Chapter 3

A utility driven bandwidth and time allocation CSS scheme for single-PU, multi-SUs overlay CRNs

3.1 Introduction

In Cognitive radio networks (CRNs), cooperative spectrum sharing (CSS) is the key functionality towards improving spectrum efficiency, where primary users (PUs) and secondary users (SUs) work cooperatively by sharing scare spectrum resources and attain mutual benefit. Unlike opportunistic CRNs, where SUs access the PUs' spectrum without their knowledge, CSS involves cooperation between PUs and SUs through the exchange of essential resource information, including key constraints. This collaboration enables both PUs and SUs to satisfy their minimum needs, fostering a willingness to participate in the collaboration. In a resource compensation-based CSS mechanism, SUs would like to relay PU's traffic for the rewards of the transmission opportunities over the PU band. Many efforts have been made to develop effective CSS models which facilitate the utilization of unused spectrum for the efficacy of secondary networks in terms of utility improvement or energy-efficient secondary communication. However,in a primary network with poor channel quality results in significant degradation of direct transmission between PU-transmitter and PU-receiver, which negatively impacts the transmission performance of PUs. Moreover, if the PU-transmitter and PU-receiver pairs are located outside of the transmission range, then direct transmission among them is quite impossible. To overcome such situation of primary network, PUs smartly select suitable SUs with superior channel conditions as cooperative relay to forward primary service towards indented receiver. And, as compensation, PUs lease (or share or allocate) either a portion of their channel access time (time sharing) or a portion of their channel bandwidth (bandwidth sharing) to the relay SUs. In time sharing, PU divides the allotted time slot into some fractions and assigns a portion for cooperative communication, while the other portion is assigned for SU's transmission. On the other hand, in bandwidth sharing, SU relays PU's traffic while simultaneously accessing the PU's spectrum to transmit its own data by utilizing an advanced physical layer technique. Many of the works related to bandwidth sharing are found leveraging the Multiple-Input Multiple-Output (MIMO) beamforming technology [17], [131] to improve the signal-to-noise ratio (SNR) of received signals, eliminate undesirable interference, focus to transmit signals towards specific locations [131].

In single-PU multi-SUs CRN scenarios, the transmission benefit of the PU is significantly influenced by two key factors: (i) the choice of SU as the relay, and (ii) the implementation of a CSS scheme between the PU and the selected SU. Additionally, for time-critical and delay-sensitive services, the service requirement of PU channel also plays a substantial role. Therefore, selecting an appropriate relay node and associated CSS scheme is crucial for the PU to meet specific service constraints. To maximize cooperative gain in terms of the PU's utility, the PU must also meet the reward constraints of the collaborating SU for successful relay-assisted cooperative communication. This involves appropriately distributing resources between the PU and SU for cooperative and secondary communication. Balancing these requirements entails a tradeoff between maximizing the PU's utility and providing sufficient rewards to the SU, as these objectives are inversely related. Therefore, it is imperative to optimally allocate PU resources during CSS to simultaneously optimize the PU's utility and the SU's reward.

In this chapter, we propose a resource exchange-based CSS scheme between the PU and a cooperative partner SU. This scheme involves sharing an appropriate portion of the PU's bandwidth or access time within the PU's frequency band, tailored to the PU's specific service requirements. We have formulated two separate optimization problems for bandwidth sharing and time sharing. Each optimization problem aims to balance the cooperative gains for the PU, the energy costs incurred by the PU, and the rewards provided to the SU in the form of transmission opportunities for secondary communication. Given the computational complexity of these optimization problems, we propose an iterative heuristic approach to achieve near-optimal resource allocation in polynomial time. To demonstrate the effectiveness and practicality of the proposed CSS schemes, we conducted a case study examining two types of PU services: best-effort service and time-sensitive service. This analysis confirmed the validity and suitability of the proposed CSS schemes in real-world applications. Additionally, we investigated the impact of different types of delays on the proposed CSS schemes, providing a comprehensive analysis to highlight the distinctions between delay-sensitive and delay-tolerant PU services.

The rest of this chapter is organized as follows: Section 3.2 defines the problem, outlines the assumptions, and introduces the symbols and notations used. The system model is discussed in Section 3.3. Section 3.4 presents the problem formulation for CSS. The proposed scheme is detailed in Section 3.5. Section 3.6 covers the case study and performance evaluation. Finally, Section 3.7 concludes the chapter.

3.2 Problem Statement

The problem is to develop cooperative spectrum sharing (CSS) schemes in a single-PU, multi-SUs overlay CRN scenario for allocating optimal chunk of bandwidth or optimal fraction of channel access time between the cooperative PU-SU pair. By formulating the CSS problem as a multi-objective optimization, the proposed scheme is able to select the most profitable SU as a relay node and balances three key objectives: maximizing PU utility, satisfying SU reward constraints, and minimizing energy consumption of PU.

3.2.1 Assumptions

- Two categories of SUs are considered: where in first category the SUs have the capability of Multiple-Input Multiple-Output (MIMO) communication [58, 106], while the SUs of the second category do not possess this capability .
- In bandwidth sharing, the SUs adopt the MIMO cooperative model as described in [77], while in time sharing, SUs adopt time-slotted system based on TDMA.

- Transmission powers of PU and SU, distance between PU and SUs and other resource constraints of SUs are known.
- The target resource constraint of PU, target reward constraint of SU, and the maximum relaying capacity of SU are known.
- Necessary control information exchange between PU and SU takes place through a dedicated common control channel [80] These control information focuses on the operational aspects of communication to ensure efficient channel usage.
- The locations of SUs and PUs are fixed in the network; that is, SUs and PUs are stationary during the partner assignment phase.
- All SUs in the network are non-malicious and the resource information provided by the SUs are trustworthy.
- The noise environment is considered to be zero mean Additive White Gaussian Noise (AWGN), and channel gain between two nodes encompasses solely the distance and path loss components [100], [97].

3.2.2 Notations and Symbols Used

To remind the symbols and notations used particularly in this chapter, the same are summarized in Table 3.1.

Symbols/Notations	Comments
$\mathcal{N}$	Set of SUs
N	Number of SUs
W	Total bandwidth of PU channel
γ	Bandwidth allocation factor
W_A	Portion of bandwidth allotted for cooperative communication
W_B	Portion of bandwidth allotted for secondary transmission
T	Total access time of PU band
β	Time allocation factor
T_A	Fraction of time allotted for cooperative communication

T_B	Fraction of time allotted for secondary transmission
$d_{PT,PR}$	Euclidean Distance between PT and PR (in m)
$d_{PT,ST}$	Euclidean Distance between PT and ST (in m)
$d_{ST,PR}$	Euclidean Distance between ST and PR (in m)
$d_{ST,SR}$	Euclidean Distance between ST and SR (in m)
R_{PT}^{tar}	Targeted transmission rate of PU
RC_{PT}^{tar}	Targeted resource constraint (in terms of time × bandwidth) of PU
RC_{ST}^{max}	Maximum PU resource (in terms of time × bandwidth) used by SU for relaying PU service
RC_{ST}^{rel}	Allotted PU resource (in terms of time × bandwidth) to SU for relaying PU service
RW_{ST}^{min}	Reward constraint (in terms of time × bandwidth) of SU on behalf of relaying PU service
RW_{ST}	Allotted reward (in terms of time × bandwidth) to SU on behalf of relaying PU service
P_{PT}	Transmission power of PU
P_{ST}^{max}	Max. relaying power of SU
$h_{PT,ST}$	Channel gain between PT and ST
$h_{ST,PR}$	Channel gain between ST and PR
$h_{PT,PR}$	Channel gain between ST and PR
$DV_{PT,ST}$	Decoding vector used by ST to obtain PT's signal
$EV_{ST,PR}$	Encoding vector used by ST to transmit PT's signal towards PR
N_0	Noise Power
$\sigma_{N_0}^2$	Noise variance
C_{PT}^{coop}	Cooperative capacity achieved by PU during cooperation with SU
EN_{PT}	Energy consumption of PU
EN_{ST}	Energy consumption of SU
$SNR_{PT,ST}$	SNR received at ST from PT
$SNR_{ST,PR}$	SNR received at PR from ST
$SNR_{PT,PR}$	SNR received at PR from PT
U_{PU}	Utility achieved by PU
TR_{PT}^{max}	Maximum transmission range of PT (in m)
λ	Per frequency transmission power rate of an ST
τ	Energy consumption rate of ST

ϕ	Gain per unit of data transfer achieved at the Maximal Ratio Combining output
ω	Negligible value ≈ 0

Table 3.1: Notations and Symbols used

3.3 System Model

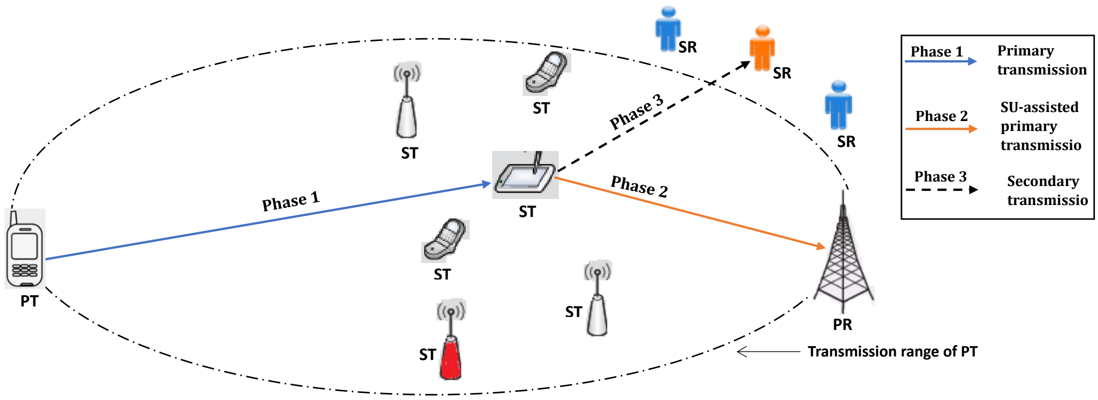


Figure 3-1: Cooperative communication among PU and SU for considered CRN scenario

We consider a cognitive radio network with a single PU transceiver pair, denoted as (PT, PR), and N heterogeneous SUs (or relays) transceiver pairs, denoted as $\mathcal{N} = \{ST_i, SR_i\}_{i=1}^{N}$. The entire cooperative communication process for the transmission of primary as well as for secondary data (or traffic) among a single PU and SU is depicted as shown in Figure 3-1. It is assumed that the secondary network can operate in both infrastructure-based (like femtocell, cognitive hotspot etc.) and ad hoc modes. Since MIMO empowered SUs can function as access points [77], it is assumed that out of N SUs few are empowered with MIMO capability. MIMO empowered SU seeks PU spectrum to improve secondary performance through MIMO cooperative relaying by facilitating simultaneous transmission of primary and secondary traffic during resource sharing. Decode-and-Forward (DF) relaying mode [77, 131] is employed by the MIMO SUs during such simultaneous transmission to separate and decode primary traffic as well as secondary traffic, that are received over multiple antennas during simultaneous transmission. Throughout the thesis, we employ interchangeable terminology, such as 'signal', 'traffic' and 'information', 'SU', 'ST' (secondary transmitter) and 'relay', as well as 'PU' and 'PT' (primary transmitter) for the sake of consistency.

We consider a time-slotted system for both cooperative and secondary data transmission. To highlight the performance gain achieved by a PU from cooperative spectrum sharing, we focus on a CRN scenario where the direct link between the PT and PR is highly attenuated, and only an ST can provide relay service for successful communication. The PT aims to improve cooperative capacity with the assistance of an appropriate ST without compromising its target service quality. As resource compensation or Reward-to-ST (RW_{ST}) for relaying PU service, the ST accesses the spectrum resource released by the PT and performs secondary transmission to the SR. The ST participates in cooperative communication only if RW_{ST} exceeds a minimum threshold RW_{ST}^{min}.

