Chapter 1

Introduction

Since the discovery of neutrinos, they have played an important role in uncovering and investigating the fundamental laws of electroweak theory, the structure of the nucleus, peering into the Early Universe, the dynamics of the sun, and even violent cosmic events like core collapse supernovae. But, despite their great abundance, there is still relatively little we know about them because they interact very weakly.

Neutrinos differ from the various Standard Model particles in several ways. The neutrino is the only known neutral fermion, so they may have their own antiparticle, leptonic CP-violation could be significant, and neutrino masses are small in comparison to the large mixing angle. Extending beyond the Standard Model is necessary for theories to account for these phenomena, and doing so could unveil physics at an extremely high energy scale. This chapter will provide additional clarification and context for the previously listed concerns by summarizing the current state of neutrino observables, recounting the fascinating history of neutrinos, and describing the neutrino oscillation phenomena. We will also discuss the possible mechanisms of neutrino mass-generating techniques.

1.1 Neutrinos: A Brief Historical Overview

The existence of a very light, electrically neutral spin $\frac{1}{2}$ particle was first proposed by Pauli in 1930 to explain the apparent non-conservation of energy observed in nuclear beta decay. It was previously believed that a two-body decay of the unstable neutron into a proton and electron would cause this process to occur. The discovery that the radiated electron's energy spectrum was continuous caused great distress. Pauli was perplexed by this discovery and, unable to attend a physics conference in T*ü*bingen, instead wrote a letter in which he suggested a brand-new, light particle as a "desperate remedy" to account for the energy that beta decay is missing. This particle was first named the neutron by Pauli; later, Fermi proposed the term neutrino. This word is taken from the Italian for little and neutral one.

When Fermi developed his first hypothesis of beta decay in 1934, he considered it as a four-fermion process in which a neutron decayed into a proton, an electron, and an anti-electron neutrino [1]. Bethe and Peierls calculated the cross section for a neutrino's interaction with nuclei that same year [2]. Their calculation revealed the cross-section to be so tiny that it was long believed impossible to detect the neutrino; Pauli wagered a case of champagne that the renowned neutrino would never be found. In 1946, Pontecorvo proposed a method using chlorine that could allow him to detect neutrinos

$$\nu +^{37} cl \longrightarrow e^- +^{37} Ar. \tag{1.1.1}$$

Furthermore, he proposed that the sun and fission reactors are great sources of neutrinos. Anyway, it was only after 1956, that Reines and Cowan identified antineutrinos emitted from a nuclear reactor [3]. The following reaction was observed as the basis for the detection:

$$\overline{\nu} + P \longrightarrow e^+ + n \tag{1.1.2}$$

Following a period of two years in 1958, Goldhaber, Grodzin, and Sunyar conducted an experiment to detect the handedness of the neutrino by measuring the circular polarization of photons [4]. The experiment proved that neutrinos were left-handed, which had a significant outcome. Since right-handed neutrinos have never been detected before, the finding by Goldhaber, Grozin, and Sunyar suggests that neutrinos are massless.

Animesh Despite this outcome, Pontecorvo considered the possibility and experimental implications of massive neutrino [5]. Neutrino oscillations, similar to K^0- $\overline{K}^0$ oscillations, could occur if neutrinos had small masses. Pontecorvo extended his theoretical and experimental work in 1962. In the first instance, flavor oscillations (between electron and muon flavor) were introduced by Maki, Nakagawa, and Sakata [6]; in the second, Lederman, Schwartz, and Steinberger's Brookhaven neutrino experiment found that neutrinos that interact with electrons and muons through charged current are, in fact, different neutrinos. In short, they were the ones who discovered the muon neutrino and won the Physics Nobel Prize in 1988.

The Homestake experiment, led by Davis and Bahcall, was the next significant experimental achievement in neutrino physics and was completed in 1970 [7]. They utilized the reaction that Pontecorvo first proposed to measure high-energy solar neutrinos in this experiment. Nonetheless, it was noticed that the observed rate of this reaction was two to three times lower than that predicted in the Minimal Solar Model and this deficit came to be known as the solar neutrino problem. With an improved understanding of neutrino properties and additional measurements made by the Sudbury Neutrino Oscillation (SNO) and Super-Kamiokande (SK) collaborations, this issue would be fully resolved. Theoretically, a significant deficit of electron neutrinos should have been expected because of the 1985 formulation of the Mikheyev-Smirnov-Wolfenstein (MSW) effect [8, 9].

In 1975 when Perl and associates at Stanford Linear Accelerator discovered the tau lepton [10], it was hinted that there might be a third generation of neutrinos, known as the tau neutrino. Later Z-decays at LEP [11] were subsequently used to confirm the three-neutrino picture. However, the Direct Observation of NU Tau (DONUT) experiment [12], which produced tau neutrinos by using the decay of charmed particles, was only directly observed in this form of neutrino in 2000.

As previously mentioned, the collaborations between SK [13] and SNO [14, 15] were the first oscillation experiments to confirm neutrino oscillations with a high degree of statistical significance. Numerous neutrino experiments since then have been carried out that have improved our understanding of the properties of neutrinos.

1.1.1 Neutrino Oscillations:

In 1957, Pontecorvo developed the theory of neutrino oscillations, which was based on a two-level quantum mechanical system [16]. When considering a two-level quantum mechanical system which is characterized by the stationary state Ψ_n with an energy eigenvalue E_n, the time evolved ket following a time t; i.e., $\Psi_n(t)$ is given by,

$$\Psi_n(t) = e^{-iE_nt}\Psi_n(0) \tag{1.1.3}$$

Thus, after time t, the stationary state will continue to be an eigenstate of the Hamiltonian with only a phase modification. However, in the case of a nonstationary state (for instance, some arbitrary superposition of the stationary states Ψ_1 and Ψ_2 with eigenvalues E_1 and E_2, respectively), it is clear that after time evolution, the probability of remaining in the initial state would, instead, be an oscillatory function of time with frequency (E_2 -E_1).

We can use equation $\nu_\alpha = \sum_{i=1}^{N}(U_\nu)_{\alpha i}\nu_i$ to find an explanation for the neutrino oscillation phenomenon by representing the neutrino flavor eigenstates ν_α (non-stationary states) as a linear combination of mass eigenstates ν_i with $(U_\nu)_{\alpha i}$ as the coefficients of linear expansion.

The probability that a neutrino flavor eigenstate ν_α moves to another flavor eigenstate ν_β traveling a distance L(= t) in vacuum after time t, i.e., the neutrino oscillation probability is given by,

$$P_{\nu_\alpha\nu_\beta} = |\nu_\beta\nu_\alpha(t)|^2 = |\sum_{i=1}^{N}\sum_{j=1}^{N}(U_\nu)_{\alpha i}(U_\nu^*)_{\beta j}|\nu_j\nu_i(t)|^2 \tag{1.1.4}$$

We can take approximation in the ultra-relativistic regime for small neutrino masses $P_i \simeq P_j \equiv P \simeq E$ and $E_i = \sqrt{P_i^2 + m_i^2} \simeq P + \frac{m_i^2}{2E}$ where m_i and E_i are the mass and energy of ν_i. Utilizing this and the orthogonality of the mass eigenstates we get

$$P_{\nu_\alpha \nu_\beta} = \delta_{\alpha\beta} - 4 \sum_{i=1<j}^{N} Re[(U_\nu)_{\alpha i}(U_\nu)_{\beta j}(U_\nu^*)_{\beta i}(U_\nu^*)_{\alpha j}] sin^2 \chi_{ij}^{osc} + \\ 2 \sum_{i=1<j}^{N} Im[(U_\nu)_{\alpha i}(U_\nu)_{\beta j}(U_\nu^*)_{\beta i}(U_\nu^*)_{\alpha j}] sin^2 \chi_{ij}^{osc} \tag{1.1.5}$$

Where,

$$\chi_{ij}^{osc} = \frac{(m_i^2 - m_j^2)L}{4E} = 1.27 \frac{\Delta m_{ij}^2}{eV^2} \frac{L/E}{m/MeV} \tag{1.1.6}$$

It is crucial to note from Eq.(1.5) that the mass square splittings, $\Delta m_{ij}^2 = m_i^2 - m_j^2$, must be non-vanishing for neutrino oscillations to occur. Therefore, the neutrino oscillation experimental observation indicates that there should be a mass difference between two neutrino mass eigenstates that is at least non-zero. In this way, the observation of neutrino oscillations confirms the massive nature of neutrinos. It is also important to note on that occasion that neutrino oscillation measurement only yields information regarding mass square splittings and that it is insensitive to the absolute neutrino mass scale. There are two mass square splittings for the three-flavor neutrino oscillations: the atmospheric splitting $\Delta m_{atmos}^2 = m_3^2 - m_2^2$ and the solar splitting $\Delta m_{solar}^2 = m_2^2 - m_1^2$. The three-neutrino oscillation spectrum consists of these two mass square splittings along with one Dirac CP phase δ_{CP}, three mixing angles- solar mixing angle θ_{12}, reactor mixing angle θ_{13} and atmospheric mixing angle θ_{23}.

