

Contents

Abstract	i
Declaration by the candidate	iii
Certificate of the supervisor	iv
Acknowledgements	v
1 Introduction	1
1.1 The q -Pochhammer symbol and Ramanujan's theta functions	1
1.2 The Rogers-Ramanujan continued fraction	3
1.3 Dissection of power series	3
1.4 Partitions of positive integers	4
1.5 Lucas sequences	6
1.6 Modular forms and Hecke operators	6
1.7 Identities for the Rogers-Ramanujan continued fraction	9
1.8 Arithmetic properties for some restricted partition functions	9
1.9 Minimal excludant and least r -gaps in partitions	12
2 Identities for the Rogers-Ramanujan Continued Fraction	14
2.1 Introduction	14
2.2 Preliminary results and lemmas	18
2.3 Proof of Theorem 2.4	23
2.4 Proof of Theorem 2.5	26
2.5 Proofs of Theorem 2.1 and Corollary 2.2	30
2.6 Proof of Theorem 2.3	33

2.7	Partition-theoretic identities	37
2.8	Concluding remarks	39
3	Congruences for k-Elongated Plane Partition Diamonds	42
3.1	Introduction	42
3.2	Proof of Theorem 3.2	46
3.3	Proof of Theorem 3.3	51
3.4	Proof of Theorem 3.4	53
3.5	Proof of the remaining congruences	54
3.5.1	Required lemmas	54
3.5.2	Proof of Theorem 3.6	56
4	Arithmetic Properties of Two Analogues of t-Core Partitions	62
4.1	Introduction	62
4.2	Preliminary results	67
4.3	Proof of Theorem 4.3	68
4.4	Proof of Theorem 4.4	70
4.5	Proof of Theorem 4.5	72
4.6	Proof of Theorem 4.6	74
4.7	Proof of Theorem 4.7	76
4.8	Proofs of Theorems 4.9 and 4.10	78
4.9	Proofs of Theorems 4.11–4.13	80
4.10	Proof of Theorem 4.14	82
4.11	Concluding remarks	85
5	Families of Congruences for 6-Regular Partitions and Partitions	
	k-Tuples with t-Cores	87
5.1	Introduction	87
5.2	Preliminaries	94
5.3	Proofs of the Theorems 5.5–5.7	96
5.4	Proofs of Theorems 5.8 and 5.9	101
5.5	Proof of Theorem 5.11	105
5.6	Proofs of (5.17) and (5.18)	107

5.7	Proof of Theorem 5.12	109
5.8	Proofs of Theorems 5.13–5.15	110
5.9	Proof of Theorem 5.16	111
5.10	Proofs of Theorem 5.17 and Corollary 5.18	113
5.11	Concluding remarks	114
6	Arithmetic Properties for Certain Restricted ℓ-Regular Partitions	117
6.1	Introduction	117
6.2	Proofs of Theorems 6.1–6.5	120
6.3	Proofs of Theorems 6.6–6.11	127
7	Arithmetic and Asymptotic Properties for Some Functions Related to the Least r-Gaps in Partitions	132
7.1	Introduction	132
7.2	Proofs of the Theorems 7.2–7.6	140
7.2.1	An alternative proof of Theorem 7.2	140
7.2.2	Proof of Theorem 7.3	141
7.2.3	Proof of Theorem 7.4	142
7.2.4	Proof of Theorem 7.5	143
7.2.5	Proof of Theorem 7.6	143
7.3	Proofs of Theorems 7.7 and 7.8	144
7.4	Proofs of Theorems 7.10 and 7.11	146
7.5	Proofs of Theorems 7.12 and 7.13	148
7.6	Proofs of the remaining theorems	153
7.6.1	Proof of Theorems 7.14–7.15	153
7.6.2	Proof of Theorem 7.16	153
7.6.3	Proof of Theorem 7.17	154
7.6.4	Proof of Theorem 7.18	154
7.7	Concluding remarks	154
	List of Publications	156
	Bibliography	157