Chapter 4

A one-to-one mapping for multiple resource allocation CSS scheme in multi-PUs, multi-SUs overlay CRNs

4.1 Introduction

Cooperative Spectrum Sharing (CSS) within Cognitive Radio Networks (CRNs) presents a promising solution to address spectrum scarcity issues by fostering collaborative assistance among users. This involves sharing scarce spectrum resources between primary users (PUs) and secondary users (SUs) to achieve mutual benefits during cooperative communication. In the context of single-PU multi-SUs CRNs, CSS aims to enhance PU,s utility and meet SU's target transmission objectives while constraining the reward associated with relay SUs. In the preceding chapter, we devised a CSS scheme for the single-PU multi-SUs CRN scenario from PU's perspective, considering the trade-off between maximizing PU's utility and minimizing SU's rewards while efficiently allocating PU resources during cooperative communication. However, practical scenarios often involve multiple PUs (or PU channels) within the networks. The existence of multiple PUs opens up numerous opportunities for CSS, attracting multiple SUs eager to form pairs with suitable PUs for CSS within the network. In such contexts, modelling a stable framework for cooperative communication between PUs and SUs that meets each other's resource constraints proves challenging. This challenge stems from the necessity to address the selection (or assignment) of appropriate cooperating partners (PU, SU pairs) and allocate optimal spectrum resources among the chosen partners to maximize the effectiveness of both the primary and secondary networks. Further, the challenge revolves around two critical trade-offs: (i) maximizing PUs utility, and (ii) maximizing SUs utility while adhering to the constraints and penalties imposed by the PUs. In the context of the trade-off involving optimizing PU utility, each PU strongly desires to be paired with the most suitable SU or relay node as a cooperative partner. This desire is rooted in the goal of enabling the maximum possible cooperative benefit and utility for the PU. Conversely, in the trade-off related to SU utility maximization, each SU strives to be paired with the most lucrative PU, aiming to maximize individual utility. However, it is crucial to simultaneously minimize the penalty imposed on the SU. Many works in the literature [44, 85, 97, 132] have addressed the trade-offs either the maximization of PU utility or maximization of SU utility in partner assignment and cooperative communication during CSS. These efforts overlook the consideration of penalty constraints during the design of CSS and the utility gain for SUs. This consideration becomes crucial when aiming to design an efficient partner assignment and cooperative communication scheme from SU's perspective in a multi-PUs, multi-SUs CRNs.

In this chapter, we propose a partner assignment and cooperative communication scheme for multi-PUs, multi-SUs CRNs, where each PU chooses most suitable SU as its cooperative partner to implement one-to-one CSS. It is worth mentioning that PUs collaborate with SUs only if the benefits of cooperation exceed those of non-cooperation, regardless of the advantages to the secondary network. Meanwhile, SUs aim to improve their utility without affecting the transmission goals of the PUs. Further, we formulate an optimization problem for the optimal allocation of PU's transmission time among selected PU-SU pairs, facilitating both cooperative and secondary transmissions. Recognizing the NP-hard nature of this problem, we propose a heuristic solution based on numerical analysis method to achieve sub-optimal resource allocations among PU-SU pairs within polynomial time. Inspired by matching theory, we identify and maintain stable PU-SU partners, ensuring each PU is matched with the most suitable SU for cooperative communication. This strategy establishes a stable one-to-one (O2O) matching framework, resulting in optimal utility for PUs and stable utility for SUs. We provide theoretical proofs to substantiate the stability and optimality of the proposed approach.

To evaluate the effectiveness of our scheme, first we compare the performance of the proposed heuristic solution with the analytical (benchmark) method and analyze its effectiveness. Secondly, we conduct a simulation-based study using the performance metrics such as avg. utility for PUs, avg. utility of SUs, and avg. satisfaction of SUs for varying number of PUs and SUs engaged in cooperative communication. The results are compared with similar schemes from existing literature to demonstrate the efficacy of the proposed scheme.

The rest of this chapter is organized as follows: Section 4.2 defines the problem, outlines the assumptions, and introduces the symbols and notations used. The system model and optimization problem formulation are discussed in Section 4.3. The proposed one-to-one matching scheme is presented in Section 4.4. Section 4.5 covers the simulation results and performance analysis. Finally, Section 4.6 concludes the chapter.

4.2 Problem Statement

The problem is to develop one-to-one cooperative spectrum sharing (CSS) schemes in a multi-PUs, multi-SUs overlay CRN scenario for allocation of optimal fractions of PU access time among the cooperative PU-SU pairs during cooperative and secondary communication. By formulating the CSS problem as a multi-objective optimization problem, the proposed scheme aims to balance trade-offs among three key objectives: optimal allocation of PU's transmission time, utility enhancement for both PUs and SUs by reduction of penalty charges on SUs. Through the incorporation of stable matching concepts, the proposed scheme successfully establishes optimal matchings for the set of PUs and stable matchings for the set of SUs while aligning them with suitable PU-SU pairs.

4.2.1 Assumptions

- SUs use time division sharing model based on TDMA for CSS over PU band.
- All PUs and SUs are equipped with a single antenna and work in half-duplex mode.
- In terms of matching theory, an open market model is considered, where transmission power of PUs and SUs, distance between PUs and SUs, targeted transmission constraint of PUs and reward constraint of SUs are known.

- The locations of SUs and PUs are fixed in the network; that is, SUs and PUs are stationary during the partner assignment phase.
- All SUs in the network are non-malicious and the resource information provided by the SUs is trustworthy.
- The noise environment is considered to be zero mean Additive White Gaussian Noise (AWGN), and channel gain between two nodes encompasses solely the distance and path loss components [100], [97].
- Necessary control information exchange between PU and SU takes place through a dedicated common control channel [80] These control information focuses on the operational aspects of communication to ensure efficient channel usage.

4.2.2 Notations and Symbols Used

To remind the symbols and notations used particularly in this chapter, the same are summarized in Table 4.1.

Symbols/Notations	**Definitions**
$\mathcal{M}$	Set of PUs
$\mathcal{N}$	Set of SUs
M	Number of PUs
N	Number of SUs
W	Total bandwidth of PU channel
T	Total access time of PU band
α, β	Time allocation factors
a_1	Duration of time invested by PT during cooperative communication
b_1	Duration of time invested by ST during cooperative communication
c_1	Duration of time invested by ST during secondary communication
P_{PT}	Transmission power of PU
P_{ST}	Transmission power of SU
U_{PU}	Utility achieved by PU
U_{SU}	Utility achieved by SU

C_{PT}^{coop}	Cooperative gain achieved by PU during cooperation with ST
$SNR_{PT,ST}$	SNR received at ST from PT
$SNR_{ST,PR}$	SNR received at PR from ST
$SNR_{PT,PR}$	SNR received at PR from PT
EN_{ST}	Energy consumption of SU
ER_{ST}	Expensive rate of SU
C_{ST}	Total capacity achieved by SU during secondary communication
c_1^{target}	Reward constraint of SU for relaying PU service
β_{PU}^{target}	Min. required β time to attain transmission constraint of PU
$P_n(\alpha, \beta, \xi)$	Penalty function set by PU for SU
N_0	Noise Power
$\sigma_{N_0}^2$	Noise variance
δ_{ST}	Amplifying factor at ST
$d_{PT,PR}$	Euclidean Distance between PT and PR (in m)
$d_{PT,ST}$	Euclidean Distance between PT and ST (in m)
$d_{ST,PR}$	Euclidean Distance between ST and PR (in m)
$d_{ST,SR}$	Euclidean Distance between ST and SR (in m)
ω	Negligible value ≈ 0
SAT_{ST}	Satisfaction of SUs

Table 4.1: Notations and Symbols used

4.3 System Model

We consider a Cognitive Radio Network (CRN) framework consisting of a set of M primary user (PU) transceiver pairs, denoted as $\mathcal{M} = \{PT_i, PR_i\}_{i=1}^{M}$. In this setup, each primary transceiver (PT_i) has the intention of transmitting its data to a dedicated primary receiver (PR_i). However, due to the transmission range between PT and PR ($d_{PT,PR}$) extending beyond the effective communication range, an intermediary node, often referred to as a relay node, becomes necessary for forwarding the information from PT to PR with the overarching objective of achieving a minimum targeted reward requirement. In the same network, we consider a set of N secondary user (SU) transceiver pairs ($|M| < |N|$),

