

# Appendix A

## $S_4$ group

The irreducible representations of  $S_4$  follow the following Kronecker products,

$$1_1 \otimes \eta = \eta, \quad 1_2 \otimes 1_2 = 1, \quad 1_2 \otimes 2 = 2, \quad 1_2 \otimes 3_1 = 3_2, \quad 1_2 \otimes 3_2 = 3_1$$

$$2 \otimes 2 = 1_1 \oplus 1_2 \oplus 2, \quad 2 \otimes 3_1 = 2 \otimes 3_2 = 3_1 \oplus 3_2,$$

$$3_1 \otimes 3_1 = 3_2 \otimes 3_2 = 1_1 \oplus 2 \oplus 3_1 \oplus 3_2, \quad 3_1 \otimes 3_2 = 1_2 \oplus 2 \oplus 3_1 \oplus 3_2$$

Now, we write the Clebsch-Gordon coefficients in particular basis

For 1-dimensional representations:

$$1_1 \otimes \eta = \eta \otimes 1_1 = \eta$$

$$1_2 \otimes 1_2 = 1_1 \sim \alpha\beta$$

$$1_2 \otimes 2 = 2 \sim \begin{pmatrix} \alpha\beta_1 \\ -\alpha\beta_2 \end{pmatrix}$$

$$1_2 \otimes 3_1 = 3_2 \sim \begin{pmatrix} \alpha\beta_1 \\ \alpha\beta_2 \\ \alpha\beta_3 \end{pmatrix}$$

$$1_2 \otimes 3_2 = 3_1 \sim \begin{pmatrix} \alpha\beta_1 \\ \alpha\beta_2 \\ \alpha\beta_3 \end{pmatrix}$$

For 2-dimensional representations:

$$2 \otimes 2 = 1_1 \oplus 1_2 \oplus 2 \quad \text{with} \quad \begin{cases} 1_1 \sim \alpha_1\beta_2 + \alpha_2\beta_1 \\ 1_2 \sim \alpha_1\beta_2 - \alpha_2\beta_1 \\ 2 \sim \begin{pmatrix} \alpha_2\beta_2 \\ \alpha_1\beta_1 \end{pmatrix} \end{cases}$$

$$2 \otimes 3_1 = 3_1 \oplus 3_2 \quad \text{with} \quad \begin{cases} 3_1 \sim \begin{pmatrix} \alpha_1\beta_2 + \alpha_2\beta_3 \\ \alpha_1\beta_3 + \alpha_2\beta_1 \\ \alpha_1\beta_1 + \alpha_2\beta_2 \end{pmatrix} \\ 3_2 \sim \begin{pmatrix} \alpha_1\beta_2 - \alpha_2\beta_3 \\ \alpha_1\beta_3 - \alpha_2\beta_1 \\ \alpha_1\beta_1 - \alpha_2\beta_2 \end{pmatrix} \end{cases}$$

$$2 \otimes 3_2 = 3_1 \oplus 3_2 \quad \text{with} \quad \begin{cases} 3_1 \sim \begin{pmatrix} \alpha_1\beta_2 - \alpha_2\beta_3 \\ \alpha_1\beta_3 - \alpha_2\beta_1 \\ \alpha_1\beta_1 - \alpha_2\beta_2 \end{pmatrix} \\ 3_2 \sim \begin{pmatrix} \alpha_1\beta_2 + \alpha_2\beta_3 \\ \alpha_1\beta_3 + \alpha_2\beta_1 \\ \alpha_1\beta_1 + \alpha_2\beta_2 \end{pmatrix} \end{cases}$$