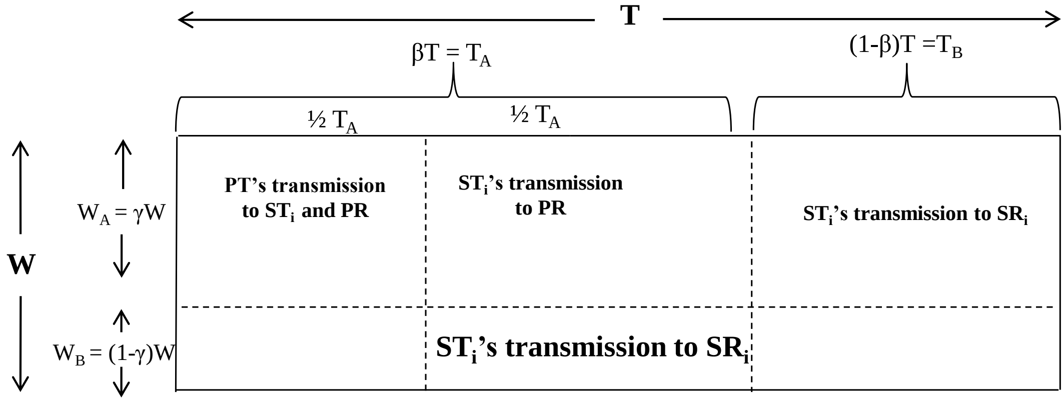
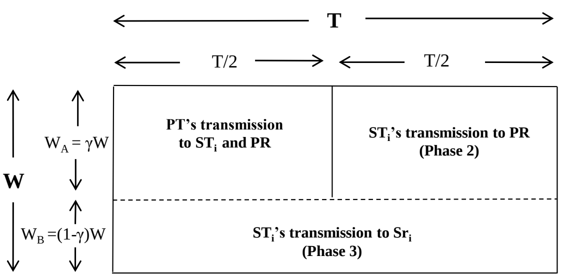


Figure 3-2: Generic time-bandwidth sharing model of PU band



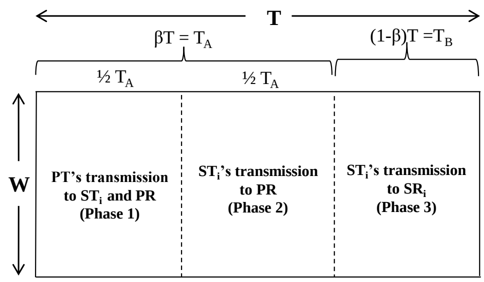


Figure 3-3: (a) Case 1: Bandwidth sharing model (b) Case 2: Time sharing model

The generic resource sharing named time bandwidth allocation model of a PU band with cooperative SU is depicted in Figure 3-2. We assume that the total bandwidth of the channel owned by PU is W MHz and the available transmission time of the channel over the bandwidth W is T sec. Depending on the characteristics of PU service requirements (as discussed in section 5) and relaying offers announced by SU (as discussed in section 4), PU decides on either to adopt bandwidth sharing (termed as Case 1) keeping the time allocation factor fixed

($\beta = 1$ as shown in Figure 3-3(a)) or time sharing (termed as Case 2) keeping the bandwidth allocation factor fixed ($\gamma = 1$ as shown in Figure 3-3(b)). In Case 1, equal time distribution ($T/2$ each) among PT-to-ST and ST-to-PR is considered, ensuring that the target transmission rate of primary data is not compromised. The details of data transmission techniques as well as the cooperative capacity achieved by the PU in both the cases are described below.

3.3.1 Case 1

For bandwidth sharing, PU prefers MIMO empowered SUs and thus consider the MIMO cooperative model as described in [77]. Here, the selected MIMO ST acts as relay and would cooperatively forward the primary signal from PT to PR in DF relaying mode, while simultaneously access the PU bandwidth to transmit secondary data towards SR. MIMO empowered SU seeks PU spectrum (available PU bandwidth) to facilitate simultaneous transmission of primary and secondary traffic during bandwidth sharing. However, in absence of MIMO capable SUs, the resource sharing will be time sharing based only. As considered by [77], let assume A_{ST} (another SU), which cooperatively works with ST and transmits the secondary information towards ST during the first phase of secondary transmission over the compensated bandwidth W_B. It is noted that PT is not bound for any type of negotiation and providing compensation to A_{ST} and PT directly deals and negotiate with ST only. However, the agreement (or partnership) between the ST and A_{ST} is out of the scope of this research work and emphasis is given towards resource sharing among PT and ST, where PT aims to allocates optimal bandwidth with ST for maximizing the cooperative capacity.

During the first $\frac{T}{2}$ time slot, PT transmits it's signal towards the selected ST over the bandwidth W_A. Using DF relaying technique and appropriate post-coding on the received signals over multiple sources, ST separates and decodes the signals received from PT and A_{ST} [17], [58]. In the next $\frac{T}{2}$ slot ST enables two of its transmit antennas; uses $ant1$ to forward the primary signal towards PR over the bandwidth W_A and $ant2$ is used to forward secondary information towards SR over the compensated bandwidth W_B. Since the secondary transmission occurs over the bandwidth W_B for T time duration, so in this case the RW_{ST} becomes W_BT. Now, the cooperative capacity (C_{PT}^{coop}) achieved by the PT through DF relaying and zero-forcing beamforming precoding technique [106] over bandwidth W_A and time T can be modeled based on the Sannon-Heartley theorem [100], [77], [17] is as follows.

$$C_{PT}^{coop} = \left((W_A \times T) log_2 \left(1 + \frac{P_{PT}|h_{PT,ST} DV_{PT,ST}^T|^2}{\sigma_{N_0}^2} + \frac{P_{ST}|h_{ST,PR} EV_{ST,PR}|^2}{\sigma_{N_0}^2} \right) \right) \tag{3.1}$$

Where, P_{PT} and P_{ST} are the transmission powers of PT and ST, $DV_{PT,ST}$ is the decoding vector used by ST to obtain PT's signal, $h_{PT,ST}$ and $h_{ST,PR}$ are the channel gain vectors from PT to ST and from ST to PR respectively, $EV_{ST,PR}$ is the encoding vector used by ST to transmit PT's signal towards PR and $\sigma_{N_0}^2$ is the noise variance. The encoding and decoding vectors used in Eq. (3.1) are calculated as described in [106]. To meet the inherent property of cooperative communication, PT never compromises its target objective (R_{PT}^{tar}) over the cooperative capacity achieved with the help of relay node. This implies that the cooperative capacity must be greater than its target objective [108], i.e.

$$C_{PT}^{coop} > R_{PT}^{tar} \tag{3.2}$$

or,

$$\left((W_A \times T) log_2 \left(1 + \frac{P_{PT}|h_{PT,ST} DV_{PT,ST}^T|^2}{\sigma_{N_0}^2} + \frac{P_{ST}|h_{ST,PR} EV_{ST,PR}|^2}{\sigma_{N_0}^2} \right) \right) > R_{PT}^{tar} \tag{3.3}$$

or,

$$(W_A \times T) > \frac{R_{PT}^{tar}}{\left(log_2 \left(1 + \frac{P_{PT}|h_{PT,ST} DV_{PT,ST}^T|^2}{\sigma_{N_0}^2} + \frac{P_{ST}|h_{ST,PR} EV_{ST,PR}|^2}{\sigma_{N_0}^2} \right) \right)} \tag{3.4}$$

This implies that $W_A T$ is the required resource (in terms of time bandwidth product) allocation that needs to be decided by PT for cooperative communication, so that it must satisfy PT constraints as well provides maximum possible cooperative benefit to PT. But at the same time it needs to satisfy the reward constraint of ST too.

On the other hand, without any SU relaying, the PU data transmission takes place over the whole time slot T, and the PU energy consumption amounts to $P_{PT}WT$ Joules [108]. However, when cooperative relaying takes place, as shown in Figure 3-3(a)), the PT transmits its signal towards ST for only $\frac{T}{2}$ fraction of time over bandwidth W_A, which implies the energy consumption is only $P_{PT}W_A\frac{T}{2}$ Joules. Therefore, the ratio of energy consumption of PT (EN_{PT}) in Case 1 over its original energy consumption value is defined as follows:.

$$EN_{PT} = \frac{P_{PT}W_A\frac{T}{2}}{P_{PT}WT} = \frac{W_A}{2W} \tag{3.5}$$

From Eq. (3.5), we can see that larger the use of W_A in the cooperative communication, higher the energy is consumed by the PT. Therefore, to keep down the EN_{PT}, PT needs to allocate minimum possible W_A during cooperative communication, but of course it should satisfy Eq. (3.4).

3.3.2 Case 2

For time sharing, time division channel sharing concept has been adopted and hence PT prefers suitable ST to relay primary signal towards PR over the shared time. Initially, PT decides to transmit its data towards ST for $\frac{1}{2}\beta T$ time duration over the entire bandwidth W. On receiving PT's signal, the selected ST employs Amplify-and-Forward (AF) relaying technique and forwards it towards PR for the next $\frac{1}{2}\beta T$ time slot. After the successful transmission, PT releases the remaining time T_B as the compensation to ST for the secondary transmission over bandwidth W. Therefore, in Case 2, the RW_{ST} becomes WT_B.

Now, the C_{PT}^{coop} achieved by PT through AF relaying technique over time T_A and bandwidth W can be modelled based on Shannon-Hartley theorem [100] as follows:.

$$C_{PT}^{coop} = \left((W \times T_A)log_2\left(1 + SNR_{PT,PR} + \frac{SNR_{PT,ST}SNR_{ST,PR}}{SNR_{PT,ST} + SNR_{ST,PR} + 1}\right)\right)$$
$$= \left((W \times T_A)log_2\left(1 + \frac{P_{PT}}{\sigma_{N_0}^2}\left|h_{PT,PR}\right|^2 + \frac{\frac{P_{PT}}{\sigma_{N_0}^2}\left|h_{PT,ST}\right|^2 \frac{P_{ST}}{\sigma_{N_0}^2}\left|h_{ST,PR}\right|^2}{\frac{P_{PT}}{\sigma_{N_0}^2}\left|h_{PT,ST}\right|^2 + \frac{P_{ST}}{\sigma_{N_0}^2}\left|h_{ST,PR}\right|^2}\right)\right) \tag{3.6}$$

But, according to the target constraint of PT, $C_{PT}^{coop} > R_{PT}^{tar}$ and this implies:

$$\left((W \times T_A)log_2\left(1 + SNR_{PT,PR} + \frac{SNR_{PT,ST}SNR_{ST,PR}}{SNR_{PT,ST} + SNR_{ST,PR} + 1}\right)\right) > R_{PT}^{tar} \tag{3.7}$$

or,

$$(W \times T_A) > \frac{R_{PT}^{tar}}{\left(log_2\left(1 + SNR_{PT,PR} + \frac{SNR_{PT,ST}SNR_{ST,PR}}{SNR_{PT,ST}+SNR_{ST,PR}+1}\right)\right)} \tag{3.8}$$

This implies WT_A is the required resource (in terms of time bandwidth product) allocation, which needs to be decided by PT for cooperative communication, so that it must satisfy PT constraints as well as provide the maximum possible cooperative benefit to PT.

However, the EN_{PT} during the transmission towards ST over $\frac{T_A}{2}$ time is found $P_{PT}W\frac{T_A}{2}$ Joules. If we formulate the same over the original energy consumption of PT then it is defined as follows:

$$EN_{PT} = \frac{P_{PT}W\frac{T_A}{2}}{P_{PT}WT} = \frac{T_A}{2T} \tag{3.9}$$

From Eq. (3.9), we can see that EN_{PT} increases with the increase of T_A. Therefore, to keep EN_{PT} low, PT needs to allocate minimum possible T_A during cooperative communication, but of course it should satisfy Eq. (3.8).

3.4 Problem Formulation

The design objective of the proposed CSS scheme is to select the most profitable ST as a cooperative relay by the PT such that it can attain maximum possible utility (U_{PU}) with an assist from the relay node. In this context, the U_{PU} for both Case 1 and Case 2 is modeled in terms of maximizing the cooperative capacity (C_{PT}^{coop}) obtained by PT (termed as profit factor of PT), for minimum possible energy consumption (EN_{PT}) and reward to ST (RW_{ST}) incurred during cooperative communication (termed as cost factor of PT), which is expressed as follows [114]:

$$U_{PU} = \frac{C_{PT}^{coop} \times \phi}{EN_{PT} \times RW_{ST}} \tag{3.10}$$

Here, ϕ is the gain per unit of data transfer achieved at the Maximal Ratio Combining output. The purpose of introducing ϕ as a constant gain parameter is to enhance the cooperative capacity received by the PU, which will be accomplished by increasing the data transfer rate for the PUs.