1.1.2 The Light of Recent Neutrino Oscillation Data:

The study of neutrinos represents one of the most rapidly evolving fields in modern particle physics research. In the last fifty years, remarkable advancements have been achieved in the field of neutrino physics, facilitated by the combined progress in oscillation studies, neutrinoless double-beta decay experiments, tritium endpoint experiments, and cosmological constraints. These mysterious particles are being studied in experiments around the world to obtain information about their oscillation parameters, including measurements of CP violation parameter δ_{CP} in the leptonic sector, which explains the matter dominance over antimatter of the universe, searching for signs of lepton number violating BSM physics in $0\nu2\beta$ experiments, and more. Another use for neutrinos is found in astrophysical measurements.

The IceCube Neutrino Observatory in Antarctica made a groundbreaking finding by detecting extremely high-energy neutrinos (around PeV), surpassing the energy levels of the LHC by approximately 250 times. This discovery opens up new avenues for investigating high-energy events through alternative means. But in our discussion, we'll limit our focus solely to the measurements related to neutrino oscillation.

The characteristics of these neutrino mixing parameters show some very interesting features:

• Since $\Delta m^2_{atmos} = m^2_3 - m^2_2$ and $\Delta m^2_{solar} = m^2_2 - m^2_1$ are separated by two orders, it makes sense to specify $R_{Mass} \equiv |\frac{\Delta m^2_{21}}{\Delta m^2_{32}}| \simeq 10^{-2}$

• We still don't know the neutrino mass ordering as the sign of the atmospheric splitting is Δm^2_{3l}.

• It is undetermined whether the atmospheric mixing angle θ_{23} lies in the first octant ($\theta_{23} < \frac{\pi}{4}$) or second octant ($\theta_{23} > \frac{\pi}{4}$).

• We still don't have precise measurements for the Dirac CP phase δ_{CP}.

• The reactor mixing angle θ_{13} is small as compared to the other two mixing

Parameters	NH (3σ)	IH (3σ)
$\Delta m^2_{21}[10^{-5}eV^2]$	$6.82 \to 8.03$	$6.82 \to 8.03$
$\Delta m^2_{31}[10^{-3}eV^2]$	$2.428 \to 2.597$	$-2.581 \to -2.408$
$\sin^2\theta_{12}$	$0.270 \to 0.341$	$0.270 \to 0.341$
$\sin^2\theta_{13}$	$0.02029 \to 0.02391$	$0.02047 \to 0.02396$
$\sin^2\theta_{23}$	$0.460 \to 0.620$	$0.412 \to 0.623$
δ_{CP}	$108 \to 404$	$192 \to 360$

Table 1.1: The 3σ ranges of neutrino oscillation parameters from NuFIT 5.2 (2022) and [17]

angles. Earlier it was thought to be zero, but it was proved to be non-zero as observed by the short baseline reactor anti-neutrino experiments in 2012.

• There is no clarification of whether neutrinos are Dirac or Majorana particles.

The Majorana nature of the neutrinos can be revealed by searches for $0\nu2\beta$, and the absolute neutrino mass scale can be determined by probing the beta decay end-point spectrum. It's important to note that the SM Higgs boson doesn't interact with neutrinos to give them mass. Researchers worldwide are working on explaining how neutrinos acquire mass. In the upcoming chapters, we've developed models for neutrino mass that align with observed oscillation parameters and make predictions for those yet to be found. This is the driving force behind the discussions in the upcoming chapters.

1.1.3 Searches To Unveil

Now that we understand the puzzles about neutrinos, we can appreciate the efficient experiments that have been carried out worldwide to explore and measure their properties.

• **Long-Baseline neutrino experiments:** DUNE stands out as a leading venture in its field. It's an expansive scientific endeavor known as the Deep Underground Neutrino Experiment (DUNE), previously referred to as the Long-Baseline Neutrino Experiment (LBNE), and is a significant international project by Fermilab. Its primary goals involve measuring CP phase δ_{CP}, determining neutrino mass ordering, and understanding the octant of θ_{23}. Additionally, DUNE aims to investigate proton decay and detect the flux of ν_e from a core-collapse supernova. With Fermilab's robust infrastructure and support, DUNE will receive the necessary neutrino beamline from the Long-Baseline Neutrino Facility (LBNF).

DUNE, an intricately crafted experiment, incorporates with a couple of detectors: one located near the Fermi National Accelerator Laboratory in Batavia, Illinois (known as the near detector), and another called the liquid argon (LAr) far detector situated over 1300 km away at the Sanford Underground Research Laboratory in South Dakota, nestled more than a kilometer below the surface. The far detector, boasting a fiducial mass of 40 kt, comprises four similar modules, each functioning as a liquid argon time-projection chamber (LArTPC). For the first time in a long-baseline neutrino experiment like DUNE, liquid argon is being utilized. This choice stems from its exceptional capacity in tracking and measuring calorimetry, resulting in heightened signal accuracy and effective identification of background discrimination. Moreover, its outstanding ability to precisely reconstruct the movement properties of particles with remarkable resolution makes it an optimal tool for meticulously measuring neutrino events spanning a wide range of energies.

The DUNE project [18] aims to measure the angle δ_{CP} with a high precision ranging from 10° to 10° degrees, and it anticipates detecting CP violation at a significant level of certainty 3Σ for around 67% of δ_{CP} values. Additionally, it seeks to comprehend the neutrino mass ordering up to a minimum value of $\Delta\chi^2 \geq 25$. India actively participates in the DUNE collaboration. Fermilab NOνA [19] and T2K in Japan [20] are two important ongoing neutrino oscillation experiments of this type.

- **Short-Baseline Experiments:**

To determine non-zero θ_{13}, short-baseline anti-neutrino experiments from reactor sources, such as Nuclear Power Plants (NPP), evaluate the survival probability of $\overline{\nu_e}$ at short distances. One of the most important discoveries of modern particle physics experiments was discovered in 2012 by the collaboration of the South Korean-based RENO [21] and the multinational Daya Bay [22], which is a short-baseline experiment based in China. The discovery was that of non-zero θ_{13} up to 5.2 and 4.9 standard deviations, respectively. Six nuclear power plants (NPPs) are currently in operation, generating a total of 2.9 (2.8 and 2.66) gigawatts of thermal power at Daya Bay (RENO). These plants produce anti-electron neutrinos $\overline{\nu_e}$, which are then detected by eight antineutrino detectors. These detectors are divided into three groups spread out over a 1.9-kilometer distance within Daya Bay. Meanwhile, RENO, another facility, has a couple of identical detectors positioned respectively 294 meters and 1383 meters away from the source. Both experiments utilize Gadolinium-doped liquid scintillator (Gd-Ls) as their method to identify inverse beta decay, represented by the equation $\overline{\nu_e} + p \to e^+ + n$, serving as the fundamental principle of detection.

- **India-based Neutrino Observatory (INO):**

India is initiating a significant venture into neutrino physics with the India-based Neutrino Observatory. The primary experiment will utilize a 50-kiloton magnetized iron calorimeter (ICAL) for the inaugural time, focusing on the investigation of atmospheric neutrinos [23]. The experiment aims to achieve two main objectives: studying matter effects to determine the ordering of neutrino mass and exploring the CP phase δ_{CP}.

- **KATRIN Experiment:**

The Karlsruhe TRItium Neutrino (KATRIN) experiment, situated at the Tritium Laboratory in Karlsruhe, aims to determine the absolute neutrino mass scale in a way that doesn't rely on any particular model. It achieves this by precisely analyzing the kinematics of electrons produced through beta decay. To accomplish this task, KATRIN utilizes a sophisticated high-resolution spectrometer known as the Magnetic Adiabatic Collimation combined with an Electrostatic Filter (MAC-E filter), which boasts a 10-meter diameter. This apparatus ensures highly accurate measurements of electron energy originating from a Tritium source [24]. Previous experiments managed to set an upper limit on the anti-electron neutrino $\overline{\nu_e}$ at around 2.3 electronvolts (eV), while KATRIN anticipates achieving measurements with an accuracy that's one step further in precision.

- **IceCube/ PINGU Experiment:**

Situated at the South Pole, the IceCube detector was designed to detect the Cherenkov light generated by interacting neutrinos coming from high-energy cosmic rays (PeV), using a vast expanse of transparent Antarctic ice measuring 1 cubic kilometer. This colossal detector consists of 5160 optical sensors or digital optical modules (DOMs) positioned 1.5 kilometers below the geographic South Pole. These sensors are arranged on 86 vertical cables known as strings, with 60 DOMs on each, where 78 are placed horizontally at intervals of 125 meters in triangular grids forming a hexagonal layout stretching across a square kilometer, while the remaining are more closely packed to form the Deep Core. Each DOM contains a 25-centimeter-long photomultiplier tube (PMT) along with data acquisition and control electronics. Additionally, 324 DOMs form the surface detector called IceTop. Following three years of thorough data collection and analysis from 2010 to 2013, IceCube identified 37 ultra-high-energy events.