For 3-dimensional representations:

$$3_1 \otimes 3_1 = 3_2 \otimes 3_2 = 1_1 \oplus 2 \oplus 3_1 \oplus 3_2 \quad \text{with} \quad \left\{ \begin{array}{l} 1_1 \sim \alpha_1\beta_1 + \alpha_2\beta_3 + \alpha_3\beta_2 \\ 2 \sim \begin{pmatrix} \alpha_2\beta_2 + \alpha_1\beta_3 + \alpha_3\beta_1 \\ \alpha_3\beta_3 + \alpha_1\beta_2 + \alpha_2\beta_1 \end{pmatrix} \\ 3_1 \sim \begin{pmatrix} 2\alpha_1\beta_1 - \alpha_2\beta_3 - \alpha_3\beta_2 \\ 2\alpha_3\beta_3 - \alpha_1\beta_2 - \alpha_2\beta_1 \\ 2\alpha_2\beta_2 - \alpha_1\beta_3 - \alpha_3\beta_1 \end{pmatrix} \\ 3_2 \sim \begin{pmatrix} \alpha_2\beta_3 - \alpha_3\beta_2 \\ \alpha_1\beta_2 - \alpha_2\beta_1 \\ \alpha_3\beta_1 - \alpha_1\beta_3 \end{pmatrix} \end{array} \right.$$

$$3_1 \otimes 3_2 = 1_2 \oplus 2 \oplus 3_1 \oplus 3_2 \quad \text{with} \quad \left\{ \begin{array}{l} 1_2 \sim \alpha_1\beta_1 + \alpha_2\beta_3 + \alpha_3\beta_2 \\ 2 \sim \begin{pmatrix} \alpha_2\beta_2 + \alpha_1\beta_3 + \alpha_3\beta_1 \\ -\alpha_3\beta_3 - \alpha_1\beta_2 - \alpha_2\beta_1 \end{pmatrix} \\ 3_1 \sim \begin{pmatrix} \alpha_2\beta_3 - \alpha_3\beta_2 \\ \alpha_1\beta_2 - \alpha_2\beta_1 \\ \alpha_3\beta_1 - \alpha_1\beta_3 \end{pmatrix} \\ 3_2 \sim \begin{pmatrix} 2\alpha_1\beta_1 - \alpha_2\beta_3 - \alpha_3\beta_2 \\ 2\alpha_3\beta_3 - \alpha_1\beta_2 - \alpha_2\beta_1 \\ 2\alpha_2\beta_2 - \alpha_1\beta_3 - \alpha_3\beta_1 \end{pmatrix} \end{array} \right.$$

where  $\alpha_i$  and  $\beta_i$  denote the elements of the first and second elements, respectively.



# Appendix B

## The Scalar Sector of the Model

The scalar potential of the model is written such that it is invariant under the symmetry  $S_4 \otimes Z_3 \otimes Z_4$  and has the following form,

$$V(H, \varphi_c, \phi_c, \varphi_\nu, \xi, \psi) = V_1(H) + V_2(\varphi_c, \phi_c) + V_3(\varphi_\nu, \xi, \psi, H) + V_4(H, \xi, \psi), \quad (\text{B.1})$$

with,

$$V_1(H) = \mu_H^2 (H^\dagger H) + \lambda_H (H^\dagger H)^2, \quad (\text{B.2})$$

$$\begin{aligned} V_2(\varphi_c, \phi_c) = & a_1(\varphi_c \varphi_c)_{11}(\varphi_c \varphi_c)_{11} + a_2(\varphi_c \varphi_c)_2(\varphi_c \varphi_c)_2 + a_3(\varphi_c \varphi_c)_{31}(\varphi_c \varphi_c)_{31} \\ & + a_4(\phi_c \phi_c)_{11}(\phi_c \phi_c)_{11} + a_5(\phi_c \phi_c)_2(\phi_c \phi_c)_2 + a_6(\phi_c \phi_c)_{31}(\phi_c \phi_c)_{31} \\ & + a_7(\varphi_c \varphi_c)_{11}(\phi_c \phi_c)_{11} + a_8(\varphi_c \varphi_c)_2(\phi_c \phi_c)_2 + a_9(\varphi_c \varphi_c)_{31}(\phi_c \phi_c)_{31} \\ & + a_{10}(\varphi_c \phi_c)_{12}(\varphi_c \phi_c)_{12} + a_{11}(\varphi_c \phi_c)_2(\varphi_c \phi_c)_2 + a_{12}(\varphi_c \phi_c)_{31}(\varphi_c \phi_c)_{31} \\ & + a_{13}(\varphi_c \phi_c)_{32}(\varphi_c \phi_c)_{32} + a_{14}(\varphi_c \phi_c)_2(\varphi_c \varphi_c)_2 + a_{15}(\varphi_c \phi_c)_{31}(\varphi_c \varphi_c)_{31} \\ & + a_{16}(\varphi_c \phi_c)_{32}(\varphi_c \varphi_c)_{32} + a_{17}(\varphi_c \phi_c)_2(\phi_c \phi_c)_2 + a_{18}(\varphi_c \phi_c)_{31}(\phi_c \phi_c)_{31} \\ & + a_{19}(\varphi_c \phi_c)_{32}(\phi_c \phi_c)_2, \end{aligned} \quad (\text{B.3})$$