From Eq. (3.10), it is seen that U_{PU} increases with the increase of C_{PT}^{coop}, which is possible for large allocation of W_A (for Case 1) or T_A (for Case 2) during cooperative communication (according to Eq. (3.1) (or Eq. (3.6)). This goes in the favour of PT, as large the use of W_A (or T_A) reduces W_B (or T_B) which further lowers down the RW_{ST} that provided to corresponding relay node. So, PT has a tendency to allocate large possible W_A (or T_A) to maximize C_{PT}^{coop} and to release minimum possible RW_{ST}. But at the same time PT needs to monitor the resultant RW_{ST}, as it must satisfies the reward constraint of ST ($RW_{ST} > RW_{ST}^{min}$). However, the EN_{PT} has the reverse characteristics with the increase of W_A (or T_A), as large of its allocation maximizes EN_{PT} (according to Eq. (3.5) (or Eq. (3.9))) and lowers down the U_{PU}. Thus, there is a contradiction, and it is necessary to balance the tradeoff between the profit and cost factors of PT by deciding the appropriate allocation of W_A (or T_A) so that the maximum possible U_{PU} can be achievable, along with the resource constraint of the cooperative partner ST should be satisfied.

3.4.1 Allocation of W_A

During the allocation of W_A for cooperative communication, PT divides the total bandwidth W based on a decision variable γ ($W_A = \gamma W$). While estimating γ for bandwidth sharing, the PT must consider few key factors : (a) The total allotted resource for cooperative communication ($W_A \times T$) should be greater than RC_{PT}^{tar}, so that Eq. (3.2) gets satisfied. (b) The total allotted resource for relay based communication ($RC_{ST}^{rel} = W_A \times \frac{T}{2}$) should not exceed the maximum relaying capacity announced by ST (RC_{ST}^{max}) and (c) EN_{PT} should keep minimum, as well as RW_{ST} should satisfy RW_{ST}^{min}. To meet condition (a), PT strives to allocate the highest achievable value of W_A by opting for a significant γ value. However, as W_A increases, the size of RC_{ST}^{rel} also grows, potentially leading to a violation of condition (b). Moreover, extensive utilization of W_A raises the energy cost EN_{PT} and diminishes W_B, which could further result in contravention of condition (c). Consequently, condition (a) conflicts with conditions (b) and (c). As a result, PT must carefully determine the value of γ in a manner that allows for the optimal allocation of W_A while satisfying conditions (a), (b), and (c), all while maximizing the achievable U_{PU}. The optimization problem associated with this is formulated as follows:

$$\begin{aligned} \arg\max_{\gamma} \quad & U_{PU} \\ \text{s.t.} \quad & (a) 0 < \gamma < 1, \quad \beta = 1 \\ & (b) RC_{PT}^{tar} < (W_A T) \\ & (c) RC_{ST}^{max} > RC_{ST}^{rel} \\ & (d) RW_{ST} \geq RW_{ST}^{min} \end{aligned} \tag{3.11}$$

3.4.2 Allocation of T_A

In time division sharing, PT decides to divide the total access time T in two time slots, viz. $T_A = \beta T$ and $T_B = (1-\beta)T$ for cooperative and secondary communication, based on a decision variable β. As previously discussed in the context of allocating W_A, PT must also consider few key points when determining the value of β for the allocation of both T_A and T_B as: (a) The total allocated resource ($W \times T_A$) must be larger than RC_{PT}^{tar}, (b) $RC_{ST}^{rel} = W \times \frac{T_A}{2}$ should not exceed RC_{ST}^{max} and (c) EN_{PT} should keep minimum and RW_{ST} must satisfy RW_{ST}^{min}. In order to meet condition (a), PT aims to allocate the maximum possible amount of T_A by selecting a large value for the parameter β. However, as T_A increases, the size of RC_{ST}^{rel} also grows, potentially leading to a violation of condition (b). Furthermore, the substantial utilization of T_A leads to an increase in EN_{PT} and a reduction in T_B which, in turn, further decreases RW_{ST}. As a result, condition (a) contradicts with both conditions (b) and (c). Consequently, PT must optimize the value of β in such a way that it enables the optimal allocation of T_A and results maximum possible U_{PU} as follows:

$$\begin{aligned} \arg\max_{\beta} \quad & U_{PU} \\ \text{s.t.} \quad & (a) 0 < \beta < 1, \quad \gamma = 1 \\ & (b) RC_{PT}^{tar} < (WT_A) \\ & (c) RC_{ST}^{max} > RC_{ST}^{rel} \\ & (d) RW_{ST} \geq RW_{ST}^{min} \end{aligned} \tag{3.12}$$

3.4.3 Nature of the Problem

To analyze whether the proposed optimization problem is easy or hard problem, we need to determine in which category the objective function of the problems belong, i.e. whether it exhibits linear or non-linear nature. Lets check the category of the objective function (for Case 1: Bandwidth sharing) as follows:

$$\begin{aligned} \max \quad & (U_{PU}) \\ where, U_{PU} = & \left(\frac{C_{PT}^{coop} \times \phi}{EN_{PT} \times RW_{ST}} \right) \\ = & \left(\frac{\Big((W_A \times T) \log_2 (1 + SNR + SNR) \Big) \times \phi}{(\frac{W_A}{2W}) \times (W_B T)} \right) \end{aligned} \tag{3.13}$$

Here, ϕ, T are constants and the term $log(1+SNR+SNR)$ is independent of the value W_A and W_B. so we can ignore them and rewrite Eq. (3.13) as :

$$U_{PU} = \frac{W_A}{W_A \times W_B}$$

Similarly, (3.14)

$$U_{PU} = \frac{T_A}{T_A \times T_B} \quad \text{(For Time sharing)}$$

Since the final derivation of U_{PU} Eq. (3.14) exhibits non-linear nature and non-linear functions are classified as hard problems. So finding the global optimal solution for a nonlinear objective function is intractable [28, 52]. To address such problems, solution techniques like Approximation algorithm, Heuristic algorithm are widely used. Therefore, to solve the proposed non linear problem, we introduce a numerical analysis based heuristic solution that provides near optimal solution for determining γ for bandwidth allocation (W_A, W_B) or β for time allocation (T_A, T_B) in polynomial time. The proof of convergence and the running time complexity of the proposed solution (Algorithm 3) are presented in the following section.

3.5 Proposed scheme

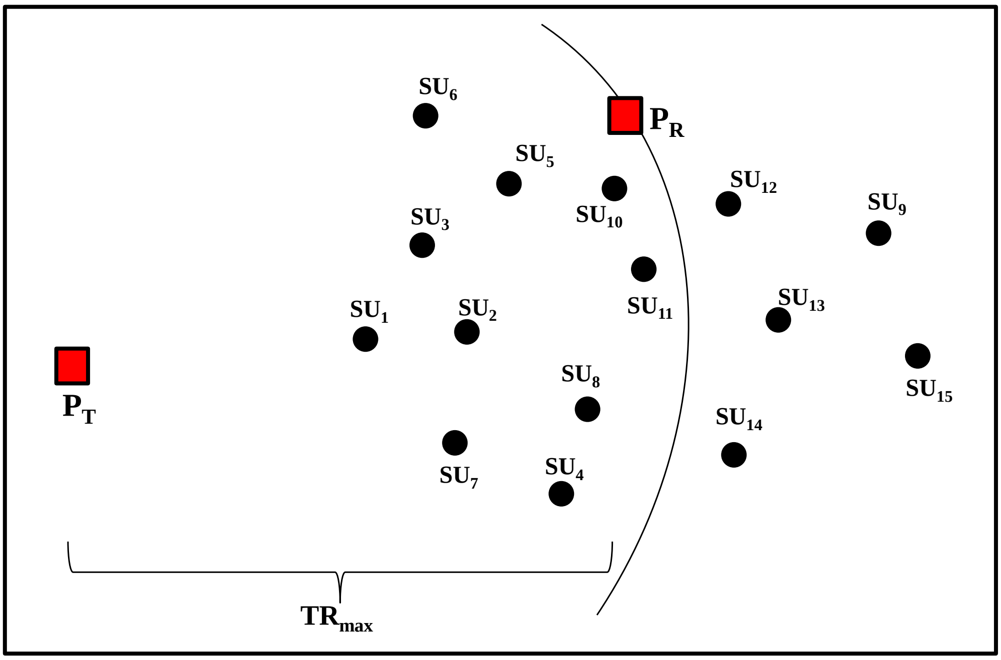


Figure 3-4: Considered PU-SU distribution for proposed scheme

The proposed scheme comprises two phases: suitable relay selection phase and optimal resource allocation phase for the cooperative communication. A scenario with randomly distributed relays as shown in Figure 3-4 is considered to analyse the relay selection technique by the PT. Here, few relays are located inside the TR_{max} and few are beyond this. Since we are modeling cooperative spectrum sharing among PT, PR and the relays, so locations of relays need to be lied well within the reachable range of PT and PR. It implies, the relays those lies outside the TR_{max} are not feasible for cooperation and out of the race from resource sharing with PT. Now, based on the targeted quality of service (QoS) requirements and resource constraints of PT (as discussed in section 5), the decision either on bandwidth sharing or time sharing has been taken by the PT itself.

To be a part of cooperative communication, each SU announces (i) its maximum relaying power capacity (P_{ST}^{max}), (ii) per frequency transmission power rate (λ Watt/Hz) (iii) energy consumption rate (τ Watt.sec) and (iv) reward or compensation constraint (RW_{ST}^{min}) in terms of time bandwidth product to the PT. However, it is noted that along with the time bandwidth product, the individual bandwidth and time constraint of ST needs to be satisfied. Since the SU_s are considered to be heterogeneous, so the resource offers announce by them are also found dissimilar based on their characteristics, locations and QoS requirements.

3.5.1 Bandwidth Sharing

In this case, PT prefers **MIMO** competence SUs for cooperative communication and analyzes the resource offers announced by them. For each offer and resource constraint of ST, PT separately calculates corresponding U_{PU} and appropriate W_A; and selects the ST as its cooperative partner for which maximum possible U_{PU} can be achievable. All necessary steps regarding suitable relay selection and W_A allocation are summarized in Algorithm 1.

Algorithm 1: Optimal relay selection and W_A allocation for cooperative communication

Input : Total bandwidth W, Total time T, $\beta = 1$, Offers of relay's (P_{ST}^{max}, RW_{ST}^{min} and λ).

Output: Optimal ST, W_A and maximum U_{PU}.

Using Shannon's theorem, PT calculates RC_{PT}^{tar} to attain R_{PT}^{tar}.

Calculates eligible relays for which $dist_{s,r} \leq TR_{PT}^{max}$.

for *each eligible relay i* **do**

  Calculates $RC_{ST,i}^{max} = \left(\frac{P_{ST,i}^{max}}{\lambda_i} \frac{T}{2} \right)$.

  Provides $RC_{ST,i}^{max}$, W and $RW_{ST,i}^{min}$ as the inputs to **Algorithm 3**.

  Obtain $W_{A,i}$, $C_{PT,i}^{coop}$, $U_{PU,i}$, $RW_{ST,i}$ and $EC_{ST,i}$ from **Algorithm 3**.

end

Extract W_A, C_{PT}^{coop}, U_{PU}, RW_{ST} and EN_{ST} for each relay node.

Find Max(U_{PU}) among the extracted values.

Find index of Max(U_{PU}).

PT selects the relay with corresponding index and allocates respective W_A with it for the cooperative communication.

3.5.2 For Time Sharing

Based on each offers and resource constraint of ST, PT calculates corresponding U_{PU} and appropriate T_A; and selects ST as cooperative partner for which maximum possible U_{PU} can be achievable. All necessary steps regarding suitable relay selection and appropriate T_A allocation are summarized in Algorithm 2.

Algorithm 2: Optimal relay selection and T_A allocation for cooperative communication

Input : Total time T, Total bandwidth W, $\gamma = 1$, Offers of relay's (P_{ST}^{max}, RW_{ST}^{min} and τ).

Output: Optimal ST, T_A and maximum U_{PU}.