While initially designed to detect extremely high-energy astrophysical neutrinos (PeV), IceCube's capabilities have been expanded to observe lower-energy neutrinos (10 GeV $\leq E \leq$ 100 GeV) for studying atmospheric neutrino oscillations. This expansion involved increasing the density of photodetectors within the ice and improving the efficiency of photomultiplier tubes (PMTs). Now, IceCube is proficient in measuring atmospheric neutrino oscillation parameters such as θ_{23} and Δm^2_{32}, rivaling the efficiency of other ongoing experiments. An analysis [25] conducted with 5174 track-like events over 953 days and data binned in the logarithm of reconstructed energy between 6 and 56 GeV revealed no preference for any mass ordering when determining θ_{23} and Δm^2_{32} using binned maximum likelihood techniques. For the normal ordering, values of $sin^2\theta_{23} = 0.53^{+0.09}_{-0.12}$ and $\Delta m^2_{32} = 2.72^{+0.19}_{-0.20} \times 10^{-3} eV^2$ were obtained, while for the inverted ordering, the results were $sin^2\theta_{23} = 0.51^{+0.09}_{-0.11}$ and $\Delta m^2_{32} = -2.72^{+0.18}_{-0.21} \times 10^{-3} eV^2$. Improving event reconstruction precision and the number of detected neutrino events can be achieved by increasing the density of photodetectors in the Deep Core region. With this in mind, the Precision IceCube Next Generation Upgrade (PINGU) has been proposed to gather better statistics and enhance precision in determining atmospheric neutrino oscillation parameters, potentially resolving neutrino mass ordering up to a 3σ confidence level after collecting data for four years. To enhance Deep Core performance, modifications include reducing the distance between digital optical modules (DOMs) from 7 meters to 3 meters, narrowing string-to-string spacing from 40-70 meters to 22 meters, and increasing the number of DOMs per string from 50 to 96. These alterations aim to make the detector sensitive to detecting low-energy events ($\leq$ 10 GeV).

- $0\nu2\beta$ **experiments:**

Exploring $0\nu2\beta$ searches stands as an intriguing field within neutrino physics because it helps to define the effective Majorana neutrino mass. In this realm, lepton number-preserving two-neutrino double beta decay DBD is allowed by the Standard Model, releasing two e^- and two $\overline{\nu_e}$ particles. Contrastingly, in $0\nu2\beta$ processes, lepton number conservation doesn't hold, resulting in the emission of two e^-, sharing the overall transition energy. This would create a distinctive peak in the sum energy spectrum of these two electrons. However, the phase space for the two-neutrino DBD is smaller than that of $0\nu2\beta$ processes, which is limited due to the smallness of the Majorana neutrino mass. Despite this, the latter serves as a useful tool for investigating lepton number-violating processes. Specifically, the reaction rate is linked to the square of the effective neutrino mass ($| m_{\nu_e}|$), making it exceptionally challenging to measure. Moreover, it is directly influenced by uncertainties in determining nuclear matrix elements. Several intricate experiments, such as CUORE and GERDA, have been devised to search for $0\nu2\beta$. A comprehensive overview of the current status of such investigations can be found in [26].

The Cryogenic Underground Observatory for Rare Events (CUORE) is in its final stages of construction. Located at the Gran Sasso Underground Laboratory, positioned 3400 meters below the surface (measured in meters water equivalent), CUORE [27] aims to detect a phenomenon called neutrinoless double beta decay ($0\nu2\beta$) using specialized cryogenic bolometers functioning at an incredibly low temperature of 10 milliKelvin (mK). This exceptionally low temperature is advantageous because it reduces the crystal's heat capacity, enabling the efficient detection of tiny energy releases caused by temperature changes resulting from $0\nu2\beta$ processes. Previously, CUORE conducted an initial exploration for $(0\nu2\beta)$ using a single setup named CUORE-0. This experiment managed to determine $T_{\frac{1}{2}} > 4.0 \times 10^{24}$ years in TeO_2, employing an exposure of 9.8 kilograms per year. Moving forward, CUORE aims to enhance this capability significantly, aspiring to reach a half-life measurement of 3.5×10^{26} years with reduced background interference.

The GERmanium Detector Array (GERDA) employs an innovative method by placing a series of highly enriched HPGe detectors—amounting to 86% directly into the liquid argon (LAr) cryogen. This setup serves a dual purpose: shielding the detectors from external γ rays while also providing the necessary cooling [28–30].

GERDA commenced its data collection in 2011 and has established a lower limit of greater than 2.1×10^{25} years for the half-life $T_{\frac{1}{2}}$ of 76_{Ge} with an exposure of 21.6 kilograms per year. When combined with findings from prior experiments, this achievement sets the current best limit for $T_{\frac{1}{2}}$ to greater than 3.0×10^{25} years.

1.2 The Standard Model of particle physics:

The Glashow-Weinberg-Salam proposal laid the foundation for the Standard Model (SM) of particle physics, a theory blending relativistic quantum field principles and local gauge theory [31–33]. This model is structured around the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, which embodies three fundamental forces: the strong nuclear force, weak nuclear force, and electromagnetic force, excluding gravity. The $SU(3)$ group delineates the strong force, characterizing the color charge of quarks and gluons, the carriers of the strong force. Meanwhile, the electro-weak force is depicted by the $SU(2) \times U(1)$ group, where L denotes left-handed chirality and Y represents weak hypercharge. The discovery of the Higgs boson[34, 35]

	$SU(3)_C$		$SU(2)_L$	$U(1)_Y$
lepton doublets	$\begin{pmatrix}\nu_{e_L}\\ e_L\end{pmatrix}$	1	2	-1
lepton singlets	e_R	1	1	-2
quark doublet	$\begin{pmatrix}U_L\\ d_L\end{pmatrix}$	3	2	$\frac{1}{3}$
quark singlets	U_R	3	1	$\frac{4}{3}$
quark singlets	d_R	3	1	$\frac{-2}{3}$
Higgs doublet	$\begin{pmatrix}\Phi^+\\ \Phi^0\end{pmatrix}$	1	2	1

Table 1.2: Charge assignments of leptons, quarks, and Higgs field under the standard model gauge group

at the Large Hadron Collider (LHC) in 2012 solidified the SM as one of the triumphant theories in particle physics.

Particles in the Standard Model can be divided into three main groups: fermions, Gauge Bosons, and scalars. Within the fermion sector, there are three types of quarks and leptons. Quarks, which carry color charges, transform in triplets, while color-neutral leptons transform as singlets under the $SU(3)_C$ Gauge group. A crucial assumption in the Standard Model is that left-handed fermion fields transform as doublets, whereas right-handed fermions transform as singlets under $SU(2)_L$. All fermions have a charge under $U(1)_Y$. Moving to the Gauge Boson category, eight gluons facilitate strong interactions, serving as massless vector fields. These gluons are electrically neutral and have charges under $SU(3)_C$. Similarly, the photon (γ) is responsible for electromagnetic force mediation, being both massless and electrically neutral. For weak interactions, the W^+, W^- and Z Bosons act as mediators. The W^+ and W^- Bosons are massive and charged, while the Z Boson is a neutral particle with substantial mass. In the Standard Model (SM), the masses of particles like fermions and Gauge Bosons, excluding neutrinos, are generated by what's known as the Higgs mechanism. The Higgs Boson, a scalar particle, holds significance as it exists as a singlet under $SU(3)_C$ and a doublet under $SU(2)_L$ within the model.

1.2.1 The Electroweak Sector:

The electroweak section of the Standard Model (SM) encompasses all the electromagnetic and weak interactions using the gauge group $SU(2)_L \times U(1)_Y$. The Higgs mechanism, spontaneous symmetry breaking, and local gauge symmetry are the three most important principles of the Standard Model. The gauge theory centers around exploring the Lagrangian density, a mathematical construct that holds all the details about the interactions and dynamics of various fields within the theory. The SM's Lagrangian remains invariant under a local gauge transformation, which is expressed as -

$$\overline{\Psi_L}' = e^{(ig\frac{\tau}{2}\theta(x)+ig'\frac{Y}{2})\Theta(x))}\overline{\Psi_L}, \overline{\Psi_R}' = e^{ig'\frac{Y}{2}\Theta(x)}\overline{\Psi_R} \tag{1.2.1}$$

In local gauge transformation, the ordinary derivative needs to be changed to what we call a covariant derivative as,

$$D_\mu = \delta_\mu + i\frac{g}{2}\tau_a W_\mu^a + i\frac{g'}{2}YB_\mu \tag{1.2.2}$$

By incorporating two gauge fields, W_μ^a and B_μ, the symmetry groups $SU(2)$ and $U(1)$ are gauged respectively. τ_a and Y serve as generators for these gauge groups, where 'a' takes values from 1 to 3, signifying the three distinct generations of leptons. The symbols g and g' stand for the coupling constants governing the strengths of electromagnetic and weak interactions, respectively. The gauge term that involves pure gauge interactions can be expressed as follows:

$$L_{gauge} = -\frac{1}{4}W_{\mu a}^a W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \tag{1.2.3}$$