$$\begin{aligned}
V_3(\varphi_\nu, \xi, \psi, H) = & \mu_{\varphi_\nu}^2 (\varphi_\nu^\dagger \varphi_\nu)_{1_1} + b_1 (\varphi_\nu^\dagger \varphi_\nu)_{1_1} (\varphi_\nu^\dagger \varphi_\nu)_{1_1} + b_2 (\varphi_\nu^\dagger \varphi_\nu)_2 (\varphi_\nu^\dagger \varphi_\nu)_2 \\
& + b_3 (\varphi_\nu^\dagger \varphi_\nu)_{3_1} (\varphi_\nu^\dagger \varphi_\nu)_{3_1} + b_4 (\varphi_\nu^\dagger \varphi_\nu)_{3_2} (\varphi_\nu^\dagger \varphi_\nu)_{3_2} + b_5 (\varphi_\nu^\dagger \varphi_\nu)_{1_1} (\xi^\dagger \xi)_{1_1} \\
& + b_6 (\varphi_\nu^\dagger \varphi_\nu)_2 (\xi^\dagger \xi)_2 + b_7 (\varphi_\nu^\dagger \varphi_\nu)_{1_1} (\psi^\dagger \psi)_{1_1} + b_8 (\varphi_\nu^\dagger \varphi_\nu)_{1_1} (H^\dagger H)_{1_1}
\end{aligned} \tag{B.4}$$

$$\begin{aligned}
V_4(H, \xi, \psi) = & \mu_\xi^2 (\xi^\dagger \xi)_{1_1} + c_1 (\xi^\dagger \xi)_{1_1} (\xi^\dagger \xi)_{1_1} + c_2 (\xi^\dagger \xi)_2 (\xi^\dagger \xi)_2 + \mu_\psi^2 (\psi^\dagger \psi)_{1_1} \\
& + c_3 (\psi^\dagger \psi)_{1_1} (\psi^\dagger \psi)_{1_1} + c_4 (\xi^\dagger \xi)_{1_1} (\psi^\dagger \psi)_{1_1} + c_5 (\xi^\dagger \xi)_{1_1} (H^\dagger H)_{1_1} \\
& + c_6 (\psi^\dagger \psi)_{1_1} (H^\dagger H)_{1_1}
\end{aligned} \tag{B.5}$$

Let us denote the *vev* of the scalars as follows:

$$\begin{aligned}
\langle H \rangle = v_H, \quad \langle \psi \rangle = v_\psi, \quad \langle \xi \rangle = (v_{\xi_1}, v_{\xi_2}), \quad \langle \varphi_\nu \rangle = (v_{\varphi_{\nu_1}}, v_{\varphi_{\nu_2}}, v_{\varphi_{\nu_3}}) \\
\langle \varphi_c \rangle = (v_{\varphi_{c_1}}, v_{\varphi_{c_2}}, v_{\varphi_{c_3}}), \quad \langle \phi_c \rangle = (v_{\phi_{c_1}}, v_{\phi_{c_2}}, v_{\phi_{c_3}}),
\end{aligned} \tag{B.6}$$

In order to calculate the *vev*  $\langle \varphi_c \rangle = (v_{\varphi_{c_1}}, v_{\varphi_{c_2}}, v_{\varphi_{c_3}})$ , we write the minimum condition,

$$\left. \frac{\partial V}{\partial \varphi_{c_i}} \right|_{\langle \varphi_{c_i} \rangle = v_{\varphi_{c_i}}} = 0, \quad (i = 1, 2, 3), \tag{B.7}$$

