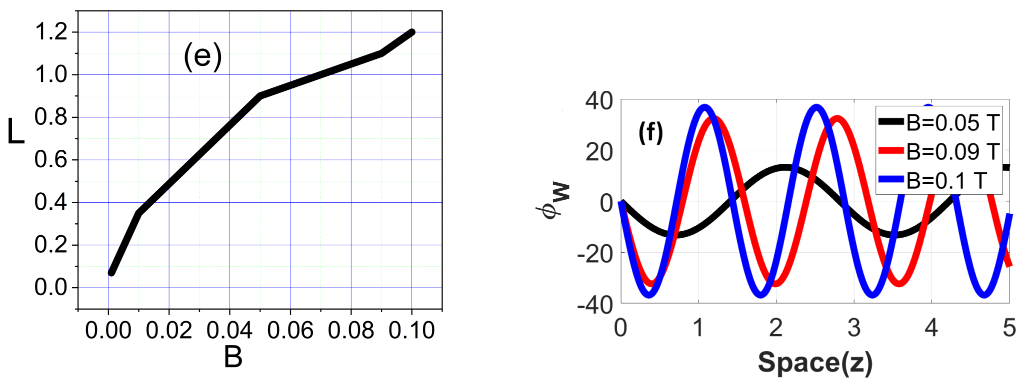

Figure 4.4: (e) Shows the effect of magnetic field (B) on the Lindemann parameter. Fig.(f) shows the variation in the strength of wake potential (ϕ_W) for a set of magnetic fields.

Γ, screening parameter κ and ion Mach number M are kept fixed at $10^{21}m^{-3}$, 350, 2, and 1.9, respectively. As the magnetic field is changed from 0.001 T to 0.1 T the dust ensemble exhibits an increase in MSD values. The diffusion exponents are calculated from the slope of MSD curves in the log-log plots of Figs. 4.4(b) and 4.4(c) and plotted across B in Fig. 4.4(d). The plots reveal that as the magnetic field increases from 0.001 T, the dust ensembles gradually transit to a disordered state. It makes a transition to a liquid state at a magnetic field around 0.05 T. The lattice correlation factor (LCF) is 0.45 for $B = 0.05$ T. At this value of the magnetic field ($B = 0.05$ T), there is also a change in the slope of the Lindemann parameter (Fig. 4.4(e)). This signifies a change in phase behavior due to the magnetic field. With the increase in B, the diffusion exponent is enhanced, indicating a transition from sub-diffusion to super-diffusion. Both MSD and the values of the diffusion exponent α exhibit a sudden change in the slope around magnetic field B = 0.05 T. These observations can be explained on the basis of the plot of wake potential shown in Fig. 4.4(f). Both the amplitude and frequency of the wake potential (ϕ_w) increase with an increase in the value of B. An increase in the strength of the magnetic field facilitates the focusing of ions flowing from bulk to the sheath region below the dust grains, thus enhancing the density of trapped ions, resulting in an increase in the amplitude of the wake potential. This anisotropic potential acting along the ion flow direction is responsible for the observed increase in MSD values and diffusion exponent. It can be inferred that by suitably tuning the magnetic field, it is possible to design a dust ensemble that is sub-diffusive or super-diffusive in its behavior.