Where,

$$W_{\mu\nu}^a = \delta_\mu W_\nu^a - \delta_\nu W_\mu^a - g\epsilon^{abc}W_\mu^b W_\mu^c, B_{\mu\nu} = \delta_\mu B_\nu - \delta_\nu B_\mu \tag{1.2.4}$$

Here, ϵ^{abc} is the structure constant for SU(2) group. The Lagrangian for fermion sector can be given as-

$$L_{fermion=\overline{L}}\gamma^{\mu}(i\delta_{\mu} - g\frac{\tau}{2}W_{\mu} - g'\frac{Y}{2}B_{\mu})L + \overline{R}\gamma^{\mu}(i\delta_{\mu} - g'\frac{Y}{2}B_{\mu})R \tag{1.2.5}$$

And the lagrangian for the Higgs field can be given as-

$$L_{Higgs} = [(i\delta_{\mu} - g\frac{\tau}{2}W_{\mu} - g'\frac{Y}{2}B_{\mu})\Phi]^2 - V(\phi) \tag{1.2.6}$$

The Yukawa Lagrangian for the quark and leptons are given as-

$$L_y = -Y_d[\overline{Q_L}\Phi d_R] - Y_u[\overline{Q_L}.\overline{\Phi}u_R] - Y_l[\overline{l_L}\Phi l_R] + h.c. \tag{1.2.7}$$

The electroweak Lagrangian in SM can be written is as follows-

$$L_{EW} = L_{gauge} + L_{fermion} + L_{Higgs} + L_{Yukawa} \tag{1.2.8}$$

The Lagrangian indicates that the scalar potential and the Higgs field play a vital role in clarifying how fermion and gauge Boson masses are generated by spontaneous symmetry breaking (SSB) and the Higgs mechanism, topics that will be covered in the following section. Concerning the Yukawa Lagrangian, it suggests that within the Standard Model (SM), the absence of a right-handed (RH) neutrino prevents the emergence of a mass term for the neutrino.

1.2.2 Origin of Gauge Boson And Fermion Mass:

The electroweak theory describes the interaction between particles using a non-abelian gauge group $SU(2)_L \times U(1)_Y$. Within this theory, the challenge lies in generating the masses for three weak interaction gauge Bosons, namely $W^+, W^-, andZ$ bosons. Interestingly, despite this, the gauge Boson responsible for electromagnetic interaction, the photon (γ), remains without mass. This peculiarity demands that Quantum Electrodynamics (QED) maintains exact symmetry, ensuring the conservation of electric charge as a fundamental quantity. Spontaneous symmetry breaking is when a system's Lagrangian maintains a certain symmetry, meaning it stays unchanged under that symmetry, but the system's ground state doesn't share that same symmetry.

The Higgs mechanism describes how the masses of both fermions and gauge bosons in the Standard Model are generated through what's called spontaneous symmetry breaking (SSB). This process necessitates a complex scalar field known as the Higgs field, which exists as an $SU(2)_L$ doublet, playing a crucial role in breaking the symmetry spontaneously.

The Higgs field can be given by-

$$\Phi = \begin{bmatrix} \Phi^+ \\ \Phi^- \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} \Phi_1 + i\Phi_2 \\ \Phi_3 + i\Phi_4 \end{bmatrix} \tag{1.2.9}$$

In this context, Φ^+ and Φ^- represent components of the Higgs field. The way scalars interact with each other is explained by the Lagrangian,

$$L_{Higgs} = (D_\mu \Phi)^\dagger.(D_\mu \Phi) - V(\Phi) \tag{1.2.10}$$

Here, D_μ is the covariant derivative. Now, the scalar potential V_Φ can be represented as-

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda(\Phi^\dagger \Phi)^2, \Phi^\dagger \Phi = \frac{1}{2}[\Phi_1^2 + \Phi_2^2 + \Phi_3^2 + \Phi_4^2] \tag{1.2.11}$$

In order for the gauge Bosons and fermions to gain their masses, the Higgs field must have a non-zero vacuum expectation value (VEV). This occurs when the Higgs potential is minimized for the given coefficients $\lambda > 0$ and $\mu^2 < 0$. When the scalar potential is minimized, it's crucial for the vacuum to maintain its charge neutrality. This is why the neutral part of the Higgs field obtains a vacuum expectation value [36–38].

Here,

$$< \Phi_0 >= \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ \nu \end{bmatrix} ; \nu = \sqrt{\frac{-\mu^2}{\lambda}}; [\Phi_1 = \Phi_2 = \Phi_4 = o, \Phi_3 = \nu] \tag{1.2.12}$$

The vacuum expectation value (VEV), is accountable for breaking the gauge symmetry $SU(2)_L \times U(1)_Y$ to $U(1)_{em}$. To describe the fluctuations around Φ_0, we can use four fields $\theta_1, \theta_2, \theta_3$ and h(x) as-

$$< \Phi(x) >= \begin{bmatrix} \theta_1 + i\theta_2 \\ \frac{1}{\sqrt{2}}(\nu + h(x)) - i\theta_3 \end{bmatrix} = e^{\frac{i\theta_a \tau_a}{\nu}} \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}}(\nu + h(x)) \end{bmatrix}; \tag{1.2.13}$$

Here we denote h(x) as the physical Higgs field. Now upon considering a $SU(2)_L$ gauge transformation in this field we will get,

$$< \Phi(x) >= \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ \nu + h(x) \end{bmatrix} \tag{1.2.14}$$

The trio of fields θ_1, θ_2, θ_3 represent the three Goldstone Bosons, responsible for providing mass to the trio of weak interaction gauge Bosons, $W^a_\mu(x)$, where 'a' takes values from 1 to 3. The mass expression for these Bosons can be obtained from the Lagrangian-

$$L_{Higgs} = M_W^2 W_\mu^+ W_\mu^- + \frac{1}{2} M_Z^2 Z_\mu Z^\mu + \frac{1}{2} M_h^2 h \tag{1.2.15}$$

Where we denote;

$$W^+ = \frac{W_\mu^1 - iW_\mu^2}{\sqrt{2}}; W^- = \frac{W_\mu^1 + iW_\mu^2}{\sqrt{2}}; Z_\mu = cos\theta_W W_\mu^3 - sin\theta_W B_W \tag{1.2.16}$$

and

$$M_W = \frac{g\nu}{2}; M_Z = \frac{g\nu}{2cos\theta_W}; M_H = 2\nu\sqrt{\lambda} \tag{1.2.17}$$

where θ_W the Weinberg angle [39] can be given as,

$$tan\theta_W = \frac{g}{g'}; \rho = \frac{M_W^2}{M_Z^2 cos^2\theta_W} \tag{1.2.18}$$

The parameter ρ holds a value of 1. This particular mechanism has successfully predicted the precise masses of the gauge bosons, directly linked to the Vacuum Expectation Value (VEV) of the complex scalar field. These predicted masses for the gauge bosons are observed to be: $M_Z = 91.1875 \pm 0.0021$ GeV and $M_W = 80.399 \pm 0.023$ GeV. Additionally, the photon field A_μ is formed as an orthogonal combination of W_μ^3 and B_μ, as expressed by the equation-

$$A_\mu = cos\theta_W W_\mu^3 + sin\theta_W B_\mu \tag{1.2.19}$$

Within the realm of particle physics, there's a gauge boson, the photon, which holds no mass and is linked to the conservation of electric charge. Throughout the entire process, the $U(1)_Q$ symmetry, connected to this photon, remains intact. Although the $SU(2)_L$ and $U(1)_Y$ gauge symmetries undergo spontaneous breaking, the selection of Φ_0 is such that the $U(1)_Q$ symmetry remains unbroken. This preservation of $U(1)_Q$ symmetry is precisely why, in the Standard Model (SM), the photon maintains its masslessness ($m_A = 0$).

Fermion mass production is also a product of the Higgs mechanism. Local gauge invariance controls the dynamics of the interaction between the gauge field and fermions, which is the source of fermion mass. This type of interaction is called the Yukawa interaction. We can write the Yukawa Lagrangian as-

$$L_Y = -Y_d[\overline{Q_L}\Phi d_R] - Y_U[\overline{Q_L}\tilde{\Phi} U_R] - Y_L[\overline{l_L}\Phi l_R] + h.c. \tag{1.2.20}$$

Here, $\Phi = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ \nu + h \end{bmatrix}$; $\tilde{\Phi} = i\tau_2\Phi^\dagger$

Once the scalar field attains its Vacuum Expectation Value (VEV), it leads to the generation of masses for both quarks and charged leptons, as given below-

$$M_u = \frac{Y_u\nu}{\sqrt{2}}; M_d = \frac{Y_d\nu}{\sqrt{2}}; M_l = \frac{Y_l\nu}{\sqrt{2}}; \tag{1.2.21}$$

Here, Y_d, Y_u, and Y_l refer to the Yukawa couplings that match with the down-type quark, up-type quark, and charged lepton respectively. However, within the Standard Model, there isn't a right-handed counterpart for the neutral lepton, which is the neutrino. Consequently, the Higgs mechanism cannot generate the mass of neutrinos.