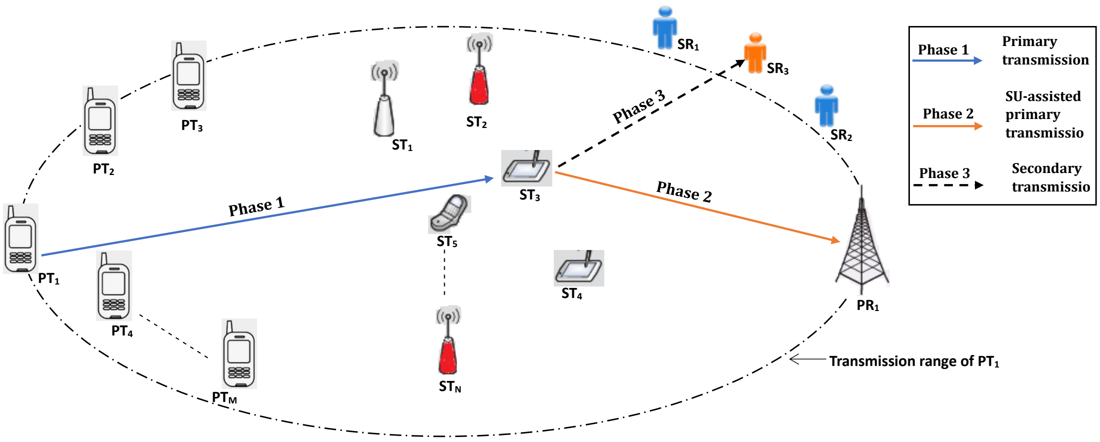

Figure 4-1: Proposed cooperative communication among selected PU-SU pair for considered CRN scenario

denoted as $\mathcal{N} = \{ST_j, SR_j\}_{j=1}^{N}$. In this context, the secondary transceivers (ST) play a crucial role as relays to assist PT in their transmissions. This assistance is provided in exchange for spectrum access opportunities for secondary communication towards secondary receivers (SR), all operating within the same spectrum band of the PUs using an overlay access paradigm. Note that PU prefers to cooperate with SU if and only if the cooperative capacity is found to be greater than the direct transmission by PU. Meanwhile, SU too accepts PU's offer only if SU can maximize its targeted data transmission rate. The entire cooperative communication among a selected PU, SU pair is depicted as shown in Figure 4-1. In the suggested scheme, each PU owns a licensed band consisting of F transmission frames, each with a duration of T time units and a bandwidth of W MHz. PUs utilize time division multiple access (TDMA), dividing each frame of duration T sec into three sub-slots (a_1, b_1, and c_1) based on decision variables α $(0 < \alpha \leq 0.5)$ and β $(0 < \beta < T)$ as as shown in Figure 4-2. SU-assisted cooperative communication occurs over the time duration $\beta{=}a_1{+}b_1$, where PT transmits data to ST during time a_1 (termed as Phase 1), and ST forwards the received primary data to PR during time b_1 (termed as Phase 2). The remaining time c_1 (termed as Phase 3), where $c_1 = (T - \beta)$ is allocated to ST for secondary transmission, compensating for relaying primary service. In our model, each ST utilizes the amplify-and-forward (AF) relaying technique to transmit primary data to PR, with transmission switching delay ignored [100, 124].

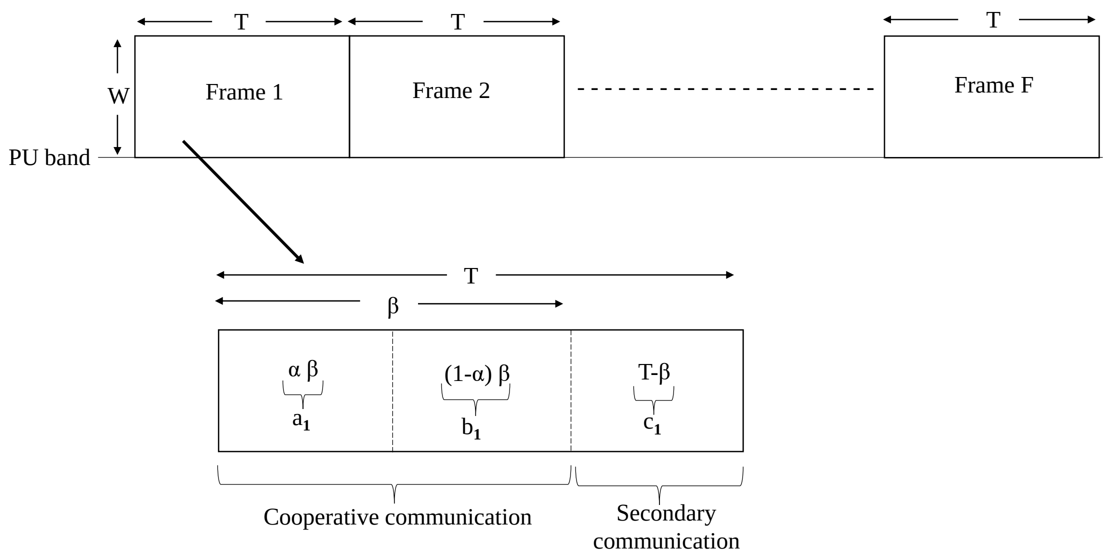

Figure 4-2: Time-slot division model of each frame for PU band

4.3.0.1 Phase 1

Let consider, X^I_{PT} is the signal transmitted by PT to ST in time duration a_1. Then, after Phase 1, the received signal at ST (denoted as $Y^I_{PT,ST}$) can be written as expressed in Eq. (4.1) [100], [114]:

$$Y^I_{PT,ST} = \sqrt{P_{PT}} h_{PT,ST} X^I_{PT} + N_{0,ST} \tag{4.1}$$

Where, P_{PT} is the transmission power of PT and it is assumed that PUs are able to adjust their transmission power level while transmitting their data. $h_{PT,ST}$ denotes the channel gain between PT and ST. $N_{0,ST}$ is the zero-mean AWGN at ST with noise variance $\sigma^2_{N_{0,ST}}$. At this point, the received SNR at ST can be written as expressed in Eq. (4.2):

$$SNR_{PT,ST} = \frac{P_{PT} \left|h_{PT,ST}\right|^2}{\sigma^2_{N_{0,ST}}} \tag{4.2}$$

4.3.0.2 Phase 2

In this phase, the ST relays the received signal from PT, i.e. $Y^I_{PT,ST}$ to PR in time duration b_1. The received signal at PR (denoted as $Y^{II}_{ST,PR}$), can be expressed as given in Eq. (4.3):

$$Y_{ST,PR}^{II} = \sqrt{P_{ST}} h_{ST,PR} \delta_{ST} Y_{PT,ST}^{I} + N_{0,PR} \tag{4.3}$$

Here, $h_{ST,PR}$ signifies the channel gain between ST and PR, δ_{ST} is the amplifying factor at ST and $N_{0,PR}$ is the zero-mean AWGN at PR with noise variance $\sigma^2_{N_{0,PR}}$. It is worth mentioning that δ_{ST} is computed as mentioned in [100] [70]. Similarly, the received SNR at PR, denoted as $SNR_{ST,PR}$ can be expressed as shown in Eq. (4.4):

$$SNR_{ST,PR} = \frac{P_{ST} \left| h_{ST,PR} \right|^2}{\sigma^2_{N_{0,PR}}} \tag{4.4}$$

At this point, we can formulate the cooperative capacity, denoted as C_{PT}^{coop} in megabits (Mb), achieved by PT in each frame through Amplify-and-Forward (AF) relaying over a total duration of $\beta = (a_1 + b_1)$ time units and utilizing a bandwidth of W MHz. This formulation is derived from the Shannon-Hartley channel capacity theorem [100], as illustrated in Eq.(4.5).

$$C_{PT}^{coop} = \left((a_1 + b_1) W log_2 \left(1 + SNR_{PT,PR} + \frac{SNR_{PT,ST} SNR_{ST,PR}}{SNR_{PT,ST} + SNR_{ST,PR} + 1} \right) \right) \tag{4.5}$$

Here, $SNR_{PT,PR}$ represents the signal-to-noise ratio received at PR from PT, which PR intercepts during Phase 1 due to the broadcast nature of wireless communications [100]. During Phase 1, the energy consumption of PT amounts to $(P_{PT} \times a_1)$ Joule. In this juncture, the utility of PT (in Mb/Joule), denoted as U_{PU}, achieved for each frame due to relay node-assisted transmission can be expressed in terms of maximizing the cooperative capacity (C_{PT}^{coop}) of PT while minimizing its energy consumption during Phase 1, as indicated in Eq.(4.6).

$$U_{PU} = \frac{C_{PT}^{coop} - C_{PT}^{direct}}{P_{PT} \times a_1} \tag{4.6}$$

Where, C_{PT}^{direct} is the capacity achieved by PT via direct transmission, and C_{PT}^{coop} must be $> C_{PT}^{direct}$ to make relay assisted transmission beneficial for the PUs.