Using Shannon's theorem, PT calculates RC_{PT}^{tar} to attain R_{PT}^{tar}.
Calculates eligible relays for which $dist_{s,r} \leq TR_{PT}^{max}$.
for *each eligible relay i* **do**
    From the offer τ_i, PT calculates corresponding power in Watt (Ψ_i) for unit time.
    Calculates $RC_{ST,i}^{max} = \left(W^{\frac{P_{ST,i}^{max}}{\Psi_i}}\right)$.
    Provides $RC_{ST,i}^{max}$, T and RW_{ST}^{min} as the inputs to **Algorithm 3**.
    Obtain $T_{A,i}$, $C_{PT,i}^{coop}$, $U_{PU,i}$, $RW_{ST,i}$ and $EC_{ST,i}$ from **Algorithm 3**.
end
Extract T_A, C_{PT}^{coop}, U_{PU}, RW_{ST} and EN_{ST} for each relay node.
Find Max(U_{PU}) among the extracted values.
Find index of Max(U_{PU}).
PT selects the relay with corresponding index and allocates respective T_A with it for the cooperative communication.

3.5.3 Algorithm 3:

The proposed Algorithm 3, based on numerical analysis technique and heuristic approach is developed to achieve the near-optimal solution for the allocation of PU resources (either bandwidth or access time) among the PU and the selected ST. Algorithm 3 jointly works for Algorithm 1 and 2 based on the inputs provided to it and delivers appropriate resource allocation (either W_A or T_A) for cooperative communication. For simplicity, we have considered the inputs provided by Algorithm 1 to Algorithm 3 and have shown the allocation of W_A (for Case 1) in the working explanation of Algorithm 3. However, to obtain appropriate allocation of T_A for Case 2, the inputs of Algorithm 2 should be provided to Algorithm 3 and the decision variable changes (γ is replaced by β) accordingly .

Time Complexity of Algorithm 3: Initially we compute the time complexity of ***while*** loop. Let, consider the length of $\gamma = [0, 1]$ is n, where 0 is the minimum value of γ and 1 is it's maximum value. At every iteration of ***while*** loop, the n is

Algorithm 3: Heuristics algorithm to obtain near-optimal resource allocation among PUs and SUs

Input : Total bandwidth W, RC_{ST}^{max}, RW_{ST}^{min} and ω.
Output: Optimal allocation of W_A and corresponding U_{PU} of PT.

1 Set flag = 0.
2 **while** *(flag!=1)* **do**
3 **Initialize** $\gamma = [0, 1]$.
4 **Initialize** low=γ[0], high=γ[1].
5 Calculate $\gamma_{mid} = \left(\frac{low+high}{2}\right)$, $\gamma_{mid_1} = \left(\frac{low+\gamma_{mid}}{2}\right)$, $\gamma_{mid_2} = \left(\frac{\gamma_{mid}+high}{2}\right)$.
6 Assign $m = (W\gamma_{mid})$, $m+1 = (W\gamma_{mid_1})$, $m+2 = (W\gamma_{mid_2})$.
7 Calculates RC_{ST}^{rel} for m, $m+1$ and $m+2$ allocation points separately.
8 **if** *($RC_{ST}^{rel,m+2} \leq RC_{ST}^{max}$)* **then**
9 Computes U_m, U_{m+1}, U_{m+2}. (Where U is the function for calculate utility of PU based on Eq. (3.10))
10 **if** *($U_m \geq U_{m+1}$ & $U_m \geq U_{m+2}$ & $RW_{ST}^{m} \geq RW_{ST}^{min}$)* **then**
11 *low*=γ_{mid_1}, *high*=γ_{mid_2}.
12 Repeat step 4 and step 5.
13 **if** *($|m-(m+1)|$ & $|m-(m+2))| \leq \omega$* **then**
14 Return U_m, RW_{ST}^{m}.
15 flag = 1.
16 **else**
17 Computes U_m, U_{m+1}, U_{m+2}.
18 Computes RW_{ST} for m, $m+1$, $m+2$ separately.
19 **end**
20 **end**
21 **if** *($U_m \leq U_{m+1}$ & $U_m \geq U_{m+2}$ & $RW_{ST}^{m+1} \geq RW_{ST}^{min}$)* **then**
22 *low*=$A[0]$, *high*=γ_{mid}.
23 Repeat step 4 and step 5.
24 **if** *($|m-(m+1)|$ & $|m-(m+2)| \leq \omega$)* **then**
25 Return U_{m+1}, RW_{ST}^{m+1}.
26 flag = 1.
27 **else**
28 Repeat steps 17 and 18.
29 **end**
30 **end**
31 **if** *($U_m \geq U_{m+1}$ & $U_m \leq U_{m+2}$ & $RW_{ST}^{m+2} \geq RW_{ST}^{min}$)* **then**
32 *low*=γ_{mid}, *high*=γ[1].
33 Repeat step 4 and step 5.
34 **if** *($|m-(m+1)|$ & $|m-(m+2)| \leq \omega$)* **then**
35 Return U_{m+2}, RW_{ST}^{m+2}.
36 flag = 1.
37 **else**
38 Repeat steps 17 and 18.
39 **end**
40 **end**
41 **else**
42 Initialize $\gamma[1] = \left(\frac{high+\gamma_{mid_2}}{2}\right)$.
43 Initialize low=γ[0], high=γ[1].
44 **GO** TO step 5 and **Repeat**.
45 **end**
46 **end**
47 From the returned U and RW_{ST} value, identify corresponding allocation point (either m or $m+1$ or $m+2$) and extract γ for the same.
48 Extract W_A for the γ and identified associate C_{PT}^{coop}, and EN_{PT}.
49 Return W_A, C_{PT}^{coop}, U, RW_{ST} and EN_{PT} to Algorithm 1.

reduced by half and runs until $\frac{n}{2} \leq \omega$ Assume, it runs maximum for k times then we can write $\frac{n}{2^k} \leq \omega$, which turns k = $O(log\frac{n}{\omega})$. For total N number of eligible relays, the overall running complexity of Algorithm 3 becomes $O(N \log \frac{n}{\omega})$, which is **polynomial** time complexity.

Convergence of Algorithm 3: According to Algorithm 3, U_m, U_{m+1} and U_{m+2} are the utilities for the bandwidth allocation point m, $m+1$ and $m+2$ respectively. The modeling of Algorithm 3 is maintained in such a way so that as the number of iteration increases, the size of γ $[0, 1]$ starts shrinking towards the point where PT can able to achieve maximum possible U_{PU}. So, at each iteration, PT calculates U_m, U_{m+1} and U_{m+2} for corresponding m, $m+1$ and $m+2$ allocation points found so far and verifies their interval. The PT allows to run the algorithm till the difference between the allocation points reaches a negligible value i.e. ω. And when it reaches the same, the flag value which is initialized as 0, turns to 1 and the algorithm terminates with maximum possible U_{PU} for corresponding allocation point, from which we can identify the bandwidth allocation point W_A. Thus, we can draw that Algorithm 3 terminates when the difference between m, $m+1$ and $m+2$ reaches ω or less than it.

Proof of Continuity: To prove the continuity of the proposed objective function, Eq. (3.11) (or Eq. (3.12)) for the decision variable γ (or β) within the interval (0,1), the concept of differentiability will be used. Since differentiability implies continuity, demonstrating that the objective function say $f(\gamma)$ (or $f(\beta)$) is differentiable within interval (0,1) will automatically establish its continuity in the said interval. Let's analyze the proof.

- Simplifying and cancelling the constant terms of the objective function Eq. (3.11), the simplified form in terms of $f(\gamma)$ is written as:

$$f(\gamma) = \frac{C}{1 - (W \times \gamma)} \tag{3.15}$$

- To check differentiability, we need to verify if the derivative $f'(\gamma)$, exists for all $\gamma \in (0, 1)$. Therefore,

$$f'(\gamma) = \frac{d}{d\gamma}\left(\frac{C}{1-\gamma}\right) \\ = -\frac{C}{(1-\gamma)^2} \tag{3.16}$$

- To check the existence of the derivative, let verify the following points :
  - The derivative $f'(\gamma) = -\frac{C}{(1-\gamma)^2}$ exists for all $\gamma \in (0,1)$, since the denominator $\frac{C}{(1-\gamma)^2}$ is always positive in this range.
  - The expression $\frac{C}{(1-\gamma)^2}$ never equals zero for $\gamma \in (0,1)$, ensuring the derivative is well-defined everywhere in this interval.

  Since, $f(\gamma)$ is differentiable for all $\gamma \in (0,1)$, it is also continuous in this interval by the principle that differentiability implies continuity.

3.6 Case analysis and Results discussions

Here, we have investigated **WHY** and **WHEN** PT decides to adopt bandwidth sharing and time sharing strategies for cooperative communication. In this regard, two types of PU services, viz. best effort service and time-sensitive service are considered and analyzed. Particularly, the best effort service includes the best delivery of primary data towards the intended PR so that the targeted objectives of PT are achieved as well as the cooperative capacity is maximized. However, the time-sensitive service includes successful delivery of PT data to PR within the strict time window allotted so far. For detail analysis, let us consider the following scenarios depicted in Figure 3-5:

Scenario 1 (For Best effort service) As shown in Figure 3-5, PT aims its best to deliver the targeted service towards PR as well as try to maximize its utility. Since PT employs overlay paradigm, so it seeks help from ST during its communication towards PR and on behalf of the help, PT provides the access right to ST over a portion of its bandwidth to achieve the secondary target towards SR.

Scenario 2 (For Time sensitive service) As shown in Figure 3-5, PT aims to achieve the targeted objective towards PR within the strict time window (r_1

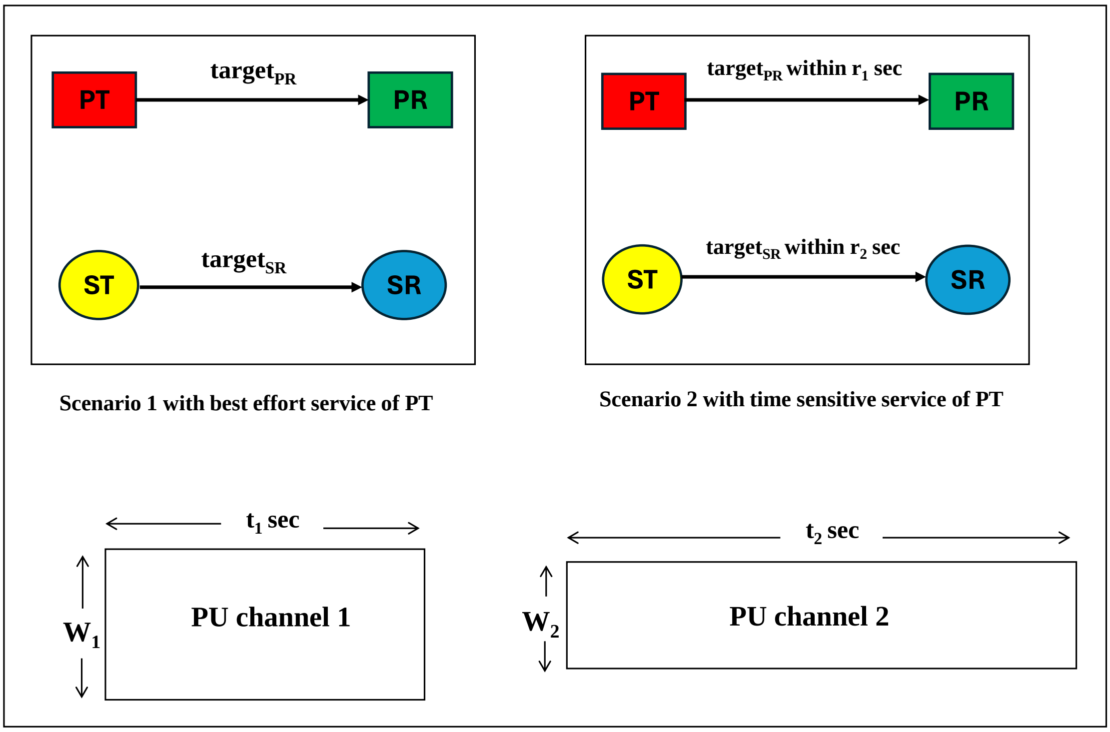


Figure 3-5: Considered scenario for case analysis

sec) defined. Like Scenario 1, PT seeks help from ST and in return provides transmission opportunity (r_2 sec) to ST for accessing its channel.