1.3 Drawbacks Of The Standard Model

While the Standard Model (SM) stands as a highly successful theory in particle physics, capable of elucidating nearly all experimental results, it is deemed incomplete due to several unresolved issues and unexplained phenomena that persist within its framework. The following are some deficiencies or shortcomings within the SM:

• Gravity stands as one of nature's four fundamental forces, yet it's notably the weakest among them. Within the realm of the Standard Model (SM), gravitational interaction remains a challenge to explain and integrate. Without a proper understanding and inclusion of gravity, any theory remains incomplete.

• The fundamental forces' strengths vary enormously in their magnitude. Within the Standard Model (SM), the masses of particles vary significantly, ranging from sub-electronvolts (for neutrinos) to well over a hundred giga-electronvolts (for the top quark). However, the SM fails to provide an explanation for the puzzle of why fermion masses display such a wide range. Furthermore, the model also lacks an explanation for the specific pattern observed in the mixing of quarks.

• Within the Standard Model (SM), numerous free parameters exist, such as quark and charged lepton masses, gauge coupling strengths, characteristics of the scalar potential, mixing angles, CP-violating phases, and more. The model lacks the ability to foresee or calculate the specific values of these independent factors since they are determined solely through experimental measurements. Due to this fact, the Standard Model (SM) cannot be regarded as a complete theory.

• The Standard Model (SM) cannot clarify whether the gauge couplings unify at a high-energy scale, as proposed by the grand unification theory (GUT). Hence, the SM is often described as an effective theory applicable at lower energies, suggesting the existence of another theory at a higher energy scale to complement its limitations.

• The tiny mass of neutrinos, confirmed by various experiments studying neutrino oscillations, cannot be explained within the Standard Model (SM). This is because the SM lacks the presence of right-handed neutrinos, which are necessary to account for the neutrino mass.

• The laws of quantum chromodynamics (QCD) suggest that there should be a strong CP symmetry. However, despite this prediction, there hasn't been any experimental proof of CP symmetry in strong interactions up until now. The issue known as the strong CP problem arises from the puzzle of why QCD appears not to violate CP symmetry. Essentially, there isn't a clear explanation within QCD itself for why it conserves CP symmetry, leading to the idea that this conservation might be an unnatural fine-tuning.

• One of the most intriguing and unresolved mysteries in high-energy physics and cosmology revolves around dark matter and its characteristics. Through measurements in cosmology and astrophysics, it has been established that nearly 27% of the universe consists of dark matter, a type of matter that doesn't emit light and differs from the ordinary matter we're familiar with. Unlike regular matter, dark matter interacts solely through gravitational forces. Additionally, about 68% of our universe is attributed to dark energy. Within the framework of the Standard Model (SM) of particle physics, there isn't a suitable set of particles that can account for or explain this "dark sector." The SM falls short of providing an explanation for the primary constituents of our universe—dark matter and dark energy.

• One more mystery in particle physics is the idea of an imbalance between matter and antimatter, referred to as the baryon asymmetry of the universe (BAU). This imbalance represents an excess of baryons over anti-baryons in the universe. However, within the Standard Model (SM) of particle physics, there isn't a satisfactory explanation for this phenomenon.

- The values of the SM's various tree-level parameters must be consistent. A radiative correction, which is the result of adding higher-order terms to the renormalization process, modifies the gauge couplings and masses. Nevertheless, there is no stable Higgs mass that corresponds to any radiative correction. This is one, of the SM's shortcomings.

- The Standard Model fails to explain whether neutrinos are four-component Dirac particles or two-component Majorana particles. This aspect regarding the nature of neutrinos remains unresolved within the model's framework.

1.4 Beyond Standard Model (BSM) Framework:

1.4.1 Neutrino Mass:

Different BSM frameworks have been put out to tackle various unresolved issues within the SM. One of the main topics of these frameworks is the mass of the neutrino. Neutrino oscillation detection is an experimental method used to prove the presence of neutrino mass, although the absolute scale of neutrino mass is still unknown. We'll briefly delve into the phenomenon of neutrino oscillation, which serves as evidence confirming the existence of realms beyond the Standard Model (BSM).

1.4.2 Neutrino Flavor Oscillation In Vacuum:

Neutrino oscillation, an idea first suggested by Bruno Pontecorvo in 1957 [40], reveals that neutrinos possess mass. There are three distinct kinds of neutrinos and antineutrinos (electron, muon, and tau) involved in weak interactions: charge current (CC) and neutral current (NC) [41]. As neutrinos travel, these three types of neutrino transform from one flavor to another, a phenomenon termed neutrino oscillation. Experimental evidence from solar, atmospheric, and long baseline experiments strongly supports three different scenarios for neutrino behavior. Initially produced as a specific flavor, neutrinos transform into a mix of mass-based states as they travel from their source [42]. Finite lepton flavor mixing and non-degenerate neutrino mass are necessary for neutrino oscillation in the vacuum.

$$\nu_\alpha = \sum_{i=1}^{N} U_{\alpha i}\nu_i \tag{1.4.1}$$

Here ν_α be regarded as flavor eigenstate with $\alpha = e, \mu, \tau$ and ν_i with i=1,2,3 are mass eigenstates.

The relationship between flavor and mass eigenstates of particles is established through a 3x3 rotation matrix as U, which is a unitary matrix. This matrix helps connect and describe how the different ways particles are observed (flavor) correspond to their specific masses (mass eigenstates).

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{PMNS} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \tag{1.4.2}$$

In the three-flavored paradigm, three mixing angles and one CP phase are used to parameterize this PMNS matrix, given by Equation 1.4.3.

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \cdot U_{Maj} \tag{1.4.3}$$

where, $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$ such that $(i, j = 1, 2, 3$ and $i < j)$. The diagonal matrix $U_{Maj} = diag(1, e^{i\alpha}, e^{i\beta})$ contains the Majorana CP phases, α and β, which become observable in case the neutrinos behave as Majorana particles. This mixing matrix can be written as $U_{PMNS} = U_l^\dagger U_\nu$, with U_l representing the matrix that diagonalizes the charged lepton mass and U_ν for the neutrino mass respectively. When a neutrino changes its type from an electron neutrino ν_e to a muon neutrino ν_μ while traveling, consider this scenario in a two-flavor case. We will get the probability amplitude of observing a certain flavor of neutrino after a certain distance L is given by [43],

$$P(\nu_e \rightarrow \nu_\mu) = sin^2(2\theta_{ij})sin^2(\frac{\Delta m_{ij}^2 L}{4E}) \tag{1.4.4}$$

where, $\Delta_{ij}^2 = m_j^2 - m_i^2$ represents the mass squared difference of the neutrinos.

1.4.3 Neutrino Flavor Oscillation In Matter:

Neutrino oscillation within matter is influenced by various factors, resulting in a unique type of oscillation called resonant oscillation. This specific phenomenon, known as the MSW effect, was initially observed and elucidated by Mikheyev, Smirnov, and Wolfenstein (MSW). In this effect, the potential experienced by different neutrino flavors is modified by charged-current interaction, and the effective potential is proportional to the densities of electrons, protons, and neutrons. The contrast in potential among different neutrino flavors is the key factor driving neutrino oscillation within matter. This difference is directly related to the number density of electrons N_e in the medium and can be expressed as-

$$V = \sqrt{2}G_F N_e \tag{1.4.5}$$

Here G_F represents the Fermi constant. The effective mass can be written as,

$$m_{\nu_e}^2 \rightarrow= m_{\nu_e}^2 + A = m_{\nu_e}^2 + \sqrt{2}G_F N_e E \tag{1.4.6}$$

The light neutrino mass can be given as-

$$m_\nu^2 = O^T M_\nu^{Diag} O + \begin{pmatrix} A & 0 \\ 0 & 0 \end{pmatrix} \tag{1.4.7}$$

Here O is an orthogonal matrix and it can be written as-

$$O = \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix} \tag{1.4.8}$$

From above, we get the mass squared matrix represented by,

$$O = \frac{m_0}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} A - \Delta m_{12} cos2\theta & \Delta m_{12} sin2\theta \\ \Delta m_{12} sin2\theta & -A + \Delta m_{12} cos2\theta \end{pmatrix} \tag{1.4.9}$$

Here, $\Delta m_{21}^2 = |m_2^2 - m_1^2|$, $m_0 = m_1^2 m_2^2 - A$ Also, the modified mass eigenvalues are-

$$m_{\nu_{1,2}} = \frac{m_0}{2} \pm \frac{1}{2}\sqrt{(\Delta m_{12} cos2\theta - A)^2 + \Delta m_{12}^2 sin^2 2\theta} \tag{1.4.10}$$

The modified mixing angle can be written as,

$$tan2\theta = \frac{\Delta m_{12} sin2\theta}{\Delta m_{12} cos2\theta - A} \tag{1.4.11}$$

Thus, depending on the electron number density in the presence of matter, the mass eigenvalues and mixing angle change.