On the other hand, let us assume that each ST has a power budget of P_{ST} Watt and it uses the same power during both Phase 2 and Phase 3. This assumption gives the assurance that during transmission, each ST treats the PU data the same way as its own data [85]. Therefore, the total energy consumption of SU (in Joule), denoted as EN_{ST}, is calculated as expressed in Eq. (4.7)

$$EN_{ST} = (P_{ST} \times b_1) + (P_{ST} \times c_1) \tag{4.7}$$

Where, $(P_{ST} \times b_1)$ represents the transmit energy invested by ST during Phase 2 for relaying PU data, and $(P_{ST} \times c_1)$ is the transmit energy utilized by ST during Phase 3 for secondary transmission. From Eq. (4.7), it is evident that if SU intends to reserve more energy for Phase 3, it should allocate less energy during Phase 2. This can be achieved by minimizing the value of b_1 for Phase 2.

4.3.0.3 Phase 3

Finally in Phase 3, SU is awarded with $c_1 = (T - \beta)$ time to access the PU band for secondary transmission towards SR as compensation for relaying primary service. At this juncture, the received signal-to-noise ratio (SNR) at SR is expressed in Eq. (4.8):

$$SNR_{ST,SR} = \frac{P_{ST} \left| h_{ST,SR} \right|^2}{\sigma^2_{N_0,SR}} \tag{4.8}$$

Therefore, the total capacity achieved by an SU, denoted as C_{ST}, for each PU frame over c_1 time and W MHz of bandwidth, as per Shannon's theorem, is expressed as in Eq.(4.9).

$$C_{ST} = \Big((c_1 \times W) log_2 (1 + SNR_{ST,SR}) \Big) \tag{4.9}$$

In this circumstance,

the utility of ST (in Mb/Joule), denoted as U_{SU}, can be formulated in terms of maximizing C_{ST} under the reasonable energy cost incurred by the ST during Phase 2 and 3 is as expressed in Eq.(4.10).

$$U_{SU} = \frac{C_{ST}}{EN_{ST} + ER_{ST}} \tag{4.10}$$

Where, ER_{ST} is the expensive rate of SU that decides based on the announced resource offers and negotiations among the PUs and SUs (discussed in following section). However, maximization of U_{PU} and U_{SU} completely depend on the values of a_1, b_1 and c_1, which are further decided by the optimal allocation of α and β values. Details of the optimal allocation of α and β variable is given in following section.

4.3.1 Optimal allocation of α and β

Based on the resource offers announced by PUs and SUs, each PU provides a chance to SUs for optimal allocation of α and β. Initially, each interested ST discloses its resource constraints: (i) P_{ST}, and (ii) c_1^{target} to PUs. Here, c_1^{target} is the minimum required compensation (in terms of access time to access PU's licensed band) asked by the ST to satisfy its targeted transmission rate. Before starting any negotiation with SUs, each PT broadcasts its β_{PU}^{target}, α_{PU} along with a penalty function $P_n(\alpha, \beta, \xi)$. Here, β_{PU}^{target} is the required β time to obtain the transmission objective targeted by PT, α_{PU} $(0 < \alpha_{PU} \leq 0.5]$ is the fraction of β time allotted for PT's transmission in Phase 1, and $P_n(\alpha, \beta, \xi)$ is the penalty function set by PT with the aim of protecting U_{PU}. Let analyze, how the penalty function operates to ensure that a guaranteed utility of PT is achieved, aiming to attain a greater C_{PT}^{coop} compared to C_{PT}^{direct}, while minimizing energy consumption as much as possible. From the formulation of PU's utility function (as given in Eq.(4.6)), it can be observed that U_{PU} increases with the increase of a_1 and b_1, but at the same time PT cannot afford a large value of a_1, as it simultaneously increases the cost factor (energy consumption) of PT. Therefore, PTs give more priority towards the increment of b_1 than a_1. On the other hand, from SU's utility function (Eq.(4.10)), it can be observed that SU's utility increases with the increase of c_1 and decreases with the increase of b_1. Since β appears as a decision variable for both c_1 and b_1, SU has a tendency to increase c_1 and decrease b_1 by: (i) reducing β up to a possible limit, say β^{new} (of course $\beta^{new} \geq \beta_{PU}^{target}$) and (ii) allocating maximum possible α, say α^{new} (of course $\alpha^{new} \leq 0.5$). Therefore, to restrict such self-awareness properties of SUs, PT models $P_n(\alpha, \beta, \xi)$ considering the following two points:

- Based on β^{new} decided by ST, the expensive rate of ST is calculated as, $ER_{ST} = \frac{1}{e^{u^v}}$, where $u = \beta^{target}$ and $v = (\beta^{new} - \beta_{PU}^{target})$. Here, the term *expensive rate* refers to the perceived costliness of a ST from the viewpoint of PUs. The ER_{ST} is crafted so that as SUs attempt to allocate a greater portion of β^{new}, their expensive rate simultaneously rises, categorizing them as more costly. Later on, based on the associated ER_{ST} value, PT sets priority among the SUs for transmission over its frames.
- Based on β^{new}, for per unit c_1^{extra} obtained by ST, PT reduces α_{PU} by ξ unit, and α^{new} is represented as in Eq.(4.11).

$$\begin{aligned}
\alpha^{new} &= \alpha_{PU} - (\xi c_1^{extra}) \\
&= \alpha_{PU} - \Big(\xi\big(c_1^{new} - c_1^{target}\big)\Big) \\
&= \alpha_{PU} - \Bigg(\xi\Big(\big(T - \beta^{new}\big) - c_1^{target}\Big)\Bigg)
\end{aligned} \tag{4.11}$$

Where, α^{new} is the newly generated time fraction for primary transmission set by the SU after reducing the β value up to β^{new}, with an intention to maximize c_1^{new}. But, in the race of maximizing c_1^{new}, SU needs to monitor the gradually reduced α^{new} as well as β^{new} values. As the former increases the size of b_1 and the later increases the size of ER_{ST}. Both the parameters directly affect the cost values of SU and reduce U_{SU}. Therefore, SU needs to decide appropriate value for β^{new} (denoted as β^*), so that acceptable ER_{ST} and α^{new} (denoted as α^*) can be obtained for cooperation, which satisfies the utility constraints of PU's as well as maximizes U_{SU}. In this context, the optimal allocation of α^{new} is formulated as given in Eq.(4.12).

$$\begin{aligned}
\alpha^{new} &= \alpha_{PU} - \Bigg(\xi\Big(\big(T - \beta^{new}\big) - c_1^{target}\Big)\Bigg) \\
\text{or,} \quad \alpha^* &= \alpha_{PU} - \Bigg(\xi\Big(\big(T - \beta^*\big) - c_1^{target}\Big)\Bigg)
\end{aligned} \tag{4.12}$$

Where, β^* is the optimal time allotted by ST for cooperative communication involving both Phase 1 and Phase 2. The optimization problem for β^* is modelled as given in Eq.(4.13).