Available PU resource Let consider, PU is the owner of two channels say (a) PU channel 1 (PU_{ch1}): with bandwidth W_1 MHz for a time window t_1 sec and (b) PU channel 2 (PU_{ch2}): with bandwidth W_2 MHz for a time window t_2 sec as shown in Figure 3-5. Further assume, $W_1 >> W_2$ and $t_2 >> t_1$.

Analysis 1 In scenario 1, PT aims its best to maximize the data transmission towards PR to achieve the targeted objective. Since PT does not have the strict time bound during the communication towards its receiver, so it prefers bandwidth sharing rather than time, so that optimal bandwidth chunks can be allotted to satisfy the considered target, reward constraint of ST as well as to maximize the C_{PT}^{coop}. In this regard, PT supports the communicating channel with large bandwidth size i.e. **PU channel 1** and applies bandwidth sharing on it to allocate optimal bandwidth chunks for $target_{PR}$ and $target_{SR}$ as shown in Figure 3-6(a). Let assume, PT finds the bandwidth allocation factor $(\gamma) = \gamma_1$ for $target_{PR}$ and allocates $W_{PR} = (W_1 \times \gamma_1)$ MHz to it. And releases the remaining bandwidth W_{SR} MHz to ST for $Target_{SR}$. However, the given time t_1 should satisfy the time constraint of each of the specified target.

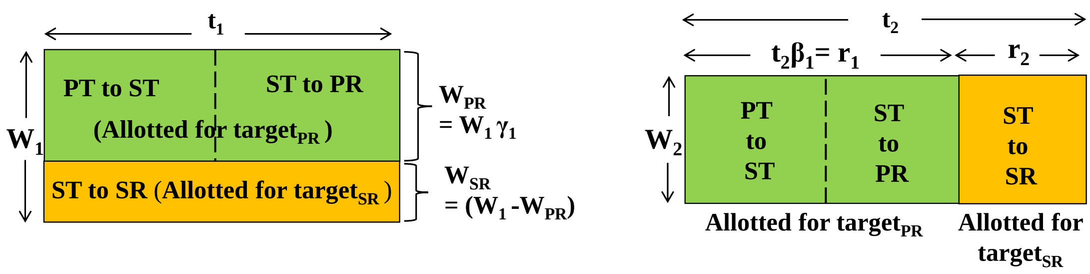


Figure 3-6: (a) Bandwidth sharing on PU Channel 2 (b) Time sharing on PU channel 1

Analysis 2 In scenario 2, PT aims to maximize the delivery of critical services towards PR, so that it yields successful data transmission within the defined time window. At the same time PT needs to provide the access right to ST over its channel for certain time. This demands optimal allocation of the total channel access time and thus PT prefers time sharing strategy rather than bandwidth to fit the targeted services over the optimally allotted time slots. To accommodate each target, PT supports **PU channel 2** with large access time ($t_2 >> t_1$) and allocates optimal time slots for $target_{PR}$ and $target_{SR}$ as shown in Figure 3-6(b). Let assume, PT finds the time allocation factor $(\beta) = \beta_1$ for $target_{PR}$ and allocates $(t_2\beta_1) = r_1$ sec to it. And, releases the remaining time=$(t_2 - r_1) = r_2$ sec to ST. However, the given bandwidth W_2 should satisfy the bandwidth constraints of all the specified targets.

The subsequent workflow diagram elucidates the PU service types and the applicable sharing strategies employed for cooperative communication.

3.6.1 Simulation setup

To maintain the inherent property of CSS [70], we focus on such a PU network where PU's direct channel gain ($h_{PT,PR}$) is much smaller than SU's relay channel gains ($h_{PT,ST}$ and $h_{ST,PR}$). As shown in Figure 3-8, a CRN network with one pair of (PT, PR) and 20 pairs of (ST, SR) distributed in a 500 × 500 m^2 square area is considered, where the green and red square boxes represent PT and PR respectively and the blue circles portrayed as relays. Here fifteen relays (1, 2,....,15) are found within TR_{PT}^{max} (>=400 meter or m) and remaining are found beyond TR_{PT}^{max}, so relay nodes (16,...,20) are not able to be the part of cooperation. To

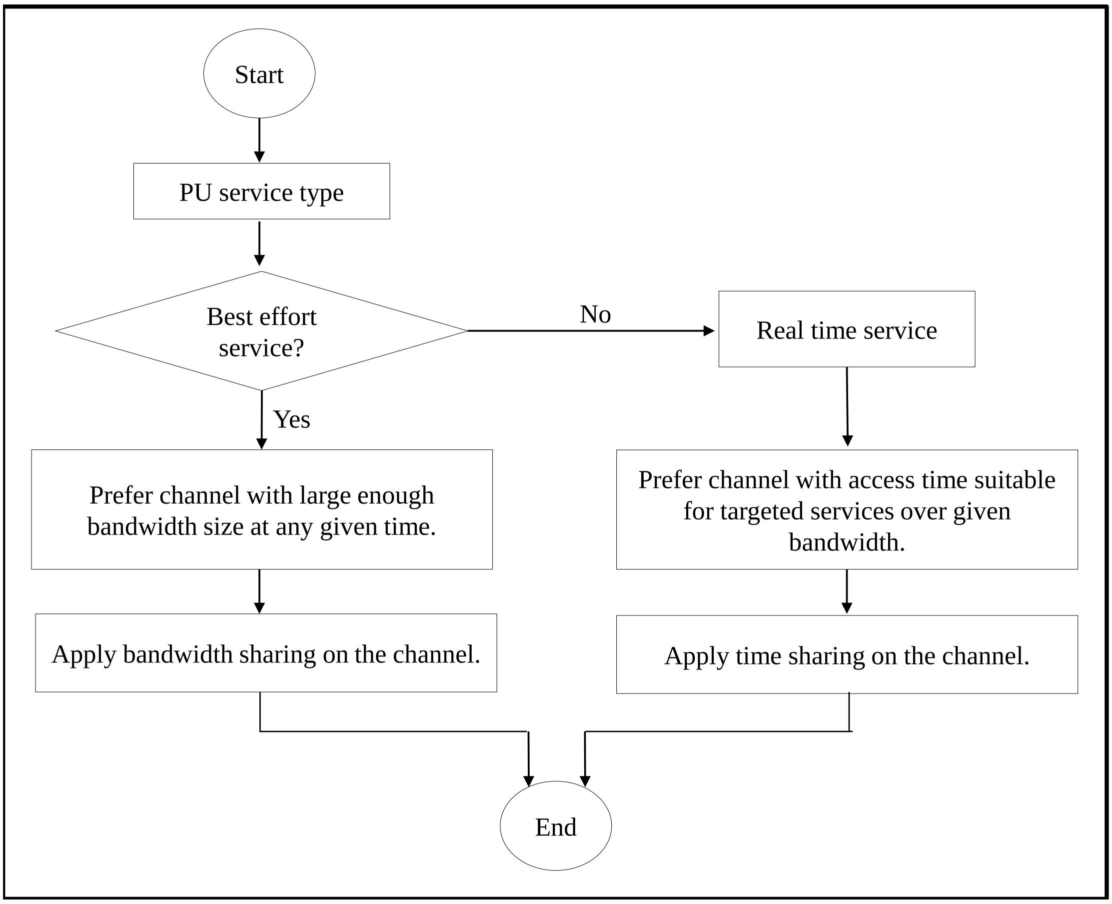


Figure 3-7: Workflow diagram of the proposed schemes

analyze the efficacy of the proposed sharing (allocation) schemes, two types of PU channels (PU_{Ch1} and PU_{Ch2}) are considered during simulation and apply both bandwidth and time sharing strategy on each of the channel. The PU channels are considered to be AWGN channels. Other relevant simulation parameters and their corresponding values are listed in Table 3.2. These meticulously chosen parameters and settings form the foundation for our comprehensive evaluation of the proposed solutions within the context of the CR network under investigation.

3.6.2 Performance Metrics

Following metrics have been used for simulation based performance analysis.

- Utility of PU (U_{PU}): Utility achieved by the PU, computed based on Eq. (3.10).
- Profit of PU (C_{PT}^{coop}): Cooperative capacity achieved by PU, computed based on Eq. (3.1) for Case 1 and Eq. (3.6) for Case 2.

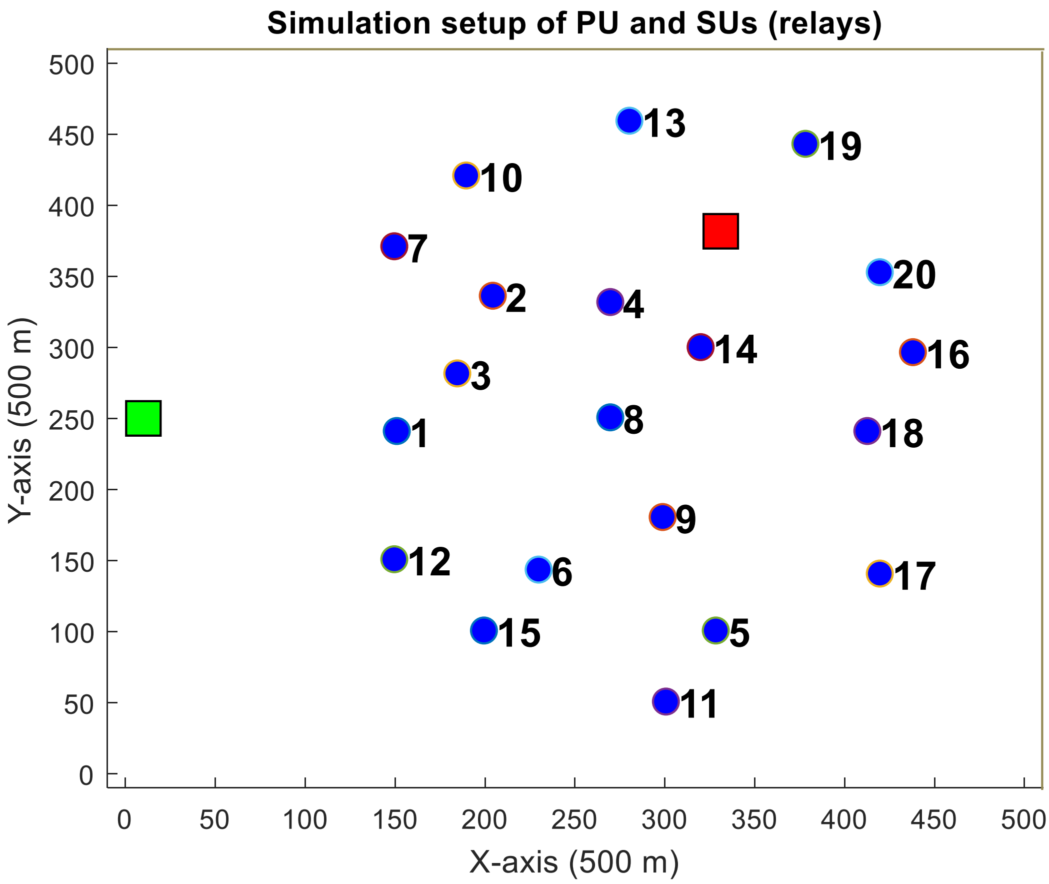


Figure 3-8: Simulation setup for the considered CRN scenario

Parameters	Values
PU_{Ch1}	$W = 1$ MHz, $T = 1$ sec
PU_{Ch2}	$W = 0.1$ MHz, $T = 10$ sec
P_{PT}	10 mW
R_{PT}^{tar}	1 Mb (over PU_{Ch1}), 5 Kb (over PU_{Ch2})
TR_{PT}^{max}	400 m
$d_{ST,SR}$	100 to 300 m (variable)
P_{ST}^{max}	10 mW to 15 mW (variable)
RW_{ST}^{min} (in terms of time bandwidth product)	0.20 to 0.30 (variable)
λ	$(1.3 \times 10^{-8}$ to $1.4 \times 10^{-8})$ Watt/Hz (variable)
τ	(0.0050 to 0.0060) $Watt \times sec$ (variable)
ϕ	0.01 [114]
$\sigma_{N_0}^2$	10^{-10} W
path loss exponent	2

Table 3.2: Simulation parameters and their values

- Cost of PU ($EN_{PT} \times RW_{ST}$): Associated costs of PU, computed based on Eq. (3.5) for Case 1 and Eq. (3.9) for Case 2.

