Abstract

A class of compact finite difference schemes has been developed on nonuniform grids to tackle the convection-diffusion equation (CDE). A nonuniform grid has the leverage of clustering grids in higher-gradient regions and spreading out grid lines at other places, thereby making the computation less expensive. Contrary to the schemes available in the literature, the present schemes are abstained from using any kind of coordinate transformation to resolve nonuniform grids. The content of the present work can be divided into three parts. In the first part, a compact discretization of generalized 3D CDE on nonuniform grids is presented. This work on nonuniform grids considers the presence of cross-derivative terms and the discretization uses only nineteen-point stencil. Extension of this newly proposed discretization to semi-linear and convection-diffusion-reaction problems is seen to be straightforward and this inherent advantage is thoroughly exploited. The scheme is found to be efficient in capturing boundary layers and preserve the nonoscillatory property of the solution. The proposed method is tested using several benchmark linear and nonlinear problems from the literature. Additionally, problems with sharp gradients are solved. These diverse numerical examples demonstrate the accuracy and efficiency of the scheme proposed. Further, the numerical rate of convergence is seen to approach four confirming theoretical estimation.

The second part of the work pertains to the implicit compact discretization of the Navier-Stokes (N-S) equations on nonuniform grids. Subsequently, the discretization is used to approximate the Boussinesq equation as well. This newly developed scheme is based on a comparatively smaller five-point stencil and leads to an algebraic system of equations with constant coefficients. The scheme carries the flow variable and its gradients as unknowns and is seen to report back

truncation accuracy of order four for linear flow problems even on a nonuniform mesh. Temporally the scheme is second-order accurate. Both primitive variable and streamfunction-vorticity formulations have been successfully tackled using the proposed scheme. Verification and validation studies were carried out to establish the efficiency of the formulation in conjunction with both Dirichlet and Neumann boundary conditions. Simulations of interior and exterior flow problems near critical Hopf bifurcation points using a comparatively smaller grid helps establish the robustness of the scheme. The numerical solution obtained by solving the Boussinesq equations for the problem of natural convection reveals the wider applicability of the scheme involving heat transfer.

The third part concerns with extending the above compact scheme to the polar coordinate system. Here, we also conceive a higher spatially third-order compact scheme on five point stencil and carry out a comparative study. Apart from validating our schemes by computing for well-known flow configurations, emphasis is also given to simulating physical flow circumstances involving mass as well as heat transfer. Convergence order of the schemes along with the grid independence of the computed solutions are also demonstrated. Subsequently, extensive investigation is performed for flow past a stationary circular cylinder. The wide range of Reynolds number in the laminar regime considered for this investigation includes the stable periodic flows, characterized by von Kármán vortex street, bulging and secondary vortex phenomena for low and moderate Reynolds numbers (Re). For higher values of Re, the critical α- and β-phenomena associated with such flows are studied in details. In all the cases, the numerical results of present study are compared to those with some previous benchmark numerical and experimental studies and are seen to be in extremely close proximity to them.