$$
\begin{aligned}
\beta^* &= \arg\max_{\beta^{new}} \quad \big(U_{SU}\big) \\
&= \arg\max_{\beta^{new}} \quad \Big(\frac{C_{ST}}{EN_{ST} + ER_{ST}}\Big) \\
&= \arg\max_{\beta^{new}} \quad \left(\frac{(T-\beta^{new})Wlog_2\big(1+SNR_{ST,SR}\big)}{\Big(\Big(\big((1-\alpha^{new}_{PU})\beta^{new}\big) + (T-\beta^{new})\Big)P_{ST}\Big) + \frac{1}{e^{u^v}}}\right)
\end{aligned}
\tag{4.13}
$$

$$
\text{s.t.} \quad \beta^{target}_{PU} < \beta^{new} < \beta^{max}_{PU}
$$

Where, $\beta^{max}_{PU} = (T - c^{target}_1)$. For each β^{new}, the corresponding α^{new}_{PU} is easily found by substituting the value of β^{new} in Eq.(4.11). However, the optimization problem for β^* (Eq. (4.13)) exhibits a non-linear nature and it is well known that solving a nonlinear system is a NP-hard problem [28, 52]. In [42], Gaganov proved that nonlinear systems with polynomial equations having rational coefficients are NP-hard. In the proposed objective function, the relationship between the decision variables α and β is nonlinear, with α being multiplied by β, and their intervals fall within rational boundaries. Additionally, there is an exponential term in the denominator that is solely dependent on β. Drawing inspiration from the findings in [42] and [28], it can be asserted that the nonlinear equation (Eq. (4.13) characterized by mixed polynomial, rational coefficient, and exponential terms is a hard problem, which is intractable and difficult to solve in polynomial time. To address such problems, solution techniques like approximation algorithms and heuristic algorithms are widely used in the literature. Therefore, for solving the proposed nonlinear optimization problem, we propose a numerical analysis-based heuristic solution outlined in Algorithm 4. The primary objectives of Algorithm 4 are to determine the values of β^* within the range $(\beta^{target}_{PU}, \beta^{max}_{PU})$ and α^* within the range (0, 0.5]. The approach outlined in Algorithm 4 operates on the premise that as the search iterations for β^* progress, the range of β^{new} i.e. $(\beta^{target}_{PU}, \beta^{max}_{PU})$ gradually narrows towards the optimal point where PT achieves its maximum utility. Simultaneously, the value of α^* can be computed using Eq. (4.12). The search operation continues in Algorithm 4 until the difference between the two search values within the narrowed range approaches zero (or negligible i.e. ω), accompanied by the attainment of the maximum achievable U_{SU}. Experimental results presented in Section 4.5 demonstrate that the proposed Algorithm succeeds in achieving a closed-to-optimal solution (β^*), while compared with the benchmark solution achieved through the analytical method (β^*_{ana}).

Algorithm 4: Computation of α^* and β^*

Input : c_1^{target}, β_{PU}^{target}, α_{PU}, T, ξ, ω.

Output: α^*, β^* computed by each SU for respective PU offer.

1 **for** *each PU offer* **do**
2   calculate $\beta_{PU}^{max} = T - c_1^{target}$.
3   **for** *each SU* **do**
4   Calculate $\beta_m = \frac{\beta_{PU}^{target} + \beta_{PU}^{max}}{2}$, $\beta_{m-1} = \frac{\beta_{PU}^{target} + \beta_m}{2}$, $\beta_{m+1} = \frac{\beta_m + \beta_{PU}^{max}}{2}$. Calculates U for β_m, β_{m-1}, β_{m+1} points, where U is the function to calculate U_{SU} based on Eq. (4.10).
5   **if** *($U_{\beta_m} < U_{\beta_{m-1}}$) && ($U_{\beta_m} > U_{\beta_{m+1}}$)* **then**
6       $\beta_{PU}^{target} = \frac{\beta_{PU}^{target} + \beta_m}{2}$, $\beta_{PU}^{max} = \beta_m$.
7   **else if** *($U_{\beta_m} > U_{\beta_{m-1}}$) && ($U_{\beta_m} > U_{\beta_{m+1}}$)* **then**
8       $\beta_{PU}^{target} = \frac{\beta_{PU}^{target} + \beta_m}{2}$, $\beta_{PU}^{max} = \frac{\beta_m + \beta_{PU}^{max}}{2}$.
9   **else if** *($U_{\beta_m} > U_{\beta_{m-1}}$) && ($U_{\beta_m} < U_{\beta_{m+1}}$)* **then**
10       $\beta_{PU}^{target} = \beta_m$, $\beta_{PU}^{max} = \frac{\beta_m + \beta_{PU}^{max}}{2}$.
11   **end**
12   **Repeat** Step 4 and 5.
13   **if** *(abs($\beta_m - \beta_{m-1}$)) && (abs($\beta_m - \beta_{m+1}$))$\leq \omega$* **then**
14   Identify $U_{\beta_{m-1}}$, U_{β_m} and $U_{\beta_{m+1}}$.
15   Find Max($U_{\beta_{m-1}}, U_{\beta_m}, U_{\beta_{m+1}}$).
16   Find corresponding β value for Maximum Utility found in step 16.
17   **else**
18   **GO TO** Step 6 and **Repeat** till Step 14.
19   **end**
20   Calculates corresponding α for the β value obtained from Step 17 using Eq (4.12).
21   **end**
22 **end**
23 All SUs compute α^* and β^* for each PU offer.

Time Complexity of Algorithm 4 : To analyse the overall time complexity of Algorithm 4, we need to investigate the running time of the ***inner for loop*** (step 3) first. Let consider the difference between β_{PU}^{target} and β_{PU}^{max} be n, which is reduced by half at each iteration and runs until $\frac{n}{2} <= \omega$ (ω is assumed to be a negligible value and used to compare the difference among the points β_{m-1}, β_m and β_{m+1}). If total number of iterations used is k, we can write $\frac{n}{2^k} <= \omega$, which implies $k = O(log\frac{n}{\omega})$. Now, for total M number of PUs, the ***outer for loop*** runs M times and for N number of PUs, the ***inner for loop*** runs N times. Thus the overall running time of Algorithm 4 becomes $O(MNlog\frac{n}{\omega})$, which is a ***polynomial time*** complexity.

Proof of Continuity: To prove the continuity of the proposed objective function, Eq. (4.13) for the decision variable $\beta \in (0, 1)$ and $\alpha \in (0, 0.5]$, the concept of differentiability will be used. Since differentiability implies continuity, demonstrating that the objective function say $f(\alpha, \beta)$ is differentiable within the said intervals of β and α, will automatically establish its continuity in the same interval. Let's analyze the proof.

- Simplifying the constant terms from Eq. (4.13), the simplified form of the utility function in terms of $f(\beta, \alpha)$ is written as:

$$f(\beta, \alpha) = \frac{(C-\beta)log(2C)}{((C-\alpha)\beta + (C-\beta))C} + \frac{1}{e^{C(\beta-C)}} \tag{4.14}$$

- To check differentiability, we need to verify if the derivative of function $f(\beta, \alpha)$ exists for all $\beta \in$ (0,1) and $\alpha \in$ (0,5]. For this, lets first calculate $f'(\beta, \alpha)$ with respect to β as shown below:

$$f'_{\beta}(\beta, \alpha) = \frac{-logC(C^2 - C\beta + 2C)}{C((C-\alpha-1)\beta + C)^2} + - \ln C \frac{C^{\beta-C}}{e^{C(\beta-C)}} \tag{4.15}$$

Again calculate $f'(\beta, \alpha)$ with respect to α as shown below:

$$f'_{\alpha}(\beta, \alpha) = \frac{(C-\beta)\beta log(2C)}{(C-\alpha\beta)^2 C} \tag{4.16}$$

- To check the existence of the derivative, lets verify the following points :
  - In the first part of Eq. (4.15), the numerator terms $logC$ is constant and $(C^2 - C\beta + 2C)$ is polynomial and continuous in β for $\beta \in (0, 1)$. Further, in denominator, the term $C((C-\alpha-1)\beta + C)^2 \neq 0$ in α and β for $\alpha \in (0, 0.5]$ and $\beta \in (0, 1)$ with $C > 1$ and hence continuous.
  - In the second part of Eq. (4.15), the numerator terms $\ln C$ is constant and $C^{\beta-C}$ is continuous in β for $\beta \in (0, 1)$.
    Further, in denominator, the term $e^{C^{\beta-C}}$ is non-zero in β for $\beta \in (0, 1)$ with $C > 0$ and hence positive and continuous.
  - In the numerator of Eq. (4.16), the term $(C-\beta)\beta$ is continuous in β for $\beta \in (0, 1)$ and $log(2C)$ is constant (for $C > 0$), hence continuous.

In denominator, the term $(C - \alpha\beta)^2 \neq 0$ in α and β for $\alpha \in (0, 0.5]$ and $\beta \in (0, 1)$ with $C > 0.5$.

Since, $f(\beta, \alpha)$ is differentiable for all $\beta \in (0, 1)$ and $\alpha \in (0, 0.5]$, it is also continuous in this interval by the principle that differentiability implies continuity.