Similarly, we have the minimum condition for  $\langle \phi_c \rangle = (v_{\phi_{c_1}}, v_{\phi_{c_2}}, v_{\phi_{c_3}})$  as follows,

$$\left. \frac{\partial V}{\partial \phi_{c_i}} \right|_{\langle \phi_{c_i} \rangle = v_{\phi_{c_i}}} = 0, \quad (i = 1, 2, 3), \tag{B.8}$$

This leads us to a system of equations and we analyze the vacuum configuration:

$$\begin{aligned}
\langle \varphi_c \rangle &= (v_{\varphi_c}, 0, 0) \\
\langle \phi_c \rangle &= (v_{\phi_c}, 0, 0)
\end{aligned}$$

We find that the vacuum alignment shown above is one of the solutions of the extremum conditions of equations (B.7 and B.8).

Just as in equations (B.7 and B.8), the *vev*  $\langle \varphi_\nu \rangle$  imposes the extremum condition on  $V$  and this leads us to the following system of equations,

$$\begin{aligned}
(2b_1 + 8b_3)v_{\varphi_{\nu_1}}^3 + (2b_2 - 4b_3)v_{\varphi_{\nu_2}}^3 + (2b_2 - 4b_3)v_{\varphi_{\nu_3}}^3 + (4b_1 + 8b_2)v_{\varphi_{\nu_1}}v_{\varphi_{\nu_2}}v_{\varphi_{\nu_3}} \\
+ (2b_5v_{\xi_1}v_{\xi_2} + b_7v_\psi + b_8v_h^2 + \mu_{\varphi_\nu}^2)v_{\varphi_{\nu_1}} + b_6v_{\xi_1}^2v_{\varphi_{\nu_1}} + b_6v_{\xi_2}^2v_{\varphi_{\nu_2}} = 0
\end{aligned}$$

$$(2b_1 + 4b_2)v_{\varphi_{\nu_1}}^2 v_{\varphi_{\nu_3}} + (6b_2 - 12b_3)v_{\varphi_{\nu_1}} v_{\varphi_{\nu_2}}^2 + (4b_1 + 2b_2 + 12b_3)v_{\varphi_{\nu_3}}^2 v_{\varphi_{\nu_2}} +$$

$$+ (2b_5 v_{\xi_1} v_{\xi_2} + b_7 v_\psi + b_8 v_h^2 + \mu_{\varphi_\nu}^2)v_{\varphi_{\nu_3}} + b_6 v_{\xi_2}^2 v_{\varphi_{\nu_1}} + b_6 v_{\xi_1}^2 v_{\varphi_{\nu_2}} = 0$$

$$(2b_1 + 4b_2)v_{\varphi_{\nu_1}}^2 v_{\varphi_{\nu_2}} + (6b_2 - 12b_3)v_{\varphi_{\nu_1}} v_{\varphi_{\nu_3}}^2 + (4b_1 + 2b_2 + 12b_3)v_{\varphi_{\nu_2}}^2 v_{\varphi_{\nu_3}} +$$

$$+ (2b_5 v_{\xi_1} v_{\xi_2} + b_7 v_\psi + b_8 v_h^2 + \mu_{\varphi_\nu}^2)v_{\varphi_{\nu_2}} + b_6 v_{\xi_2}^2 v_{\varphi_{\nu_3}} + b_6 v_{\xi_1}^2 v_{\varphi_{\nu_1}} = 0$$

(B.9)

The above system of equations has several solutions, one of the solutions being

$$v_{\varphi_{\nu_1}} = v_{\varphi_{\nu_2}} = v_{\varphi_{\nu_3}} = v_{\varphi_\nu} = \frac{1}{\sqrt{6}} \cdot \sqrt{-\frac{2b_5 v_\xi^2 + 2b_6 v_\xi^2 + b_7 v_\psi^2 + v_8 v_h^2 + \mu_{\varphi_\nu}^2}{b_1 + 2b_2}} \quad (\text{B.10})$$

with  $v_{\xi_1} = v_{\xi_2} = v_\xi$ . Another solution exists with  $v_{\varphi_{\nu_1}} \neq v_{\varphi_{\nu_2}} \neq v_{\varphi_{\nu_3}}$ , which has a very long expression and we choose such a solution to obtain the mass matrix taken under consideration in our model.