So, we can see that the probability expression is dependent on the neutrino energy E, the mass square difference, the mixing angle θ, and the propagation distance L. Therefore, only a non-zero mass squared difference and a finite mixing of flavors corresponding to a non-zero mixing angle can cause neutrino oscillation. Different experiments have yielded somewhat accurate oscillation parameters, but the absolute neutrino mass scale remains unknown.

Furthermore, the subject of neutrino mass ordering remains unresolved. Various oscillation experiments have verified that the solar mass square difference, Δm_{12}^2 is consistently positive, meaning that $m_2 > m_1$. We do not, however, know the sign of the atmospheric mass square difference, or Δm_{23}^2. Because of this, depending on the sign of Δm_{23}^2, there are two alternative orderings or hierarchies of neutrino masses.

- Normal mass hierarchy : $m_3 > m_2 > m_1$
- Inverted mass hierarchy : $m_2 > m_1 > m_3$

1.4.4 Type Of Neutrino Mass: Dirac and Majorana:

The mathematical framework of the Standard Model (SM) reveals that the mass and mixing of neutrinos are determined by a specific term in the equation. This term emerges from the interaction between the left-handed (LH) and right-handed (RH) parts of the particle field. However, because the SM does not include right-handed neutrinos, it cannot generate the mass of neutrinos. In theory, there's a possibility to include right-handed neutrinos in the Standard Model (SM). If this were to happen, it would introduce two different kinds of neutrino mass expressions within the electroweak Lagrangian. These are known as the Dirac and Majorana mass terms. With the Dirac mass term, the conservation of the lepton number remains intact, while the Majorana mass term violates the lepton number by two units. The Dirac mass comes about when the right-handed neutrino coupling with the active lepton following the Higgs field obtains a Vacuum Expectation Value (VEV) of 174 GeV. Now, we can write the Dirac Lagrangian as-

$$-L_{Dirac} = \sum_{i,j} \overline{\nu_{iL}} M_d \nu_{jR} + h.c. \tag{1.4.12}$$

Here M_d is the 3×3 mass matrix. We can write $M_d = Y_\nu m_\nu$, Y_ν is the Yukawa coupling.

To achieve a neutrino mass at sub eV scale, the Yukawa coupling needs to be approximately around 10^{-12}, a value that lacks a natural explanation. This needs a fine-tuning of the theory. Meanwhile, Dirac particles are described by four-component Dirac spinors.

Ettore Majorana suggested that since the SM only contains the LH neutrino, the mass term may be expressed as follows: $\nu_L^C = C\overline{\nu_L^T}$, where C is the charge conjugation matrix. When a particle is its antiparticle, it is referred to as Majorana. Within the Standard Model, among all particles, neutrinos stand out as the only neutral ones that could be identified as Majorana fermions. The Majorana

Lagrangian can be represented as-

$$-L_{Majorana} = -\frac{1}{2} M_R \nu_L \nu_L^C \tag{1.4.13}$$

Because of, the hermitian conjugate (a mathematical operation) of Majorana particles is identical, $\frac{1}{2}$ appears in the Lagrangian equation. The inclusion of this mass term isn't permitted within the Standard Model because it violates the lepton number by $\Delta L = \pm 2$. To explain the existence of neutrino mass, we need to explore realms beyond the Standard Model.

1.4.5 Mechanisms Of Neutrino Mass Generation

Neutrino oscillation observed experimentally suggests that leptonic mixing and neutrino mass lead to novel and fascinating physics in BSM. Numerous frameworks have been put out by theoretical physicists to handle the neutrino mass and mixing. Among these proposals, the development of the seesaw mechanism stands out as a crucial advancement in the realm of Beyond Standard Model (BSM) physics. Different types of seesaw mechanisms such as type-I [44–48], type-II [49–53], type-III [54, 55], inverse seesaw(ISS) [56–58] and radiative seesaw [59–63]. The left-right symmetric model (LRSM) is another significant BSM framework where type-I and type-II seesaws naturally emerge [64–68]. The Grand Unified Theory (GUT) [69, 70], a significant expansion of the Standard Model, provides explanations for neutrino mass and various phenomena.

- **Type-I Seesaw:**

The simplest extension of SM to realize the small neutrino mass through the dimension five operators is the type-I [71, 72] seesaw mechanism. This mechanism involves extending the SM by introducing a gauge singlet fermion, which, through its interaction with lepton doublets and scalar Higgs particles via Yukawa interaction, produces the neutrino mass. We can write the Majorana mass term as-

$$-L_{Type-I} = Y_\nu \overline{N_R} \tilde{\Phi}^\dagger L + \frac{1}{2} M_R \overline{N_R} N_R^C + h.c. \tag{1.4.14}$$

Here, Y_ν is the nonsymmetric and non-hermitian Yukawa matrix and M_R is the RH neutrino mass matrix

After the breaking of electroweak symmetry, the Higgs particle acquires a vacuum expectation value (VEV) and contributes to the Dirac neutrino mass ($M_D = Y_\nu \nu$), with V representing the VEV of the Higgs particle. The mass scale M_R is notably higher than M_D, mainly because the gauge singlet ν_R remains decoupled from the electroweak scale, allowing it at a much higher energy scale. Now, we can write the light neutrino mass matrix as-

$$L_{mass} = \frac{1}{2} \begin{pmatrix} \overline{\nu_L}^C & \overline{N_R} \end{pmatrix} \begin{pmatrix} 0 & m_D^T \\ m_D & M_R \end{pmatrix} \begin{pmatrix} \nu_L \\ N_R \end{pmatrix} \tag{1.4.15}$$

After diagonalization, we can write the light neutrino mass as $M_{light} = M_D^T M_R^{-1} M_D$, while the mass of the heavier neutrinos is represented by $M_{heavy} = M_R$, M_R stands for the matrix describing the masses of the right-handed neutrinos, and m_ν represents the matrix for the masses of the lighter neutrinos. The seesaw mechanism gets its name because it is obvious from the formulation of mass matrices that the heavier the M_R, the lighter the M_ν.

- **Type-II Seesaw:**

Here, the scalar SU(2) triplet is added to the SM in the type-II [50, 51, 73], and the neutrino mass is generated by this scalar triplet. Under the SM gauge group, the Higgs triplet $\Delta = (\Delta_1, \Delta_2, \Delta_3)$ is transformed as (1,3,+1).

We can write the matrix representation of the triplet Higgs as,

$$\Delta = \frac{1}{\sqrt{2}} \sum_i \sigma^i \Delta_i = \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \Delta^0 & -\frac{\Delta^+}{\sqrt{2}} \end{pmatrix} \tag{1.4.16}$$

Here, $\Delta^0 = \frac{\delta_1 + i\delta_2}{\sqrt{2}}$, $\Delta^+ = \delta_3$, $\Delta^{++} = \frac{\delta_1 - i\delta_2}{\sqrt{2}}$ are three complex scalar and σ^i are the Pauli matrices.

Now the Lagrangian for type-II seesaw can be written as,

$$-L_{Type-II} = Y_\Delta L^T Ci\sigma_2 \Delta L + M_\Delta^2 Tr[\Delta^+\Delta] + \frac{1}{2}(\lambda_\Delta M_\Delta)\tilde{\Phi}^\dagger \Delta^+ \Phi + h.c. \quad (1.4.17)$$

In the above equation, Y_Δ is the Yukawa coupling and M_Δ is the mass of triplet Higgs. When the neutral component of the triplet Higgs gains VEV through the breakdown of electroweak symmetry, the neutrino mass is generated. The VEV can be written as follows- $< \Delta >= \nu_\Delta = \frac{\lambda_\Delta \nu^2}{M_\Delta}$.

So, the neutrino mass generated in type-II seesaw mechanism is ,

$$M_\nu = \frac{Y_\Delta \nu_\Delta}{\sqrt{2}} \quad (1.4.18)$$

- **Type-III Seesaw:**

To produce the neutrino mass in a type-III seesaw, three extra hyper chargeless color singlet fermion triplets are added with SM [74, 75]. The SU(2) representation of the triplet $\Sigma = (\eta_1, \eta_2, \eta_3)$ can be given as,

$$\Sigma = \frac{1}{\sqrt{2}} \sum_i \sigma^i \Delta_i = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} & \Sigma^+ \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} \end{pmatrix} \quad (1.4.19)$$

Here, $\Sigma^0 = \eta_3$ and $\Sigma^\pm = \frac{\eta_1 + i\eta_2}{\sqrt{2}}$. Neutrino mass is generated in Type-III is as follows:

$$-L_{TypeIII} = Y_\Sigma \tilde{\Phi}^\dagger \Sigma^a L + \frac{1}{2} M_\Sigma Tr[\Sigma^a \Sigma^b] + h.c. \quad (1.4.20)$$

In the above equation, Y_Σ is the dimensionless Yukawa coupling matrix and M_Σ is the mass of triplet fermion. In this framework, the light mass of neutrinos is determined by-

$$-m_\nu = m_D M_\Sigma m_D^T \quad (1.4.21)$$

Here, $m_D = \frac{Y_\Sigma \nu}{\sqrt{2}}$. Since the triplet fermion does not process symmetry of any kind, its mass can approach the theory's cut-off scale.