Perpendicular and parallel diffusion coefficients are plotted across B in Fig. 4.5(a) and 4.5(b) respectively. Due to the presence of magnetized wake, MSD shows a strong dependence on B even for its low values. A complete novel characteristic of transport of dust ensembles is observed for 0.001 T $< B <$ 0.01 T, keeping M fixed at 1.9, where the decay exponent γ (defined as $D_{\perp} \propto B^{\gamma}$) is found to be > 2. This result is in contrast with the previous findings where cross-field diffusion remains indifferent to variation in B in this regime [183]. We emphasize that such a weak magnetic field does not affect dust particles via Lorentz force but rather manifests itself through the modification in the background plasma response function. The novel behavior of the diffusion coefficient may be explained on the basis of symmetry breaking caused by flowing magnetized ions. The magnetic field is not too strong in this regime to confine the dust particles but is sufficient to affect the wake formed behind dust grains. The magnitude of the effective potential gradually shifts from Yukawa dominant to wake dominant, resulting in the enhancement of MSD and the decay exponent. Fig. 4.3(c) shows the comparison of the amplitudes of wake and Yukawa potential in the ion flow direction. As we increase the magnetic field from 0.001 T to 0.1 T, the wake potential gradually increases and exceeds Yukawa. The trend is reversed beyond 0.09 T. The cross-field diffusion coefficient ($D_{\perp}$) now decreases with B, exhibiting a negative slope. This is also the regime where the system transitions to a disordered state, exhibiting super-diffusion. In this regime, the Lorentz force term starts to become effective and shows its influence on the diffusion property of the system. Moreover, the wake potential becomes dominant in this range of magnetic fields and causes the dislocation of dust particles from the ordered state. This result is in agreement with the findings of Ott and Bonitz [183]. They observed that for small values of Γ, i.e., in the weakly coupled regime, the magnetic field suppresses cross-field diffusion. However, in our simulation, we observe that the combination of Lorentz force and magnetized wake results in a $\frac{1}{B^2}$ dependence of the cross-field diffusion coefficient instead of Bohm diffusion. From this analysis, the dependence of $D_{\perp}$ on the magnetic field may be classified into three regimes - (a) a novel regime, characterized by a solid-like state, ultra-low magnetic field ($B < 0.01$ T) where $D_{\perp} \propto B^{\gamma}$ with $\gamma > 2$. In this regime, even a small change in magnetic field results in a large deviation from Yukawa potential due to an increase in asymmetric wake potential. Here transport of particles is found to be sub-diffusive. It is worth noting that the effect of the magnetic field in this regime is mediated purely by flowing ions and not due to the usual Lorentz force on plasma particles. (b) a plateau region where $D_{\perp} \propto B^{\simeq 0.1}$ for 0.01 T$< B < 0.09$ T. (c) a classical region where $D_{\perp} \propto B^{-2}$. In this regime diffusion is the result of combined action of Lorentz force and strong wake potential. Variation of parallel diffusion with B is shown in Fig. 4.5(b). Parallel diffusion is weak in comparison to perpendicular diffusion. It increases with B up to 0.05 T beyond which, it shows a negative slope. The change in slope occurs at a magnetic field that coincides with a point of transition from crystalline to liquid state. Like cross-field diffusion, $D_{\|}$ also further gets suppressed beyond $B = 0.09$ T. In the work by Ott and Bonitz [183], parallel diffusion is found to remain indifferent for low values of magnetic field. In contrast, here we observe a significant increase in $D_{\|}$ for the very weak magnetic field which is attributed to the influence of magnetized wake.

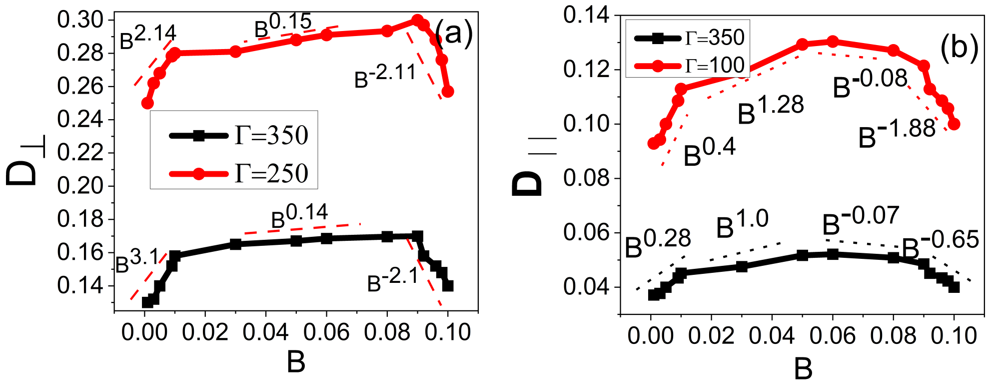

Figure 4.5: Diffusion coefficients (a) perpendicular ($D_{\perp}$) and (b) parallel ($D_{\|}$) to the magnetic field versus field strength (B in Tesla) for a range of coupling parameters when $\kappa = 2$, M = 1.9 and $n_n = 10^{21} m^{-3}$.

It is the ion flow velocity that drives the attractive oscillatory wake potential and modulates its amplitude and velocity as shown in Fig. 4.6(c). The results of MSD with time for a range of normalized ion flow velocity in the supersonic regime are illustrated in Fig. 4.6(a) and 4.6(b) for B = 0.05 T, $\kappa = 2$, and $\Gamma = 350$. With an increase in the value of M, the orderliness of the dust ensemble increases, and MSD values gradually decrease. The ion focusing gets distorted as the velocity of flowing ions increases. As a result, the strength of wake potential becomes weaker and Yukawa potential becomes more dominant. This results in a stronger correlation among the grains and MSD values decrease. The plot of diffusion exponent with M in Fig. 4.7(a) brings out some interesting properties of diffusion shown by the system. The values of α indicate supper-diffusion for $1.0 < M < 1.2$; whereas α decreases to sub-diffusion beyond M = 1.2 and shows an abrupt change in slope at $M = 1.5$. In order to understand the mechanism behind the transition from supper-diffusion to sub-diffusion, we have calculated the spring constants from the expression of potentials. The spring constant is defined as, $K = \frac{d^2\phi}{dr^2}$. Where ϕ is the effective potential as a function of the distance r. The effective