## List of Publications

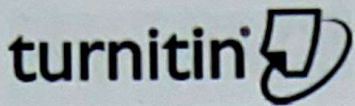
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1. **Thapa, B.**, Francis, Ng. K. Resonant leptogenesis in minimal inverse seesaw ISS (2, 2) model. *arXiv: 2312.04399* (2023) (Under Review).
2. **Thapa, B.**, Barman, S., Bora, S., and Francis, Ng. K. A minimal inverse seesaw model with  $S_4$  flavour symmetry. *Journal of High Energy Physics*, **2023**, 154 (2023).
3. **Thapa, B.**, Francis, Ng. K. Connecting low-energy CP violation, resonant leptogenesis and neutrinoless double beta decay in a radiative seesaw model. *Nuclear Physics B*, **986**, 116054 (2022).
4. **Thapa, B.**, Francis, Ng. K. Resonant leptogenesis and  $TM_1$  mixing in minimal type-I seesaw model with  $S_4$  symmetry. *European Physical Journal C*, **81**, 1061 (2021).
5. Barman, A., Francis, Ng. K., **Thapa, B.**, and Nath, A. Non-zero  $\theta_{13}$ , CP-violation and neutrinoless double beta decay for neutrino mixing in the  $A_4 \times Z_2 \times Z_3$  flavour symmetry model. *International Journal of Modern Physics A*, **38**, 2350012 (2023).
6. Bora, H., Francis, Ng. K., Barman, A., and **Thapa, B.** Neutrino mass model in the context of  $\Delta(54) \times Z_2 \times Z_3 \times Z_4$  flavour symmetries with inverse seesaw mechanism. *Physics Letters B*, **848**, 138329 (2023).
7. Bora, H., Francis, Ng. K., Barman, A., and **Thapa, B.** Majorana neutrinos in Double Inverse Seesaw and  $\Delta(54)$  flavour models. *International Journal of Modern Physics A*, **39**, 2450066 (2024).

## Papers Presented in Conference/ Workshop

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1. B. Thapa, Ng. K. Francis, **International Meeting on High Energy Physics (IMHEP-II)**, Institute of Physics, Bhubaneswar, February 16-22, 2023.
2. B. Thapa, Ng. K. Francis, **One-Day National Symposium**, Department of Physics & Student's Science Council, Tezpur University, March 10, 2023.
3. B. Thapa, Ng. K. Francis, **XIII Biennial National Conference of Physics Academy of North East (PANE-2022)**, Department of Physics, Manipur University, November 8-10, 2022.
4. B. Thapa, Ng. K. Francis, **National Conference on Emerging Trends in Physics (NCETP)**, Department of Physics, Tezpur University, June 16, 2021.



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### Abstract

The Standard Model (SM) of particle physics achieved remarkable success in modern physics when the Higgs boson was discovered in 2012, solidifying its status as one of the most successful theories. It gave us a remarkable insight into the fundamental nature of the universe. However, the SM has many shortcomings and is far from providing a complete picture. For instance, the SM encounters a challenge in understanding the properties of neutrinos.

Contrary to the prediction from the SM, evidence from various neutrino oscillation experiments suggests that neutrinos possess non-zero masses and their flavours mix. This is concrete experimental proof of Beyond the Standard Model (BSM) physics. BSM frameworks usually include the extension of the SM particle content, scalars, and/or fermions. The seesaw mechanism is one such framework that explains the smallness of neutrino mass. The seesaw mechanism can explain the origin of neutrino masses and the matter-antimatter asymmetry of the universe. In the simplest seesaw framework (type-I seesaw mechanism), the heavy right-handed neutrinos introduced can potentially decay out of equilibrium in the early universe. The decay of these particles can cause a lepton asymmetry, which eventually gets transformed into a baryon asymmetry by processes involving the violation of baryon (B) and lepton (L) numbers, known as sphaleron processes. However, in standard thermal leptogenesis with hierarchical masses of the right-handed neutrinos, the observed value of BAU can be explained if their mass scale is  $O(10^9)$ .

Neutrino oscillation suggests neutrino transition between different flavour states as they travel through space. Theoretically, the neutrino flavour mixing

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