• **Inverse seesaw:**

Here, the Standard Model is expanded by introducing one or more generations of right-handed neutrinos ν_R along with a singlet fermion (S) [57, 58]. We can write the Lagrangian for the Inverse seesaw as,

$$L = -\frac{1}{2} n_L^T C M n_L + h.c, \tag{1.4.22}$$

The Lagrangian results in the subsequent formation of a mass matrix-

$$M = \begin{pmatrix} 0 & M_d & 0 \\ M_d^T & 0 & M_R \\ 0 & M_T^R & \mu \end{pmatrix} \tag{1.4.23}$$

Here, μ, M_d and M_N are complex matrices. Following the block diagonalization of the 9x9 matrix under the assumption that $\mu < M_D < M_R$, we will obtain the final light neutrino mass as follows:

$$M_\nu = M_d^T (M_R^T)^{-1} \mu M_R^{-1} M_d \tag{1.4.24}$$

It can be observed from the above equation that, the neutrino mass in this one generates from the double suppression of the RH neutrino mass.

Due to this reason, the inverse seesaw is recognized as a low-scale seesaw. Within this specific type of seesaw, the origin of neutrino mass can be beautifully and simply explained.

• **Left-Right Symmetric Model (LRSM):**

The LRSM (Left-Right Symmetric Model) extends the standard model's gauge group straightforwardly. It achieves the restoration of parity at a high energy level and organizes fermions into the $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge group, which is testable in current experiments [64, 76, 77]. In the framework of the Left-Right Symmetric Model (LRSM), the common type-I and type-II seesaw mechanisms emerge naturally. Additionally, within LRSM, various other issues such as the parity violation of weak interaction, neutrinos massless, CP (charge-parity) problems, and hierarchy issues can also be resolved. In this model, the source of the electric charge is expressed by the equation: $Q = T_{3L} + T_{3R} + \frac{B-L}{2}$ [78]. Here, T_{3L} and T_{3R} represent the generators of $SU(2)_L$ and $SU(2)_R$, while B-L denotes the operator for the baryon minus lepton number charge. The quarks and leptons (left-handed and right-handed) that transform within the Left-Right symmetric gauge group are described as follows:

$$Q_L = \begin{bmatrix} u \\ d \end{bmatrix}_L, Q_R = \begin{bmatrix} u \\ d \end{bmatrix}_R, \Psi_L = \begin{bmatrix} \nu_l \\ l \end{bmatrix}_L, \Psi_R = \begin{bmatrix} \nu_l \\ l \end{bmatrix}_R \tag{1.4.25}$$

Here, under $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, the quarks are assigned quantum numbers $(3, 2, 1, \frac{1}{3})$ and $(3, 1, 2, \frac{1}{3})$, while the leptons are assigned quantum numbers $(1, 2, 1, -1)$ and $(1, 1, 2, -1)$ respectively.

In the Left-Right Symmetric Model (LRSM), the Higgs part comprises a bi-doublet characterized by the quantum numbers $\Phi(1, 2, 2, 0)$ and $SU(2)_{L,R}$ triplets, $\Delta_L(1, 2, 1, -1)$ and $\Delta_R(1, 1, 2, -1)$.

The equation that describes the Yukawa Lagrangian in the sector involving leptons is as follows-

$$L = h_{ij} \overline{\Psi}_{L,i} \Phi \Psi_{R,j} + \tilde{h}_{i,j} \overline{\Psi}_{L,i} \tilde{\Phi} \Psi_{R,j} + f_{L,ij} \Psi^T_{L,i} C i\sigma_2 \Delta_L \Psi_{L,j} + f_{R,ij} \Psi^T_{R,i} C i\sigma_2 \Delta_R \Psi_{R,j} + h.c. \tag{1.4.26}$$

Here, the indices i and j, which are summed over, refer to the different members within a family. These indices, specifically ranging from 1 to 3 as i and j = 1, 2, 3, correspond to the three distinct generations of fermions. Here, $\tilde{\Phi} = \tau_2 \Phi^* \tau_2$ and γ_μ are the Dirac matrices and $C = i\gamma_2\gamma_0$ is the charge conjugation operator. The Majorana Yukawa couplings $f_L = F_R$ (for left-right symmetry) give rise to Majorana neutrino mass after electroweak symmetry breakdown when the scalar triplets Δ_L and Δ_R gain non-zero VEV, leading to a 6×6 neutrino mass matrix, which is provided as follows when discrete parity symmetry is taken into consideration-

$$M_\nu = \begin{bmatrix} M_{LL} & M_D \\ M_D^T & M_{RR} \end{bmatrix} \tag{1.4.27}$$

Here,

$$M_D = \frac{1}{\sqrt{2}}(k_1 h + k_2 \tilde{h}), M_{LL} = \sqrt{2}\nu_L f_L, M_{RR} = \sqrt{2}\nu_R f_R \tag{1.4.28}$$

where, M_D, M_{LL} and M_{RR} are the Dirac neutrino mass matrix, left-handed and right-handed mass matrix respectively. Now assuming $M_L << M_D << M_R$, the light neutrino mass, generated within a type I+II seesaw can be written as,

$$M_\nu = M_\nu^1 + M_\nu^{II} \tag{1.4.29}$$

,

$$M_\nu = M_{LL} + M_D M_{RR}^{-1} M_D^T = \sqrt{2}\nu_L f_L + \frac{K^2}{\sqrt{2}\nu_R} h_d f_R^{-1} h_D^T \tag{1.4.30}$$

Here the first and second term in the above equation corresponds to type-II seesaw and type-I seesaw mediated by RH neutrino respectively. Here,

$$h_D = \frac{(k_1 h + k_2 \tilde{h})}{\sqrt{2}k}, k = \sqrt{|k_1|^2 + |k_2|^2} \tag{1.4.31}$$

Within the framework of the Left-Right Symmetric Model (LRSM), both the type I and type II seesaw mechanisms can be expressed using the term M_{RR}, which naturally emerges at a high-energy scale due to the spontaneous breaking of parity. In LRSM, the Majorana Yukawa couplings, represented by f_L and f_R, are equal (i.e., $f_L = f_R$), and the vacuum expectation value (VEV) for the left-handed triplet ν_L can be formulated as follows:

$$\nu_L = \frac{\gamma M_L^2}{\nu_R} \tag{1.4.32}$$

So, the above equation can be written as,

$$M_\nu = \gamma(\frac{M_W}{\nu_R})^2 M_{RR} + M_D M_{RR}^{-1} M_D^T \quad (1.4.33)$$

We can write the dimensionless parameter γ as-

$$\gamma = \frac{\beta_1 k_1 k_2 + \beta_2 k_1^2 + \beta_3 k_2^2}{(2\rho_1 - \rho_3)k^2} \quad (1.4.34)$$

In this context, the terms β and ρ represent dimensionless parameters found in the equation defining the Higgs potential.

• **Lepton number violation: Neutrinoless Double Beta Decay:**

The true nature of neutrinos still remains a mystery—whether they exist as Dirac particles or Majorana particles. Majorana particles are unique as they are their own antiparticles, causing a breach in lepton number symmetry. On the other hand, Dirac particles conserve the lepton number. However, distinguishing between these two states cannot be accomplished through neutrino oscillation experiments since they lack sensitivity to Majorana parameters. To uncover lepton number violation (LNV), we need alternative and more sensitive experiments. Among these, the most prominent method to confirm lepton number violation is through the neutrinoless double beta decay (NDBD) experiment.

The Neutrinoless Double Beta Decay (NDBD) [79–81] is a sluggish radioactive process of second order, initially proposed by Wendell H Furry in 1939 and can be written as-

$$N(A, Z) \rightarrow N(A, Z + 2) + 2e^- \quad (1.4.35)$$

This situation involves what could be seen as a double beta decay, which is only feasible for atomic nuclei with both even proton and even neutron numbers. During this process, a nucleus with a specific proton count Z and mass number A undergoes decay by releasing two electrons and two antineutrinos, transitioning into a nucleus with a higher proton count of Z + 2 and the same mass number

A. If we observe Neutrinoless Double Beta Decay (NDBD), it will undoubtedly confirm that there's a violation in lepton number and that neutrinos possess Majorana characteristics by their very nature. The effective Majorana mass $m_{\beta\beta}$ can be written as-

$$m_{\beta\beta} = m_1 c_{12}^2 c_{13}^2 + m_2 S_{12}^2 c_{13}^2 e^{2i\alpha} + m_3 S_{13}^2 e^{2i\beta} \tag{1.4.36}$$

Here, $C_{ij} = cos\theta(ij)$, $S_{ij} = sin\theta(ij)$ are respective oscillation angles and α and β are the Majorana phase.