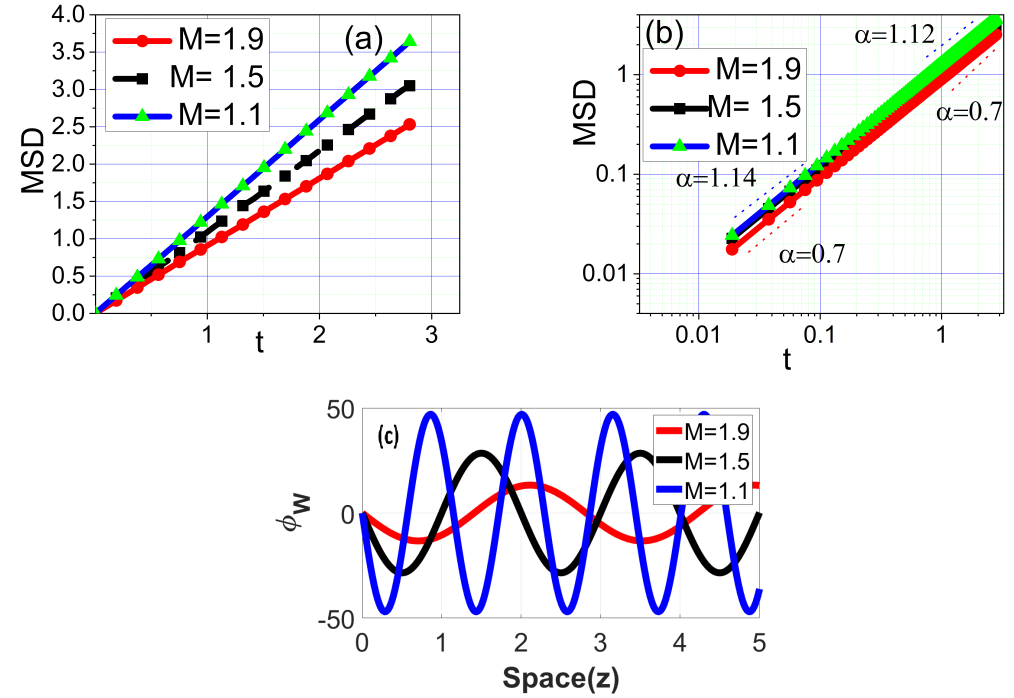

Figure 4.6: (a) The figure depicts the evolution of MSD for a range of M values when B = 0.05 T, Γ = 350, κ = 2, and $n_n = 10^{21}m^{-3}$. (b) The straight line fit of the MSD curve is represented through the dashed lines in the figure.(c) The variation in the strength of wake potential (ϕ_W) for a set of Mach number for B = 0.05 T, Γ = 350, κ = 2, and $n_n = 10^{21}m^{-3}$.

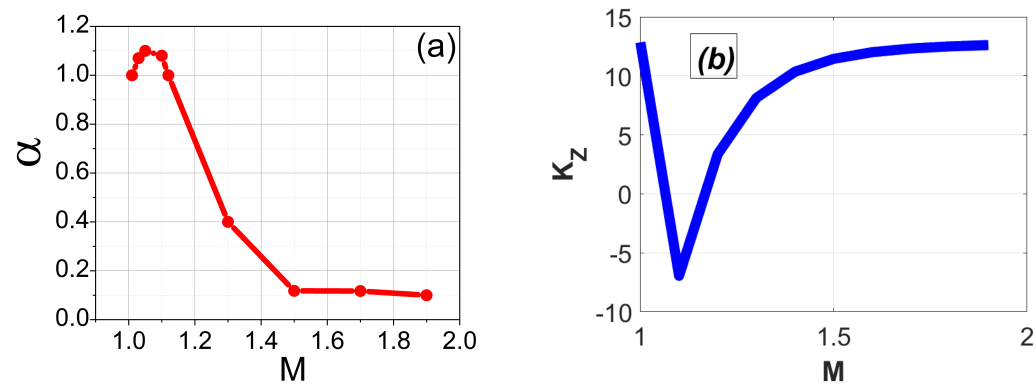

Figure 4.7: (a) Diffusion exponent α vs M when $B = 0.05$ T, Γ = 350, κ = 2 and $n_n = 10^{21}m^{-3}$ (b) Effective spring constants K_Z vs M for $B = 0.05$ T, Γ = 350, κ = 2 and $n_n = 10^{21}m^{-3}$.

spring constant along the direction of ion flow is $K_z = K_z(Yukawa) + K_z(wake)$. The spring constant gives an idea about the strength of the dominant interaction operating in the system. It is clearly seen from Fig. 4.7(b) that effective spring constant K_z shows a dip corresponding to M=1.1 and this means that attractive wake potential dominates over repulsive Yukawa potential. This increases the disorderliness of the system and drives the system to exhibit super diffusion. As M increases beyond 1.2, Yukawa potential dominates over the wake and the system goes to a sub-diffusive regime. Around M = 1.5, Yukawa becomes maximum and brings in regularity to the system, and diffusion beyond this almost becomes steady.