- W_A and T_A allocation: Optimal chunk of bandwidth and time obtained from Algorithm 3 based on the inputs provided by Algorithm 1 and 2 respectively.
- U_{PU} analysis considering delays for Case 1 and Case 2 based on Eq. (3.17) and Eq. (3.18) respectively.

3.6.3 Experiment on PU_{ch1}

In first phase of simulation, we have considered the channel PU_{Ch1} and analyzed the effect of both bandwidth and time allocation strategy on it for cooperative communication.

3.6.3.1 PU's utility analysis for associated profit and cost values (for Case 1: bandwidth sharing)

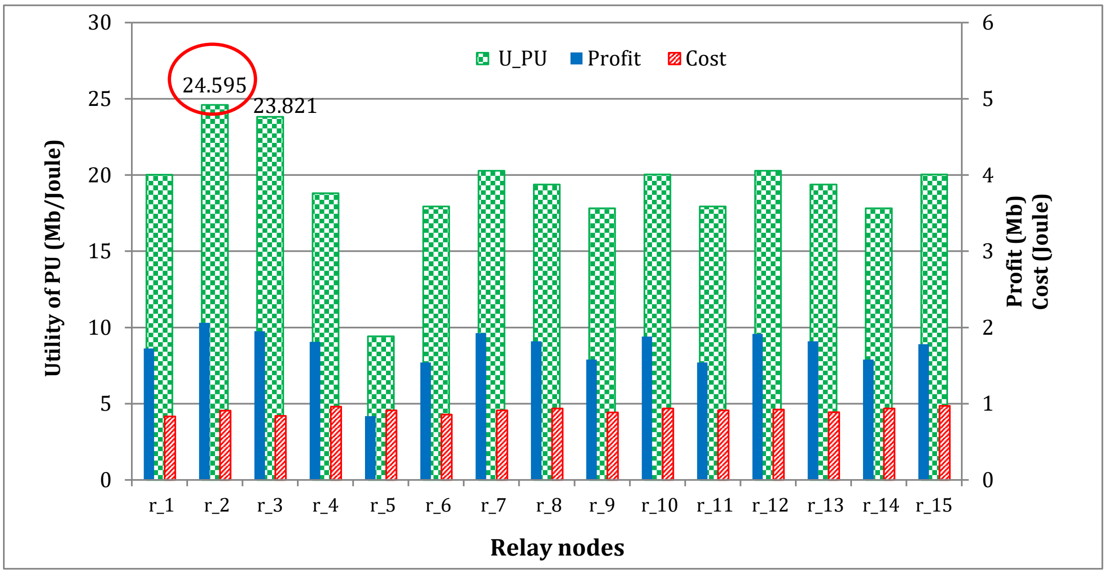


Figure 3-9: U_{PU} achieved for corresponding profit and cost value (for Case 1)

Initially, we implement bandwidth sharing strategy on PU_{ch1}, and analyze its efficacy. Figure 3-9 shows the graph of achieved utility of PU along with the profit ($C_{PT}^{coop} \times \phi$) and overall cost value ($EN_{ST} \times RW_{ST}$) achieved from Algorithm 1, associated with each relay node during the cooperative communication. Among 15 relays, the utility of PU is found maximum for relay 2 ($U_{PU,2} = 24.59$), where the $profit_2$ and $cost_2$ are found 2.06 and 0.91 respectively. Further, PT finds the next highest utility for relay 3 ($U_{PU,3} = 23.82$) with $profit_3 = 1.95$ and $cost_3 = 0.84$. If we analyze the profit and cost values of both the relays separately then it is found that though PT finds maximum U_{PU} with relay 2, the cost for relay 2 is found relatively high than of relay 3. This might be due to extra consumption of EN_{PT} and higher RW_{ST} associated with relay 2. In such circumstances, selection of relay depends on the target constraint of PT. For example, if PT sets cost minimization is in high priority, then among the two relay nodes PT selects relay 3 with low cost value. On the other hand, if utility maximization under a regional cost value is prioritized then relay 2 is selected by the PT. Since our proposed model gives emphasis on utility maximization of PT, so relay 2 is chosen by the PT as its cooperative partner and allocate optimal bandwidth chunk (W_A) with it for cooperative communication.

3.6.3.2 Analysis of PU's utility vs. allocated bandwidth chunk (W_A) for cooperative communication

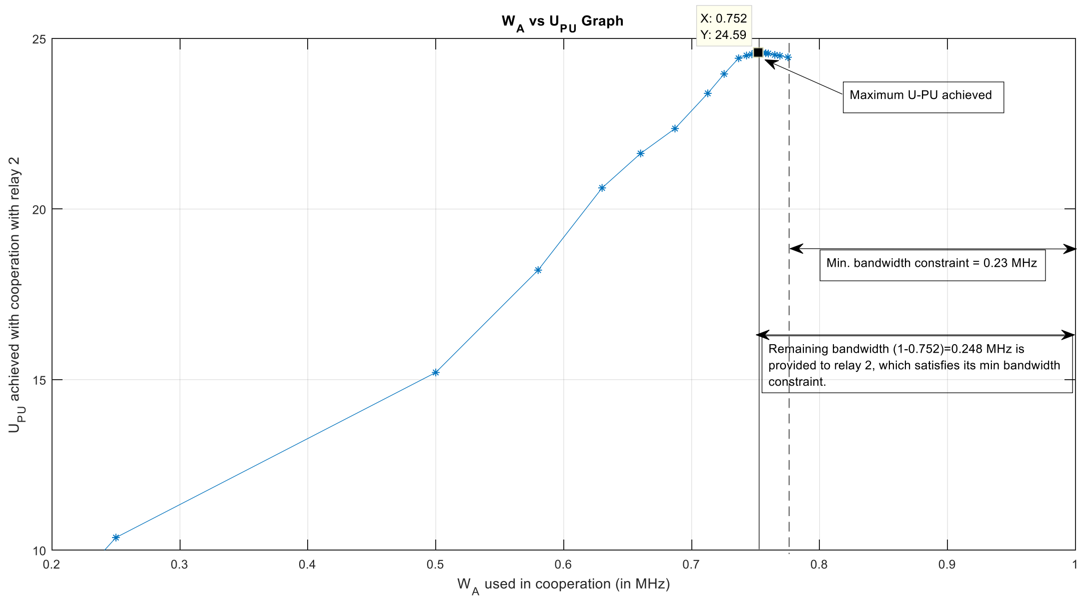


Figure 3-10: W_A allocation by PU with relay 2 (for Case 1)

The allocation of bandwidth chunk (W_A) by the PT with relay 2 for cooperative communication is shown in Figure 3-10. According to Algorithm 3, PT tries to choose optimal value of γ from the range (0,1), with the aim to maximize C_{PT}^{coop} as well as U_{PU}. Till the use of $\gamma = 0.752$ i.e. $W_A = (0.752 \times W) = 0.752$ MHz, PT obtains gradual increment in U_{PU} with $W_B = 0.248(1 - 0.752)$ MHz and $EC_{PT,2} = 0.376$. Further the increment of W_A in the cooperation, lowers down the generated W_B as well as increases $EC_{PT,2}$ that results in reduction of $U_{PU,2}$ from 24.59 to 24.569. So, Algorithm 3 terminates at $\gamma = 0.752$ and allocates $W_A = 0.752$ MHz for cooperative communication among the PT and relay 2. Further, PT releases $RW_{ST,2} = (0.248 \text{ MHz } \times 1 \text{ sec}) = 0.248$ (which is $>$ $RW^{min}_{ST,2} = 0.23$) to relay 2 for secondary access.

Cross case analysis: Here, we examine the effect of employing time-allocation strategy on PU_{Ch1} for cooperative communication. In this context, the total PU access time, 1 sec, should be optimally allocate for both cooperative and secondary communication. Algorithm 2 yields channel information as inputs to Algorithm 3, determining an optimal value for $\beta = 0.752$ with relay 2. Subsequently, $T_A = (\beta \times T) = (0.7521 \times 1 \text{ sec})= 0.752$ sec is allocated for cooperative communication. As a compensation, the remaining time $T_B = (1-0.752) = 0.248$ sec is released for relay 2 to access the entire 1 MHz channel bandwidth, resulting the value of $RW_{ST,2} = (0.248 \text{ sec} \times 1 \text{ MHz}) = 0.248$. While the generated $RW_{ST,2}$ satisfies $RW^{min}_{ST,2}$, it must also meets individual resource constraints separately. This implies that, in addition to the bandwidth requirement, it must adhere to the time constraint set by ST for successful communication with SR. In this case, the allocated $T_B = 0.248$ sec by PT does not meet the minimum time constraint of relay 2, thus violating its reward constraint. Even if PT opts for the relay node with the next highest U_{PU}, relay 3, a violation of the time constraint persists. Consequently, the time-sharing strategy on PU_{Ch1} fails to meet the access time requirements specified by the relay nodes and is therefore not applicable to PU_{Ch1}.

3.6.4 Experiment on PU_{Ch2}

In second phase of simulation, we have considered the channel PU_{Ch2} and analyzed the effect of both time and bandwidth allocation strategy on it for cooperative communication.

3.6.4.1 PU's utility analysis for associated profit and cost values (for Case 2: time sharing)

On applying the time sharing strategy on PU_{ch2}, the graph as shown in Figure 3-11 is found for the achieved U_{PU} along with corresponding profit and cost incurred. Among all the eligible relays, the utility of PU is found maximum for relay 3 ($U_{PU,3} = 17.66$), where the $profit_3$ and $cost_3$ are found 1.854 and 0.5265 respectively. On the other hand, PU finds the next highest utility for relay 2 ($U_{PU,2} = 17.48$) with $profit_2 = 1.869$ and $cost_2 = 0.5345$. Though PU finds max-

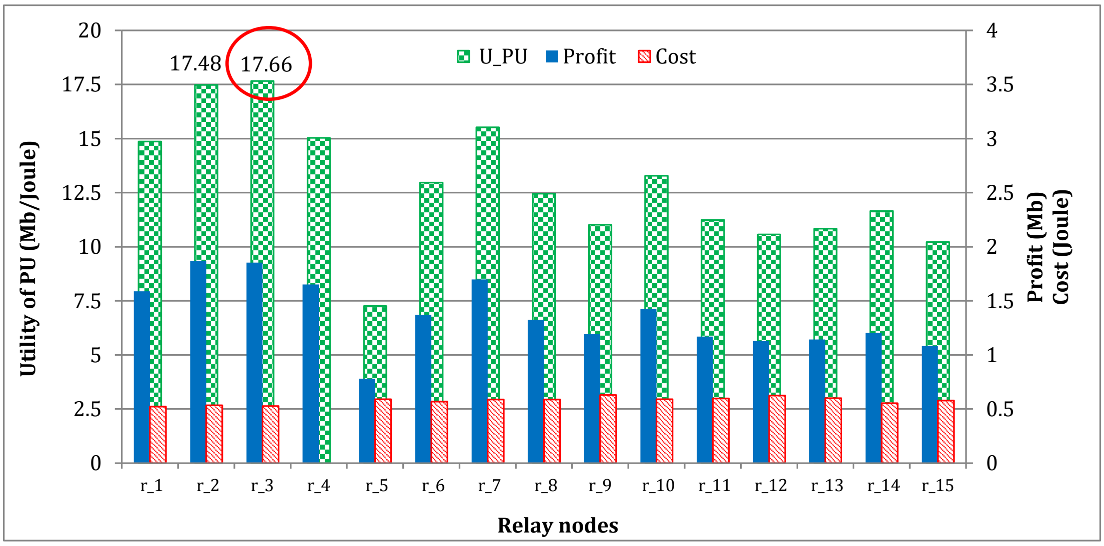


Figure 3-11: U_{PU} achieved for corresponding profit and cost value (for Case 2)

imum U_{PU} with relay 3 but the individual profit it finds maximum with relay 2. However, the cost value associated with relay 2 is seen relatively high than with relay 3, which results in little increment of $U_{PU,3}$. So, PU selects relay 3 as its cooperative partner, even it provides relatively less profit than with relay 2 and allocates appropriate time with it.