4.4 Proposed one-to-one mapping model among PUs and SUs

The objective of the proposed model is to identify the most appropriate PU-SU pair for the construction of one-to-one mapping, facilitating the establishment of cooperative communication within the set of PUs and SUs. In the process of forming the one-to-one mapping model, each SU endeavors to partner with its most preferred PU, with the intention of maximizing its own utility. Similarly, each PU strives to have the most lucrative SU as its cooperative partner to optimize its utility. This results in a mutually beneficial scenario, creating a win-win situation for both PUs and SUs. To analyse such mutually beneficial relationships between the users of two disjoint sets in the field of resource sharing among the competitive as well as cooperative users, matching theory is proven to be an effective framework [22, 43, 71].

4.4.1 Matching Theory

Cooperative partner selection and resource allocation for cooperative as well as for secondary communication among the selected partners become challenging due to heterogeneous characteristics and conflict of interests associated with PUs and SUs. Matching theory is found to be widely used in such scenario like stable partner (PU-SU pair) assignment and optimal resource allocation to analyse and handle the interactions among two disjoint sets of users with cooperative or competitive behaviours. Some of the relevant definitions of matching theory that need to be investigated to establish a mutually beneficial relationship among the selected PU-SU pairs are given below [44, 47, 71]:

- ***Definition 1*** *One-to-One matching: A One-to-One matching between sets M and N such that $i \in M$ and $j \in N$, can be represented by a One-to-One* *matching $\mu(.)$, where $\mu(i) = j$ (i is matched with j) if and only if $\mu(j) = i$ (j is also matched with i). Further, $\mu(i) = i$ and $\mu(j) = j$ indicate i and j stay single.*

- ***Definition 2*** *Stable matching: In a multi-PUs and multi-SUs scenario, the stable matching between the PU-SU pair (also termed as unblock PU-SU pair) can be established if both of them satisfy the following two properties :*
  - ***Property 1:*** *Any i and j of matching μ is willing to maintain the current partnership rather than stay single.*
  - ***Property 2:*** *Neither i nor j of matching μ can increase their individual utility further, via unilateral deviation (choosing a new partner by betraying current partnership).*

  Otherwise, the PU-SU pair is called as blocking pair that results in an unstable matching.

- ***Definition 3*** *Optimal matching: In a multi-PUs multi-SUs resource sharing scenario, it is possible to construct the optimal matching either from PU's or from SU's perspective. There always exists an optimal matching for each $i \in M$, where every i achieves the maximum possible utility (PU-optimal matching). Similarly, for each $j \in N$, there always exists an optimal matching (SU-optimal matching). But the point to be noted is that a matching or an equilibrium that is optimal for the users of one set will not be optimal for the users of opposite set. That means, users of two different sets cannot achieve optimal equilibrium at the same time.*

Drawing from the principles of matching theory, a matching game based solution strategy has been devised to establish a stable one-to-one assignment of cooperative PU-SU pairs for cooperative communication. In the development of this solution, we make the assumption that SUs act in a self-interested manner, seeking to pair with their most preferable PU. In this context, SUs employ the results derived from Algorithm 4 (specifically, β^* along with its corresponding α^* and U_{SU}) as the primary inputs for the matching game. Utilizing the obtained values of β, α, and U_{SU}, SUs create a Preference List (PL) of PUs with a length of M, arranging the achieved U_{SU} in descending order. The matching game is assumed to operate in a round-by-round fashion, up to M rounds. In each round, each SU approaches its most preferred PU with a tuple (α^*, β^*), intending to establish a cooperative communication partnership. Simultaneously, PUs select the most profitable SU, rejecting any previously accepted requests from earlier rounds. The step-by-step procedure for the one-to-one matching model is outlined in Algorithm 5.

Algorithm 5: Stable PU-SU pair formation for cooperative communication

---

Input : Provide (α^*, β^*) to each PU.
Output: Formation of stable PU-SU pair.
1 **Initialize:** Matching among the (PU, SU) is null, i.e. $\mu(i) \in M = \mu(j) \in N = \phi$
2 **Preference List (PL) creation by SUs:**
3 Based on (α^*, β^*), each SU computes U_{SU}.
4 Prepares PL for PUs with decreasing order of U_{SU} found so far.
5 According to PL, each SU offers corresponding (α^*, β^*) request, to its most preferred PU.
6 **One-to-One matching:**
7 **while** *(each PU of set M is not mapped with perfect SU)* **do**
8   PU calculates U_{PU} and corresponding ER_{SU} based on received (α^*, β^*) values.
9   PU accepts the request of SU with highest achievable U_{PU} and rejects the rest.
10   For two similar U_{PU} values, PU accepts the request with minimum ER_{SU} value.
11   **if** *(SU is rejected by PU)* **then**
12   **Repeat (until all entries in PL are processed)**
13   SU updates its PL by substituting the next preferred PU as its current preference and offers corresponding (α^*, β^*) request to it.
14   PU updates its current holding request with the new one if and only if:
15         $(U_{PU_{new}} > U_{PU_{hold}})$ or
16         $(U_{PU_{new}} = U_{PU_{hold}}$ && $ER_{SU_{new}} < ER_{SU_{hold}})$
17   Otherwise, reject the new request.
18   **else**
19   PU_i and SU_j announce as stable partners for cooperation.
20   **end**
21 **end**

Time Complexity of Algorithm 5: To analyse the overall time complexity of proposed algorithm, we need to investigate the running time of ***while loop*** (Step 7) along with its inner ***conditional IF statement*** (Step 11). Lets analyse the worst case scenario of the algorithm along with its worst case time complexity, where PU_i receives requests from all the N SUs at $Round_1$. Out of these N requests, PU_i selects the SU offer for which maximum U_{PU_i} can be achievable and rejects the others. In $Round_2$, assume the remaining $(N-1)$ SUs send respective request to PU_{i+1} and after selecting the most profitable one, PU_{i+1} rejects the remaining $(N-2)$ requests. This process continues for each entry in PL of SUs i.e. up to M preferences (where M is total number of PUs). Therefore, the worst case running time complexity of the ***while loop*** along with the ***IF statement*** is $= N + (N-1) + (N-2) + \dots\dots\dots + (N-M) = O(N \times M)$, which is a ***polynomial time*** complexity.

4.4.1.1 Stability of the matching game model

Theorem 1: Algorithm 5 converges to a stable matching, even if the SU pairs with its least preferred PU.

Proof: In each round of the matching game, each SU strives to be paired with its most preferred PU. If a SU faces rejection from its current preferred PU, it proceeds to send requests to the next preferred PU, and this process continues until the SU receives a positive response from a requested PU. Let's consider the worst-case scenario, where SU_j has the opportunity to form a pair with its least preferred PU_i and attains a utility of $U_{SU_j}^{least_i}$. In this state, SU_j has no further PU options to explore for maximizing the achieved utility. Consequently, SU_j decides to be paired with PU_i instead of remaining unpaired, thereby satisfying Property 1 and 2 for SU_j.

However, in the assignment process, a PU only abandons its currently mapped SU and accepts a new one if the latter provides a higher U_{PU} than the former. This implies that if PU_i accepts the request of SU_j, it must offer the highest U_{PU_i} among all the previous requests it has received so far. If this condition is met, PU_i willingly accepts SU_j as its cooperative partner, thereby satisfying Property 1 and 2 for PU_i. Consequently, no blocking pairs emerge in any iterative step of the algorithm, demonstrating that the final assignment or matching of PU-SU pairs is stable.

Theorem 2: The outcome of the one-to-one matching obtained from Algorithm 5 converges to an optimal matching for the set of PUs.

Proof: In each round of Algorithm 5, PU_i may receive multiple requests or offers from various SUs. Evaluating these requests, PU_i calculates the corresponding U_{PU_i} and selects the SU that offers the maximum U_{PU_i}. However, when new offers are received in subsequent rounds, PU_i only accepts them if U_{PU}^{new} is greater than $U_{PU}^{current}$. Otherwise, it rejects the new request. This implies that if PU_i accepts an offer in $Round_r$ with utility $U_{PU_i}^{Round_r}$, it must be the highest utility obtained by PU_i so far. This underscores that PU_i consistently prefers and accepts the SU offer with the maximum profit, irrespective of the round. Hence this proves that the proposed algorithm converges to an optimal matching for each PU.