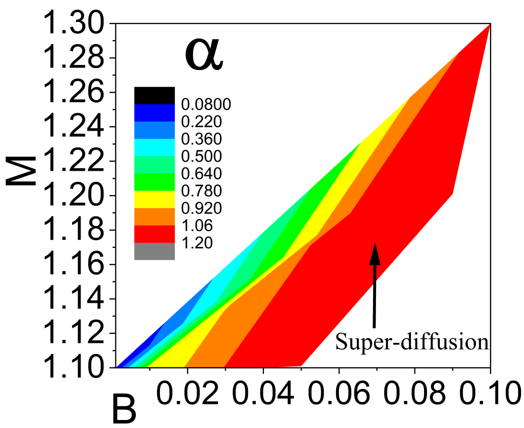

Figure 4.8: Diffusion exponent α as a function of magnetic field strength (B in Tesla) and Mach number (M) for $\Gamma = 350$, $\kappa = 2$, and $n_n = 10^{21} m^{-3}$.

Fig. 4.8 depicts the range of Mach number (M) and Magnetic magnetic field (B) for which the dust ensemble exhibits super-diffusion. The red region as indicated in the figure represents super-diffusion (α >1). The simulation performed in this work clearly reveals that super-diffusive property is exhibited when effective potential becomes wake-dominant. The asymmetric wake potential drives the dust ensemble towards a disordered and super-diffusive state.

4.5 Conclusions:

The perturbation of the Debye sphere around the dust grains due to the streaming magnetized ions modifies the dielectric response of the plasma medium and leads to the appearance of the new anisotropic interaction potential. Previous research predominantly focused on the impact of magnetic fields on diffusive characteristics in dusty plasma by examining the influence of the Lorentz force. However, due to the relatively small charge-to-mass ratio, the Lorentz force remains negligible, even at moderate magnetic field strengths, when considering the dynamics of dust particles. In our current investigation, we aim to illustrate how the anisotropic magnetized wake affects the self-diffusion of dusty plasma, even within the regime of modest magnetic field strengths. The uniqueness of the present work is that we have used modified effective interaction potential while performing Langevin dynamics simulation and this helps us to see the effect even for small to moderate values of magnetic field on dust dynamics. In this comprehensive study, we provide an in-depth analysis of how the magnetized wake impacts diffusion and the phase behavior of a strongly coupled three-dimensional dusty plasma system using Langevin dynamics simulations. Our research has revealed several noteworthy and novel findings, enriching our understanding of the intricate interplay between magnetic fields and dusty plasma dynamics. These findings have the potential to advance our knowledge of plasma physics and contribute to a broader understanding of complex phenomena in astrophysical and laboratory settings. The observations can be summarized as follows-

(1) The ion flow velocity is the driving force behind the generation of the attractive oscillatory wake potential, and it also serves to modulate both its amplitude and velocity. For a range of normalized ion flow velocity (M) lying between 1.0 to 1.2, the attractive wake potential is predominant, and the dust ensemble exhibits super-diffusion.

(2) We have identified a novel regime characterized by the dominance of the wake potential, which exhibits super-diffusive behavior when the magnetic field is appropriately adjusted. Super-diffusion is observed when the magnetic field is increased beyond 0.09 T for suitable values of Mach number in the supersonic

regime.

(3) The behavior of the dust ensemble undergoes a transition from sub-diffusion to super-diffusion depending on whether the dominant factor in the effective potential is the Yukawa component or the wake component. The dust ensemble shows sub-diffusion when the effective potential is Yukawa dominant and super-diffusion when it is wake-dominant. The magnetized wake plays a crucial role in driving the system from sub-diffusion to super-diffusion.

(4) Both the perpendicular ($D_\perp$) and parallel ($D_\parallel$) diffusion coefficients are influenced by magnetic fields, even when the field strength is weak. The specific nature of how these diffusion coefficients (perpendicular and parallel) depend on the magnetic field (B) is influenced by several factors, including the state of the system, ion flow velocity, and the effective interaction potential. A novel regime of dependence of cross-field diffusion coefficient $D_\perp$ as B^γ with $\gamma > 2$ is observed for ultra-low magnetic field B (< 0.01 T). On the other hand combined effect of Lorentz force and wake potential leads the system to exhibit B^{-2} variation for a relatively large magnetic field (> 0.09 T).