3.6.4.2 Analysis of PU's utility vs. allocated time slot (T_A) for cooperative communication

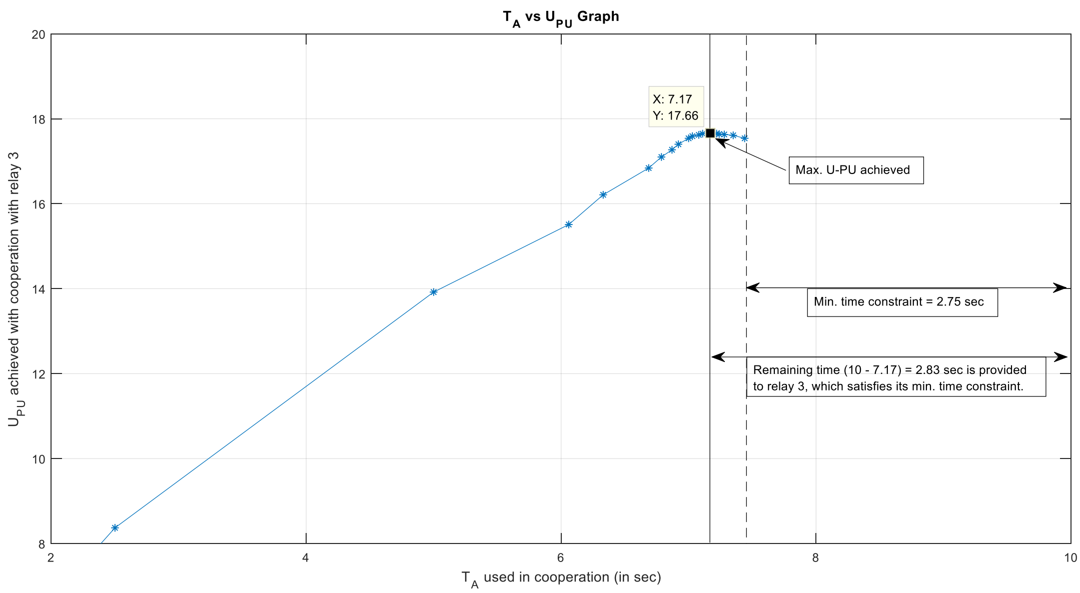


Figure 3-12: T_A allocation by PU with relay 3 (for Case 2)

The allocation of time slot (T_A) by the PT with relay 3 is shown in Figure 3-12. According to Algorithm 3, PT tries to choose optimal value of β from the range of (0,1), with the aim to maximize its U_{PU} value. Till the use of $\beta = 0.717$ i.e. $T_A = (0.717 \times T) = 7.17$ sec, PT obtains gradual increment in U_{PU} with $T_B = (10 - 7.17) = 2.83$ sec and $EN_{PT} = 0.3585$. Further increment of T_A in the cooperation, lowers down the generated T_B as well as increases EN_{PT} that results in reduction of $U_{PU,3}$ from 17.66 to 17.539. So, Algorithm 3 terminates at $\beta = 0.717$ and allocates $T_A = (0.717 \times 10) = 7.17$ sec for cooperative communication. Further, PT releases $RW_{ST,3} = (0.1$ MHz $\times 2.83$ sec$) = 0.283$ (which is $> RW_{ST,3}^{min} = 0.275$) to relay 3 for secondary access.

Cross case analysis: Here, we have examined the impact of bandwidth sharing on PU_{Ch2}. Here, the total PU bandwidth of 0.1 MHz should be optimally allocated for both cooperative and secondary transmission. When Algorithm 1 provides channel information to Algorithm 3, PT identifies the maximum U_{PU} with relay 7 for $\gamma = 0.699$ and allocates $W_A = (\gamma W) = (0.69990 \times 1) = 0.0699$ MHz for cooperative communication. Concurrently, PT allows relay 7 to access the remaining bandwidth ($W_B = 0.1 - 0.0699) = 0.0301$ MHz for a duration of 10 sec, resulting the value of $RW_{ST,7} = (0.0301$ MHz $\times 10$ sec$) = 0.301$. While the generated $RW_{ST,7}$ satisfies $RW_{ST,7}^{min}$, it must also fulfill individual bandwidth and time constraints separately. However, in this case, the size of W_B (0.0301 MHz) allocated by PT does not meet the bandwidth constraint of relay 7, thus violating its reward constraint. Consequently, relay 7 declines to become the cooperative partner of PT. Even if PT selects the next suitable relay nodes, the same violation is observed. This outcome indicates that bandwidth sharing on PU_{Ch2} fails to satisfy the bandwidth constraints of the relays and is therefore not applicable to PU_{Ch2}.

Discussion: Thus, based on the analysis of both proposed allocation techniques, the conclusion can be drawn that the PU strategically maps its suitable channel to the appropriate allocation technique, depending on the requirements of the PU's target services and channel characteristics. This strategic mapping aims to achieve the maximum possible utility through cooperation with relay nodes. Consequently, the proposed techniques introduce flexibility in resource allocation for cooperative communication between PUs and SUs. However, in scenarios where PU channels exhibit similar resource characteristics, such as bandwidth ($W_1 \approx W_2$) and access time ($t_1 \approx t_2$), the PU has the option to choose either channel for the required strategy.

3.6.5 Delay analysis for the proposed resource sharing models

In a cooperative communication network, a data packet may experience variety of delays including transmission delay, propagation delay, processing delay, queuing delay [88], [32], [11] during its journey towards the intended destination. In case of our proposed relay based cooperative communication scheme, different types of delays are encountered when the PU data packet passes through the first hop (PT-to-relay transmission) as well as through the second hop (relay-to-PR transmission). However, we have considered the first three types of delays, viz., transmission delay (t_d), propagation delay (p_d) and processing delay (pr_d), while estimating the total delay exhibit in the proposed scheme as shown in Figure 3-13. Here we have ignored the queuing delay, as in CRN scenario it is basically caused due to waiting for an available PU channel by the SUs or opportunistic access of relay links by the PUs [60], [99] which is not an issue in our case. Therefore, the total delay incurred in the proposed cooperation schemes is calculated as: $total_d = \Big((t_{d1} + t_{d2}) + (p_{d1} + p_{d2}) + pr_d\Big)$.

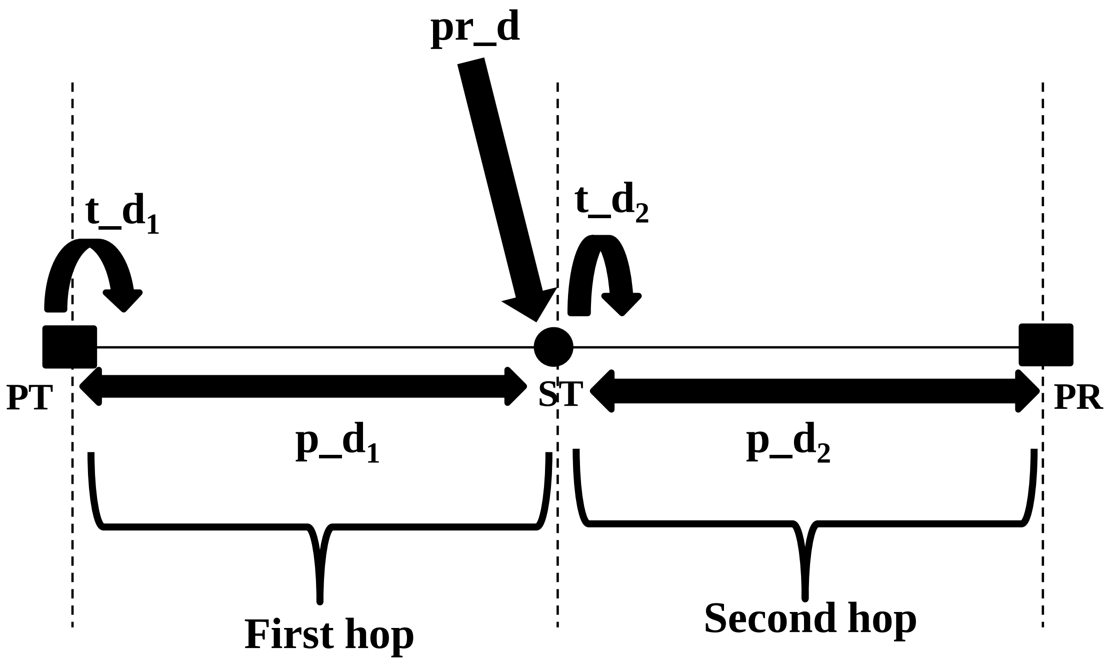


Figure 3-13: Different delays incur in the proposed scheme

t_d depends on the size of transmitted packet (size of PU data in our case) and the quality of transmission link (transmission rate of PU channel in our case). Further, p_d depends on the distance between the nodes in each hope (distance between PT-to-relay and relay-to-PR in our case) and the propagation speed of the transmitted medium (free space in our case). Since the considered distance among PT and relays during simulation is very less (in meter or m), so a negligible p_d is incurred in the entire communication session. On the other hand, different values of pr_d are observed when the relay nodes employ AF relaying (Case 2) and DF relaying (Case 1), as the relay node with former technology simply amplify the received source signal and forward it to the destination. However, the relay node with DF technology first decodes the source message, then re-encodes it and finally forwards towards the destination. Additionally in the proposed bandwidth sharing scheme the relay nodes are empowered with MIMO technology, so along with decoding the PU data, the relay needs to separate the PU data as well as secondary data that are received simultaneously over multiple antennas. So, comparatively large processing delay is incurred in DF relaying than that of the AF relaying technology.

3.6.5.1 Delay analysis in the bandwidth sharing model (for case 1)

Lets analyse the impact of above delays in the proposed bandwidth sharing scheme. In reference to Figure 3-3(a)), it is observed that the total time T is equally divided in two half for PT-to-relay and relay-to-PR transmissions. Due to the incurred delays, the allotted time $\frac{T}{2}$ for each hop gets reduced. This impacts the cooperative capacity (C_{PT}^{coop} in Eq. (3.1)) achieved by the PU as it is directly proportional to the total allotted time T. However, both the cost values of PT do not get affected by the incurred delays, as EN_{PT} (Eq. (3.5)) is independent of time T and the RW_{ST} is independent of cooperative communication. Therefore, the attainable utility of PT with associated delays (U_{PU_d}) is realized as follows:

$$\begin{aligned} U_{PU_d} &= \frac{profit}{cost} \\ &= \left(\frac{C_{PT}^{coop} \times \phi}{EN_{PT} \times RW_{ST}} \right) \\ &= \left(\frac{W_A \times (T - total_d) log_2(1 + SNR + SNR) \times \phi}{EN_{PT} \times RW_{ST}} \right) \end{aligned} \tag{3.17}$$

In earlier results it has been found that PT performs bandwidth sharing over PU_{ch1} by selecting relay 2 as its cooperative partner and obtains $U_{PU} = 24.5198$ (refer to Figure 3-9 and Figure 3-10). Now in the same case we integrate the associated delays and compare the achievable U_{PU_d} with U_{PU} obtained so far.

In this regard, the overall transmission rate of PU_{ch1} is assumed to be 100 Mbps and PT needs to transmit a data packet of size 1 Mb towards PR. So, the obtained t_d during the transmission PT-to-relay 2 as well as relay 2-to-PR is found to be 0.008 sec. Again, from the simulation, it is obtained that the $dist_{pt,relay2} = 200$ m and $dist_{relay2,PR} = 146$ m. So, the p_d in each hop is found to be 6.7×10^{-7} sec and 4.8×10^{-7} sec respectively. Further inspired by [30], the pr_d that incurs due to DF relaying in bandwidth sharing scheme is considered as 0.083224 sec. Therefore, the total delay incurs in the proposed bandwidth sharing model is: $total_d = (0.008 + 0.008) + (6.7 \times 10^{-7} + 4.8 \times 10^{-7}) + 0.0832 = 0.0992$ sec and the obtained $U_{PU_d} = 22.0904$ (Based on Eq. (3.17)). Since the associated delays affect only the profit factor of U_{PU_d}, so the obtained U_{PU_d} decreases 10% than the initial U_{PU}.