Summary: Analysis of the proposed algorithm (Algorithm 5) reveals that under the best-case scenario, where both PU and SU have the opportunity to pair with their top preferences, the proposed solution converges to an optimal match for both PUs and SUs. However, in the worst-case situation, a SU may opt to accept the offer from its least preferred PU, achieving a specific utility (say U_{SU}^{least}) that satisfies the SU's target constraint but cannot be further improved. In this circumstance, instead of remaining unpaired, the SU accepts the offer and establishes a stable match with its least preferred PU. Consequently, in conclusion, the proposed one-to-one partner selection algorithm is found to be optimal for the set of PUs and stable for SUs.

4.5 Simulations results and performance analysis

The performance of the proposed schemes, namely the optimization scheme (Algorithm 4) and the one-to-one (O2O) matching scheme (Algorithm 5), is evaluated through simulations conducted in a MATLAB environment. We consider a cognitive radio network (CRN) comprising M primary users (PUs) and N secondary users (SUs), where M is less than N ($M < N$). The PUs and SUs are randomly distributed within a square area measuring 1000×1000 m^2. The distances between PU transceiver pairs and SU transceiver pairs are set to approximately $800m$ and $500m$, respectively. Other simulation parameters and their values used to perform simulation are shown in TABLE 4.2. These meticulously chosen parameters and settings form the foundation for our comprehensive evaluation of the proposed solutions within the context of the CR network under investigation.

Parameters	Values
M	5 to 15
N	5 to 30
F	2 to 4 (variable)
T	10 sec
W	1 MHz
P_{PT}, P_{ST}	[0.02 to 0.05] Watt
β_1^{target}	4 to 5 sec (variable)
c_1^{target}	2.5 to 3.5 sec (variable)
$d_{PT,PR}$	600 to 800 m (variable)
$d_{ST,SR}$	300 to 500 m (variable)
$\sigma^2_{N_{0,ST}}$, $\sigma^2_{N_{0,PR}}$, $\sigma^2_{N_{0,SR}}$	10^{-10} Watt
path loss exponent	2

Table 4.2: Simulation parameters and their values

4.5.1 Performance Metrics

Following metrics have been used for simulation based performance analysis.

- Average β^*: Optimal β time (in sec) obtained by N SUs for cooperative communication, based on Eq. 4.13.
- Average utility of PUs (Avg. U_{PU}): Utility achieved by M number of PUs that computed as given in Eq. 4.6.
- Average utility of SUs (Avg. U_{SU}): Utility achieved by N number of SUs that computed as given in Eq. 4.10.
- Average satisfaction of SUs (Avg. SAT_{ST}): The satisfaction level obtained by N number of SUs that computed as given in Eq. 4.17.

4.5.2 Performance analysis of proposed optimization scheme

The simulation results of the proposed optimization scheme (Algorithm 4) are compared with the following schemes as listed below.

- *(i) Analytical approach:* The benchmark result (in terms of finding optimal value) is obtained by identifying the critical point (local maximum β or β^*_{ana}) that provides the maximum possible value for the proposed objective function (Eq. (4.13)). The critical point is determined using standard techniques for solving *maximum/minimum* problems, involving the calculation of the derivative of the objective function and performing first or second derivative tests for specific intervals [112].
- *(ii) Greedy based approach:* To implement the greedy-based approach, we adopt a time slot model similar to the one described in [40], where the cooperative time is equally divided between the *PU* and the relay node. Subsequently, the relay is rewarded with a dedicated time period to access the PU band. Initially, each PU broadcasts offers consisting of the total cooperative time period and compensation for the relay in the network. Upon receiving these offers, SUs greedily select the PU offer that provides them with the maximum possible reward, without analyzing other factors. SUs then respond to the PU with two parameters: the corresponding distance

from the PU and the relaying power budget. Based on the replies from SUs, the PU calculates the corresponding profit and cost values associated with each SU and selects the SU that yields the maximum profit.

- *(iii) Direct Transmission:* In this scenario, the primary transmitter PT directly communicates with its PR without any assistance from relay nodes.

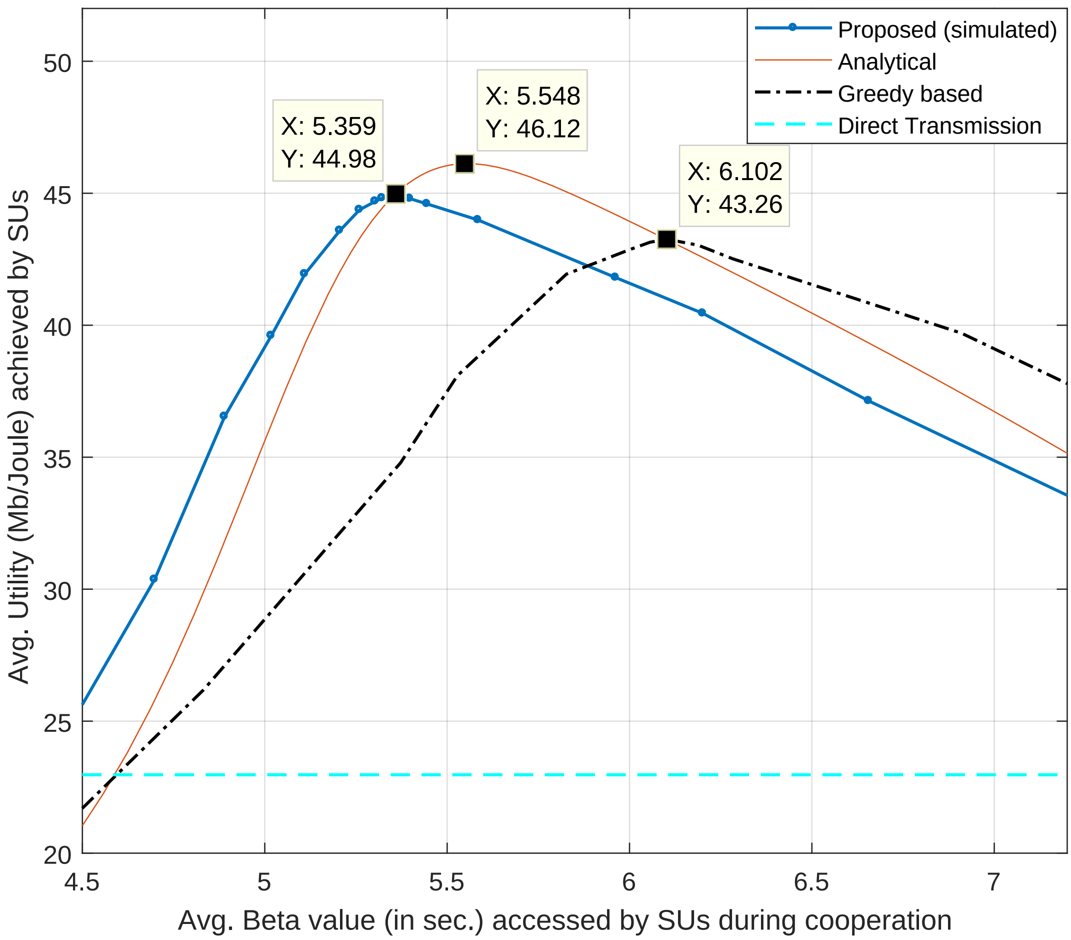

Figure 4-3: Performance comparison analysis of avg. SUs utility vs. avg. β^*

Figure 4-3 depicts the graph of average U_{SU} versus average β^* obtained from Algorithm 4 (shown in blue) for a cognitive radio (CR) network with $N = 30$ SUs and $M = 10$ PUs. The graph illustrates that the proposed solution achieves an average β^* of 5.359 sec for the specified CR network, achieving 96% accuracy compared to the benchmark result ($\beta^*_{ana} = 5.548$ sec).

Additionally, regarding the utility of SUs, the proposed solution attains an average U_{SU} of 44.98, which is 97% accurate when compared to the benchmark utility ($U_{SU_{ana}} = 46.12$) obtained through the analytical method. However, it is observed that the performance of the greedy approach slightly deteriorates compared to the proposed one. This is because, in the greedy approach, when SUs select PU's offers for cooperative communication, some of the PU's offers may highly satisfy SU constraints while others may not, resulting in a reduced average $U_{SU_{grdy}}$ of 43.26. In conclusion, the direct transmission approach exhibits the poorest performance compared to all the other considered approaches.

