CONTRIBUTIONS TO IDENTITIES FOR THE ROGERS-RAMANUJAN CONTINUED FRACTION, ARITHMETIC PROPERTIES OF CERTAIN RESTRICTED PARTITION FUNCTIONS, AND THE LEAST r-GAPS IN PARTITIONS

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Summary

In the Chapter 2 of this thesis, we prove some new identities for the Rogers–Ramanujan continued fraction. In the process, we also find some new relations for the Rogers-Ramanujan functions by using dissections of theta functions and the quintuple product identity.

In the next chapter, we extend some individual congruences found by Andrews and Paule [8] and da Silva, Hirschhorn, and Sellers [49] for $d_k(n)$ to their respective families as well as find new families of congruences for $d_k(n)$. We also present a refinement in an existence result for congruences of $d_k(n)$ found by da Silva, Hirschhorn, and Sellers [49] , and prove some new families of congruences modulo 5, 7, 8, 11, 13, 16, 17, 19, 23, 25, 32, 49, 64, and 128.

Using the theory of modular forms, we find the arithmetic densities of $\overline{a}_t(n)$ and $\overline{b}_t(n)$ modulo arbitrary powers of 2 and 3 for certain values of t and $\overline{b}_t(n)$ modulo p_i^k where $t = p_1^{a_1} \cdots p_m^{a_m}$ where $p_i \geq 5$ are primes, in Chapter 4. We further prove some infinite families of congruences for $\overline{a}_3(n)$ and $\overline{b}_3(n)$ modulo arbitrary powers of 2 and 3 and $\overline{b}_2(n)$ modulo 2.

In Chapter 5, we establish some new infinite families of congruences modulo 3 and some individual congruences modulo 9 for $b_6(n)$. In the process, we also deduce some Kolberg-type congruences. We obtain several Ramanujan-type congruences modulo powers of 5 for partition k-tuples with 5-cores, for $k = 2, 3, 4$. We also discover some new infinite families of congruences modulo powers of primes for partition k-tuples with p-cores, where p is a prime.

We find the arithmetic densities of $\mathrm{pod}_\ell(n)$ for $\ell = 3,\ 5,\ 7,\ 13,\ 17$ and $\mathrm{ped}_t(n)$ for $t = 13,\ 17$ modulo 2 and arbitrary powers of 2 in Chapter 6. We also prove some new multiplicative relations for $\mathrm{pod}_5(n)$, $\mathrm{pod}_9(n)$, $\mathrm{ped}_5(n)$, and $\mathrm{ped}_9(n)$ modulo small powers of 2 with the aid of the theory of Hecke eigenforms.

Finally, in Chapter 7, we study some arithmetic functions related to the sum of least r-gaps and establish their connections with certain known partition functions such as Andrews' singular overpartitions [4]. Next, we obtain Hardy-Ramanujan-type asymptotic formula for two such functions. We also explore arithmetic properties for these functions.