3.6.5.2 Delay analysis in the time sharing model (for case 2)

Secondly, in case of time sharing model (refer to Figure 3-3(b)), the appropriate allotted time T_A which is further divided equally for PT-to-relay and relay-to-PR transmissions, gets reduced due to the associated delays. Unlike in the bandwidth sharing scheme, this impacts the achievable profit (C_{PT}^{coop} in Eq. (3.6)) and incurred cost (EN_{PT} in Eq. (3.9)) of PT. However, another cost parameter, RW_{ST} does not get affected by the delays as it is independent of T_A. Therefore, the attainable U_{PU_d} in this case is realized as follows:

$$\begin{aligned} U_{PU_d} &= \frac{profit}{cost} \\ &= \left(\frac{DR_{coop} \times \phi}{EN_{PT} \times RW_{ST}} \right) \\ &= \left(\frac{W \times (T_A - total_d) log_2(1 + SNR + \frac{SNR}{SNR+SNR+1}) \times \phi}{\frac{T_A - total_d}{2T} \times RW_{ST}} \right) \end{aligned} \tag{3.18}$$

From Figure 3-11 and Figure 3-12, it has been found that PT performs time sharing with relay 3 over PU_{ch2} and obtains $U_{PU} = 17.6618$. To compare the U_{PU_d} with the obtained U_{PU}, it is assumed that the data transmission rate of PU_{ch2} is 1 Mbps and PT needs to transmit a data packet of size 5 kb towards PR. So, the obtained t_d during the transmission PT-to-relay 3 as well as relay 3-to-PR is found to be 0.04 sec. Again from the simulation setup, the $dist_{PT,relay3} = 187$ m and $dist_{relay3,PR} = 150$ m are observed, and thus the incurred p_d in each hop is found to be 6.2×10^{-7} sec and 5×10^{-7} sec respectively. However, in the amplify and forward technique, a very negligible pr_d is incurred to amplify and forward the received PU data towards PR. So, we have considered the pr_d in this case equals to 1 ms (0.001 sec). Therefore, total delay incurred in the proposed time sharing model is: $total_d = (0.04+0.04)+(6.2\times10^{-7}+5\times10^{-7})+0.001 =$ 0.0817 sec and the obtained $U_{PU_d} = 17.6063$ (Based on Eq. (3.18)). Since the associated delays affect the profit as well as one of the cost factors of PT, so a very little reduction (0.33%) in U_{PU_d} is observed as compared to initial U_{PU}.

3.6.6 Performance Comparison with existing methods

The performance analysis of the proposed bandwidth and time sharing schemes are compared with the following conventional schemes as listed below.

- *(i) Greedy approach:* Here, we implement a greedy approach, where PT greedily selects the relay node that offers highest P_{ST}^{max}; Without analyzing the constraint RW_{ST}^{min} and other parameters like λ, τ, $dist_{PT,ST}$, $dist_{ST,PR}$, which play major role in deciding the profit as well as cost of PT during the cooperation process.
- *(ii) Random selection approach:* Here, PT randomly selects any one relay node among the group of eligible relays, without analyzing the parameters associated with the relay node.
- *(iii) Non-cooperation approach:* Here, PT directly communicates to its PR without assist any of the relay node.

3.6.6.1 For Case 1: Bandwidth sharing

In Figure 3-14, the performance of the proposed bandwidth sharing scheme over PU_{ch1}, is compared with the above discussed schemes in terms of U_{PU} and incurred costs value. In this case, PT obtains maximum $U_{PU} = 24.59$ along with the cost values $EN_{PT} = 0.376$ and $RW_{ST} = 0.248$. However, according to approach (i), PT greedily selects relay node 6, as it offers highest $P_{ST}^{max} = 13.8$ mW. But it does not analyze the λ value offered by relay 6, which is found relatively large. This reduces the size of $RC_{ST,6}^{max}$, which affects cooperative resource allocation with relay 6. Further the RW_{ST}^{min} of relay 6 is also found significantly high that results large $RW_{ST,6}$ and increases the cost of PT. These all impact the allocation of W_A and reduces corresponding U_{PU} as shown in the graph. On the other hand in (ii), PT randomly selects relay node 5 as it's

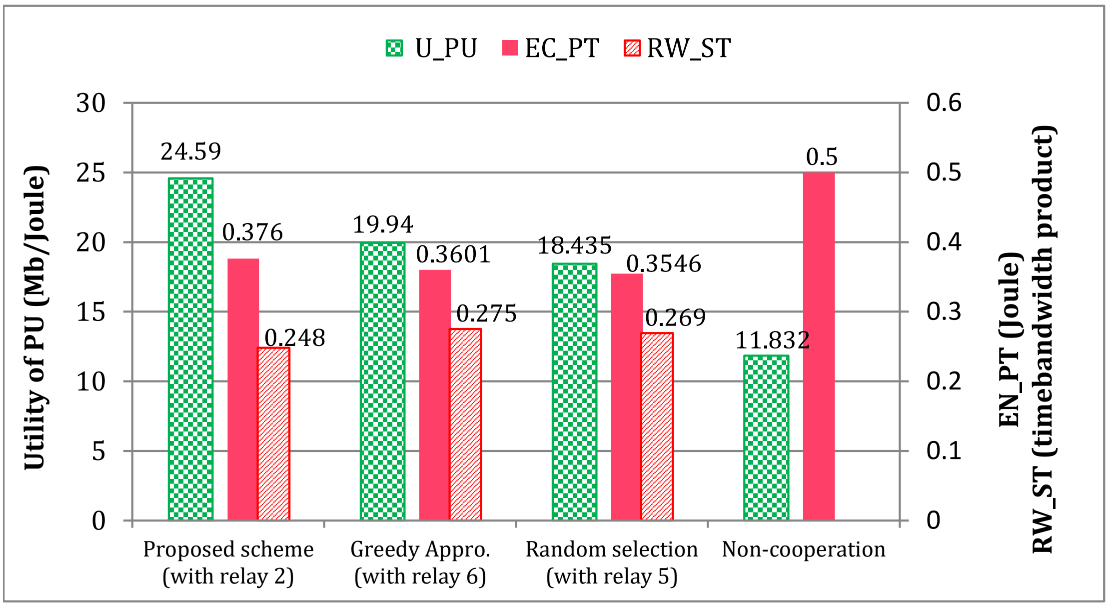


Figure 3-14: Performance comparison of the proposed scheme vs. conventional schemes

cooperative partner. But due to large $dist_{PT,ST}$, $dist_{ST,PR}$ and RW_{ST}^{min} associated with relay 5, PT cant able to gain high U_{PU}. Finally, through the non-cooperation approach PT directly communicates towards PR that incurs high EN_{PT}. This increases the cost value of PT and yields very less U_{PU} as shown in the Figure 3-14.

3.6.6.2 For Case 2: Time sharing

Similarly, in Figure 3-15, the performance of the proposed time sharing scheme over PU_{ch2}, is compared with the above discussed approaches in terms of U_{PU} and incurred costs value. In this case, PT obtains maximum $U_{PU} = 17.66$ with the cost values $EN_{PT} = 0.376$ and $RW_{ST} = 0.248$. However, in (i) PT selects relay node 4, as it offers highest $P_{ST}^{max} = 11.84$ mW. But it ignores the value of τ, $dist_{PT,ST}$, and RW_{ST}^{min}, which are found sufficiently large that increase the cost value of PT as well as lower down $U_{PU} = 19.94$. On the other hand, according to (ii), PT randomly selects relay node 1 as it's cooperative partner. But due to less P_{ST}^{max} offered by it, PT can not able to gain high U_{PU} with relay 1.

Discussion: Hence, through the comparative analysis, it is evident that both the suggested resource allocation methods outperform the considered approaches, ensuring the highest achievable U_{PU} while adhering to the specified resource con-

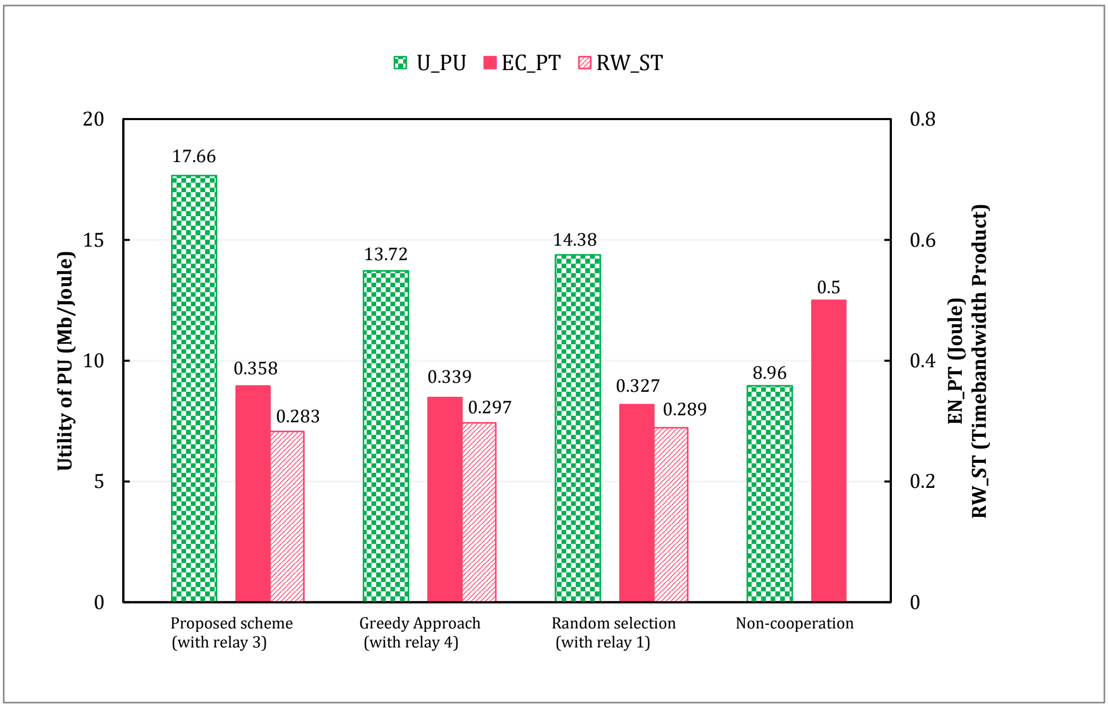


Figure 3-15: Performance comparison of the proposed scheme vs. conventional schemes

straints for the chosen relay node. In the case of the non-cooperation approach, where no relay node is involved, only one of the two cost values, EN_{PT}, is associated and is notably higher than in other approaches. Nevertheless, in both the proposed schemes, when compared to approaches (i) and (ii), PT is identified as less energy-efficient. This inefficiency stems from the allocation of a relatively larger resource size in the proposed bandwidth allocation ($W_A = 0.752$ MHz) and time allocation ($T_A = 7.17$ sec). This is because EN_{PT} is directly proportional to W_A or T_A (according to Eq. (3.5) or Eq. (3.9)).

3.7 Conclusion

In this chapter, a relay based cooperative resource sharing scheme is proposed and investigated in a single-PU multi-SU overlay CRN scenario. Here, we target to solve two problems: (i) how PU selects optimal relay node as its partner for cooperative communication, and (ii) how PU performs optimal resource allocation scheme (either bandwidth or time) during cooperative communication with the help of the selected relay node. We have formulated the CSS problem as a multi-objective optimization problem for maximization of PU utility as well as to satisfy the reward constraint of each selected relay node. Due to the computationally hard nature of the formulated optimization problem, a numerical analysis based heuristic solution is proposed that results in polynomial time sub-optimal solution of resource allocation for cooperative communication. The key idea of the proposed heuristic solution is to shrink the shareable resource size towards the optimal allocation point that maximizes the PU utility. The formal proof of convergence of the proposed algorithm is provided to demonstrate its efficacy. While comparing with the conventional approaches, the proposed scheme is found to be more efficient for PU in terms of utility maximization and delivers the minimum possible reward to the relay node. However, the proposed schemes is found to be less energy efficient than the greedy approach. Moreover, a case study is presented to assess the efficiency of the proposed resource allocation scheme, tailored to meet the quality of service (QoS) needs of the PU.

After working on development of a CSS scheme for single PU channel-multi SUs scenario, our next investigation aims to develop a CSS mechanism capable of achieving a balanced trade-off between optimal resource allocation and stable partner selection in a multi-PU channels, multi-SUs CRN scenario.