4.5.3 Performance analysis of average utility of PUs and SUs

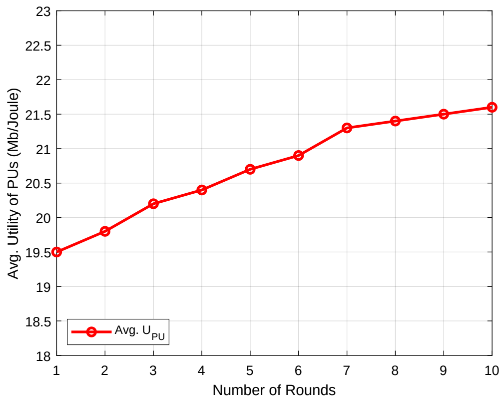

Figure 4-4: Avg. utility of PUs

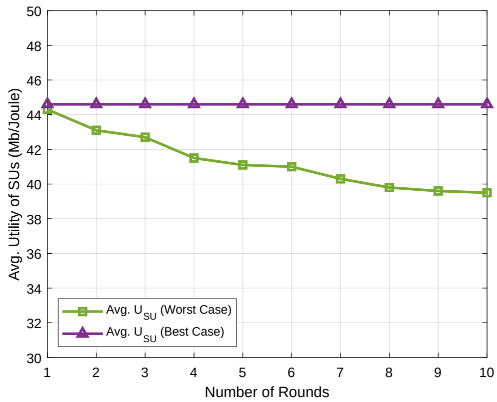

Figure 4-5: Avg. utility of SUs

The investigation of the average utility for both primary users (U_{PU}) and secondary users (U_{SU}) under the proposed one-to-one matching scheme is conducted for a cognitive radio network scenario with 10 PUs ($M = 10$) and varying numbers of SUs ($N = 10, 20, 30$) involved in different round. It is noted that, although PUs and SUs collaborate in our proposed framework, their utilities are independent, as each follows distinct utility functions based on their quality-of-service requirements and transmission objectives.

Figure 4-4 illustrates the average U_{PU} for the network under consideration and explores how it varies with the increasing number of SUs during the assignment rounds. As the rounds progress, PUs receive requests from new SUs and either accept the request that offers the highest utility or reject it. Consequently, the average U_{PU} either gradually increases or reaches saturation, as shown in graph (red colour line).

On the other hand, the graphs representing the average U_{SU} in Figure 4-5 exhibit opposite trends compared to the previous case. In the best-case scenario, SUs are paired with their top preferred PUs, resulting in maximum possible average U_{SU} as shown by the purple color graph. However, in the worst-case scenario, when more SUs are involved in the assignment rounds, SUs may end up paired with their least preferred PUs due to repeated rejections from higher-ranked options, or may remain unmatched, resulting in an U_{SU} of 0. This leads to a decrease in the average U_{SU}, as indicated by the green line in the graph.

4.5.4 Performance analysis of average utility of SUs for varying network size

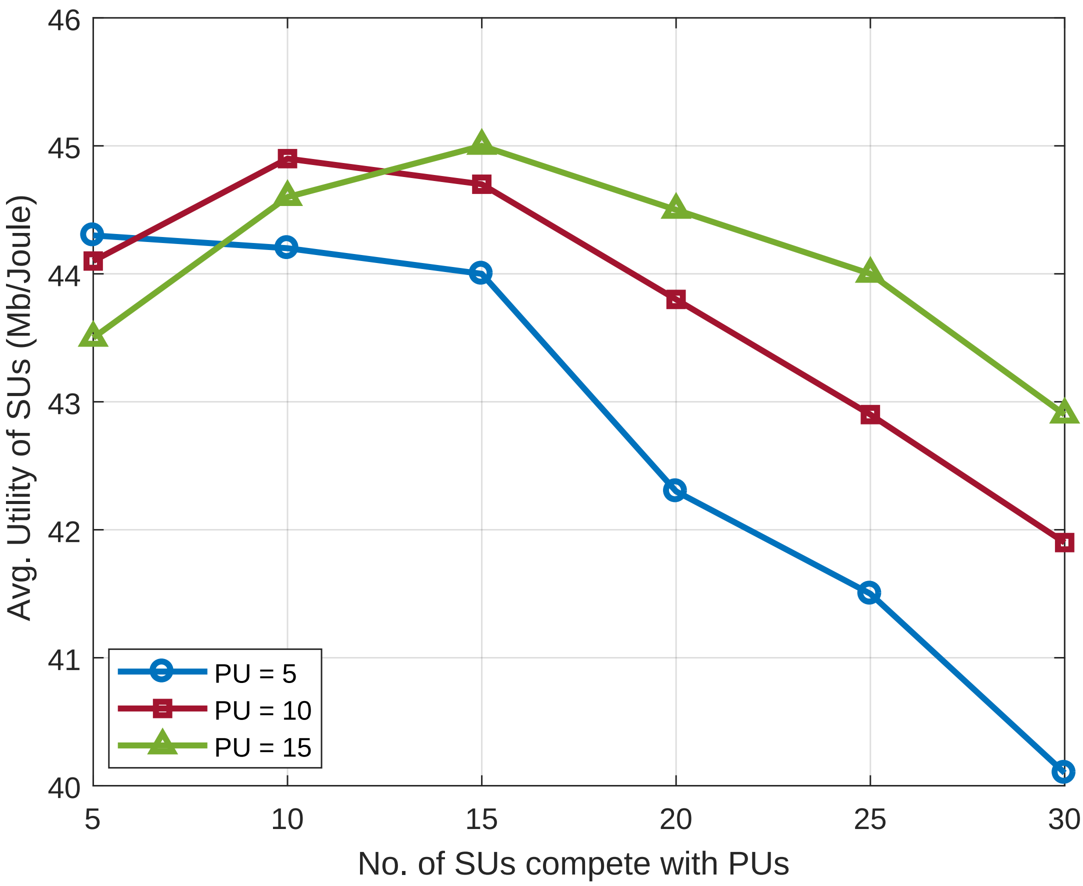

Figure 4-6: Avg. utility of SUs for different number of SUs and PUs

Figure 4-6 illustrates the graphs of average U_{SU} for a network with an increasing number of SUs, where N ranging from 5 to 30, while maintaining a fixed number of PUs i.e. M at 5, 10, and 15, respectively. As the number of SUs increases during the assignment process, there is a heightened competition among them to be paired with favorable PUs, particularly evident when M is smaller. Consequently, U_{SU} begins to decline as PUs have more options to choose from among SUs.

In the graph depicting $M = 5$ PUs, the U_{SU} reaches its maximum when only 5 SUs are involved in the assignment process. Moreover, as the number of

SUs increases, PUs have the opportunity to negotiate with more SUs and select the most profitable ones by rejecting others. Therefore, in the worst-case scenario, SUs may either accept offers from PUs even if they provide low utility (of course it should > targeted transmission constraint of SU) or choose to exit the assignment process. This leads to a gradual decline in the graph of U_{SU} for $M = 5$, as depicted in blue line graph. However, the remaining two graphs for $M = 10$ and $M = 15$ initially show an increase in U_{SU} when the number of involved SUs is small (i.e., $N = 11$ and $N = 17$, respectively); thereafter, U_{SU} begins to deteriorate with the increasing number of SUs in the assignment process.

4.5.5 Performance analysis of avg. satisfaction of SUs for varying network size

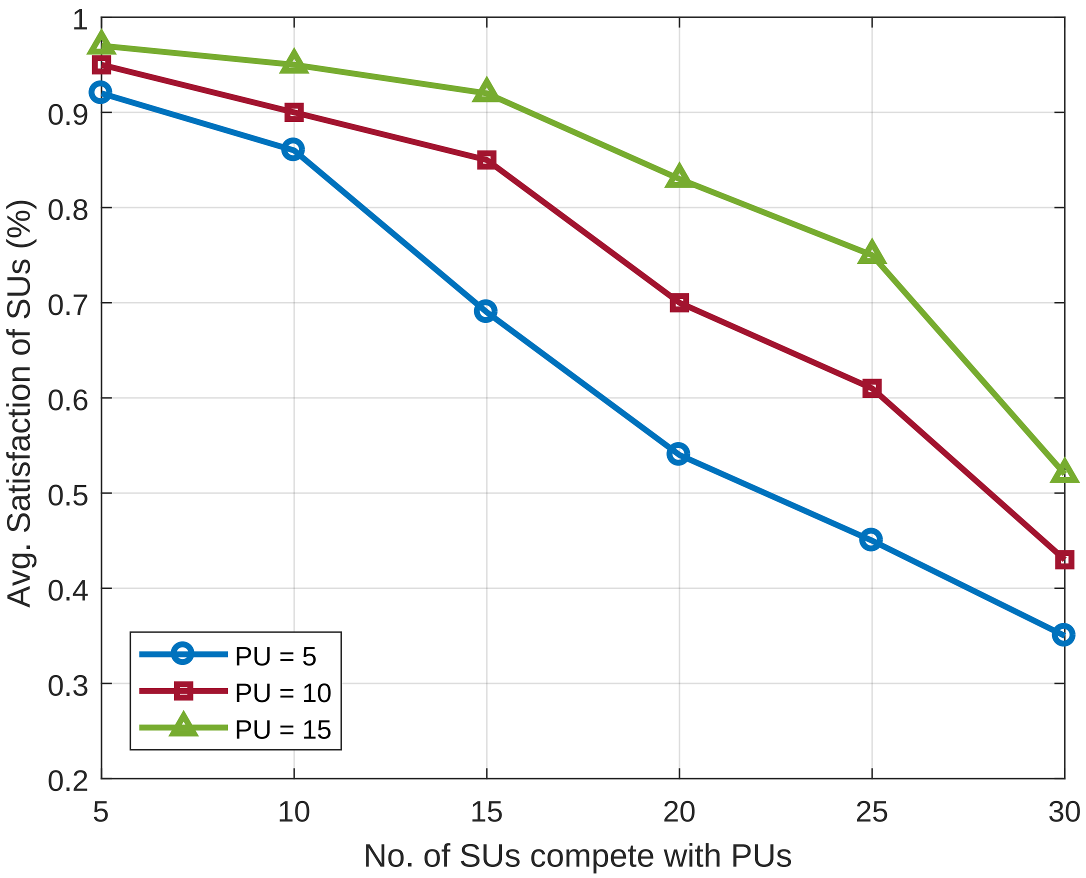

Figure 4-7: Avg. satisfaction of SUs for different number of SUs and PUs

Figure 4-7 illustrates the average satisfaction level of SUs with increasing the number of SUs participating in the assignment process. Inspired by [94], the satisfaction of a SUs (SAT_{ST}) for an assigned PU-SU pair can be determined based on the position of the assigned PU in the preference list of the SU, which is expressed as: $SAT_{SU} = \frac{(M+1)-p}{M}$. Here, M represents the total number of PUs, and p denotes the position of the assigned PU in the preference list of the SU. Similarly, the avg. SAT_{ST} for a one-to-one mapping model with M primary users and N secondary users is formulated as:

$$SAT_{ST} = \frac{\sum_{j=1}^{N}(M+1) - p_j}{M \times N} \tag{4.17}$$

In Figure 4-7, the graphs depict the avg. SAT_{ST} for a CRN scenario, as considered in the previous case (Figure 4-6). When the number of PUs in the assignment process is 5, along with 5 SUs, the avg. SAT_{ST} reaches approximately 95% satisfaction. However, as the number of SUs increases from 5 to 30 in the assignment process, the pool of options for PUs to select the most suitable SUs also expands. Consequently, SUs are restricted from forming preferred pairs with PUs, leading to a drastic reduction in their satisfaction level, as indicated by the graph (blue line).

Similar trends in the graphs of avg. SAT_{ST} are observed for two additional cases where the number of PUs in the assignment process is increased to 10 and 15, respectively. However, due to the involvement of a greater number of PUs, the satisfaction level of SUs has improved in both cases, reaching up to 20% (red line) and 48% (green line) satisfaction, respectively, when compared to the earlier case with 5 PUs.

4.5.6 Performance analysis of proposed approach vs. conventional approaches

To validate the performance of the proposed scheme, the conventional random selection scheme and an existing scheme as mentioned in [118] are considered and compared the utility of SUs as shown in Figure 4-8.

To perform the comparison analysis, a CR network with $N = 20$ SUs and some fix number of PUs ($M = 5$, $M = 10$ and $M = 15$) is considered. The findings illustrated in Figure 4-8 show that the utility of SUs increases progressively as the number of PUs in the network grows. In the proposed model, a greater exchange of resource offers occurs among PUs and SUs when more PUs are participating in the network. This scenario allows SUs to negotiate more effectively for favorable PU offers and optimally allocate α and β values accordingly, leading to comparatively higher utility values for SUs (depicted by the blue bar graphs).

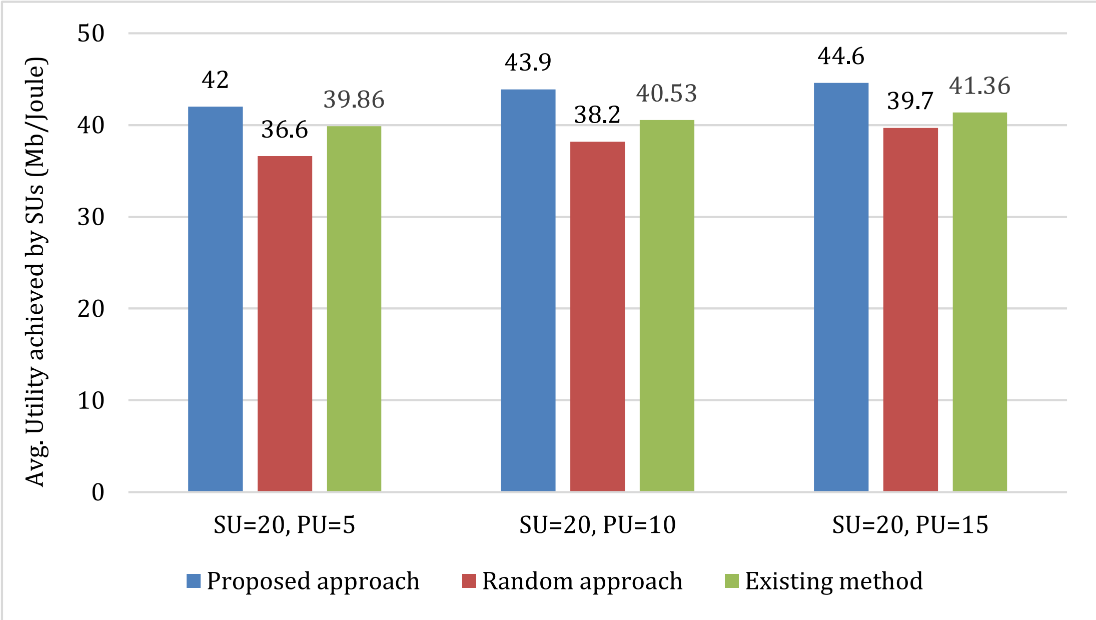

Figure 4-8: Performance comparison of proposed approach with existing and random approaches

Further, in the existing work [118], the partner selection and access time optimization model was developed from the perspective of PUs. In this model, PUs solely optimized the access time for cooperative and secondary communication, while SUs must determine their relaying power levels, from which PUs select SUs for cooperation. As the number of PUs in the network increases, more SUs got selected for cooperation, enhancing the utility for both PUs and SUs. However, the rate of enhancement in SU utility was limited than the proposed method due to the lack of SU involvement in the optimization process, and the preferences of SUs concerning PUs are not considered in this analysis (depicted in green bar graphs). Finally, in the case of the random selection approach, the choice of cooperative partners is arbitrary. Consequently, there is no guarantee that all the selected PU-SU pairs are suitable for cooperative communication. This leads to a decrease in the utility values of SUs compared to those in the proposed model, as illustrated by the graph (depicted by the red bar graphs).

4.6 Conclusion

In this chapter, we addressed two key objectives of cooperative spectrum sharing (CSS): modeling cooperative partner selection and allocating resources among the chosen partners in a multi-PUs, multi-SUs cognitive radio network (CRN) environment. We formulated the CSS problem as a multi-objective optimization and introduced a heuristic solution based on numerical analysis. This solution allocates portions of PU transmission time among cooperative partners near-optimally and within polynomial time. Additionally, we proposed an algorithm based on the stable matching concept to select stable (PU, SU) pairs for cooperative communication. The algorithm converges to (i) an optimal matching for PUs, ensuring each PU maximizes its utility from its cooperative SU partner, and (ii) a stable matching for SUs, preventing SUs from being paired with their least preferred PUs. Simulation results show that the proposed cooperative scheme significantly enhances utility for the PU network and improves SU utility compared to a random selection approach.

After establishing one-to-one mapping between PUs and SUs for CSS, our next focus is to explore many-to-one mapping among SUs and PUs for enhancing overall SU utility and improve secondary network efficacy. The next chapter presents the cooperative strategies employed by SUs to bolster secondary network utility.