Up to now, the NDBD (Neutrinoless Double Beta Decay) process hasn't been observed experimentally. Various dedicated experiments such as KamLAND-Zen [82], GERDA [83], NEMO-3 [84], EXO-3 [85], CUORE-3 [86], Legend-1K [87], and MAJORANA [88] have been conducted specifically to detect this particular decay.

The range of effective Majorana neutrino mass is supported by experimentally allowed is-

$$m_{\beta\beta} < (0.061 < 0.165)eV \tag{1.4.37}$$

The Neutrinoless Double Beta Decay process holds significant importance in both theoretical and experimental Beyond Standard Model (BSM) physics. While the actual detection of this decay remains a distant goal, the NDBD process, which violates lepton numbers, stands as a key motivation driving research in BSM physics. In the near future, experiments such as KATRIN are anticipated to measure the neutrino's mass, which would mark a remarkable achievement in this field.

- **Baryon Asymmetry of the Universe(BAU)**

The imbalance between matter and antimatter, referred to as matter-antimatter asymmetry or baryon asymmetry of the Universe (BAU), stands as a significant subject in particle physics. The prevalence of matter surpassing antimatter also suggests the potential existence of physics beyond the Standard Model. In the aftermath of the Big Bang, when the universe was extremely hot and dense, it was expected to generate equal amounts of particle-antiparticle pairs due to an abundance of radiation. However, observations revealed an imbalance, indicating an excess of particles compared to antiparticles, crucial for the existence of the universe. This lingering asymmetry remains a vital and unsolved astrophysical mystery to this day. The information gathered from baryon acoustic oscillation, WMAP (Wilkinson Microwave Anisotropy Probe), and Planck data has led to the determination of the baryon to photon ratio in terms of number density [89]is-

$$\eta_B = \frac{n_B - \overline{n}_B}{n_\gamma} = (6.1 \pm 0.18) \times 10^{10} \tag{1.4.38}$$

Here η_B, $\overline{n}_B$ and η_γ are the baryon number density, anti-baryon number density, and photon density.

The above expression in terms of entropy can be written as-

$$Y_B = \frac{\eta_B - \eta_{\overline{B}}}{S} = (8.75 \pm 0.23) \times 10^{11} \tag{1.4.39}$$

These findings are strongly backed by the theories of Big Bang nucleosynthesis (BBN). Several theories suggest that neutrinos might play a crucial role in explaining this asymmetry. Sakharov proposed [90] the most fundamental condition for the creation of the universe's baryon asymmetry (BAU) many years ago.

• **Baryon Number Violation:** The baryon number (B) needs to be violated simply as the Baryon Asymmetry of the Universe (BAU) cannot be generated if the baryon number remains conserved.

• **Charge(C) and Charge-Parity (CP) violation:** To create the Baryon Asymmetry of the Universe (BAU), there's a necessity for the violation of C and CP symmetries. This violation means that the production of left-handed particles can't perfectly balance with the production of their corresponding right-handed antiparticles (which are the CP-conjugated state of the left-handed particles) in any given process. CP violation ensures this imbalance by ensuring that the numbers aren't equal. Meanwhile, C violation is necessary to prevent the generation of right-handed particles from compensating for the generation of left-handed ones in these processes.

• **Departure From Thermal Equilibrium:** Only when the rate at which the baryon number violating process is slower than the universe's expansion is this departure from thermal equilibrium realized. Stated differently, the heavy particle will separate into its constituent sub-products before it can participate in the inverse decay of the same process. It is a prerequisite for generating the BAU.

Although the SM has the previously stated Sakharov requirements, they are insufficient to account for the observed BAU. So, to explain the observed BAU, we need new physics beyond SM. Numerous processes, including leptogenesis, electroweak baryogenesis, GUT baryogenesis, the Affleck-Dine mechanism, and others, have been proposed to explain the BAU.

1.4.6 Leptogenesis:

Physicists have proposed several intriguing theoretical models to explain the observed imbalance between matter and antimatter. One popular mechanism, called leptogenesis, explains this asymmetry. In leptogenesis, the imbalance in leptons is created prior to the electroweak phase transition. Subsequently, this lepton asymmetry is transformed into a Baryon Asymmetry of the Universe (BAU) through a process called sphaleron, which violates the combined quantity of baryon number (B) and lepton number (L). The sphaleron process changes any initial lepton or B-L asymmetry into a baryon asymmetry. The source of the baryon asymmetry can be linked to leptons, including neutrinos. Heavy right-handed neutrinos, proposed as counterparts to the light neutrinos that account for neutrino mass, are believed to have existed during the early stages of the universe.

The decay of this massive neutrino could potentially generate more matter than antimatter. Initially proposed by Fukugita and Yanagida, this concept of leptogenesis involves the decay of a heavy neutrino with a Majorana mass, occurring at temperatures higher than the critical temperature T_C= 100-200 GeV, which sets the stage for the creation of matter-antimatter asymmetry (BAU) as defined by Sakharov. This type of decay fulfills all the necessary conditions outlined by

Sakharov for generating baryon asymmetry in the Universe. The combination of both tree-level and one-loop-level diagrams results in crucial CP asymmetry. Additionally, the Yukawa interactions, which are responsible for these processes, take place relatively slowly at temperatures above the electroweak scale, causing a departure from thermal equilibrium.

A heavy neutrino breaks down into a lepton and a Higgs doublet, expressed as $N_i \to L+\Phi^C$. The CP conjugate process, $N_i \to L^C+\Phi$, also takes place both at the tree level and the loop level, leading to a violation of the lepton number. The CP asymmetry parameter ϵ_{CP} originates from the interaction between the tree level and loop level decay amplitudes, involving self-correction, and is mathematically defined as follows:

$$\epsilon_{CP} = \frac{\Gamma(N_i \to L + \Phi^C) - \Gamma(N_i \to L^C + \Phi)}{\Gamma(N_i \to L + \Phi^C) + \Gamma(N_i \to L^C + \Phi)} \tag{1.4.40}$$

The mass range of the right-handed neutrinos varies depending on the specific framework being studied. In the left-right symmetric model, it falls within the TeV scale, while in the radiative seesaw model, it's around the GeV scale. Additionally, in certain Grand Unified Theory (GUT) concepts, the mass extends up to 10^{16} GeV. Several forms of leptogenesis exist, including thermal leptogenesis, resonant leptogenesis, vanilla leptogenesis, and others. These various types have the potential to generate the observed baryon asymmetry in the universe.

1.4.7 Lepton Flavor Violation :

The occurrence of Charged Lepton Flavor Violation (CLFV) implies the presence of new physics beyond the Standard Model (SM). The discovery of charged lepton muons dates back to the early 1940s, initiating a continuous quest for CLFV. Even in this era of precise experiments, the search for CLFV persists as it could potentially lead us toward understanding Beyond Standard Model (BSM) physics. The experimental confirmation of neutrino oscillation has already demonstrated instances of lepton flavor violation while neutrinos propagate, solidifying the evidence for the substantial mass of neutrinos. This realization suggests the likelihood of lepton flavor violation extending into the charged lepton sector. Although the exact mechanism behind LFV remains unknown, this area of study is interconnected with phenomena like neutrino mass, CP violation, and the exploration of new physics beyond the Standard Model.

Numerous ongoing and upcoming experiments are specifically focused on investigating processes that violate lepton flavor, such as two-body decay ($l_\alpha \to l_\beta \gamma$) [91] and three-body decay ($l_\alpha \to 3l_\beta$) [92]. Among these, a thorough analysis is being conducted on various muon decay channels like $\mu - e$, N, $\mu \to eee$, $\mu \to e\gamma$ and $\mu^- e^- \to e^- e^-$, as muon decay experiments hold significant prominence [93–95].

1.4.8 Dark Matter:

The two basic concerns of contemporary cosmology that remain unsolved in the SM are the nature and origin of dark matter and matter-antimatter asymmetry. The existence of dark matter is firmly rooted in experimental observations, pointing directly toward realms beyond the Standard Model. Fritz Zwicky [96]initially proposed the concept of dark matter in 1933, and since then, numerous observations have confirmed the existence of this enigmatic, non-baryonic, and non-luminous form of matter.

• **Galaxy Cluster:** These are extremely large entities that hold a vast quantity of gas within the intergalactic space. The initial evidence supporting Dark Matter (DM) comes from observing the velocity dispersion of the Coma Cluster [97]. Typically, when considering only the visible matter, scientists expect the velocity dispersion of this cluster to be around 80 km/sec. However, the measured velocity dispersion is approximately 2000 km/sec, revealing a significant difference. This substantial mismatch strongly suggests the existence of non-luminous matter, contributing to the gravitational effects observed in the cluster.

• **Galaxy Rotation Curve:**

The rotational curves of spiral galaxies provide one of the most remarkable and important findings of the peculiar gravitational effects of dark matter [98].

Nearly all of the observable mass in these kinds of galaxies is concentrated in the disc and the bulge. The rotating velocity v of galaxies under gravitational pull at a distance R from the galactic center can be obtained from classical theory as-

$$v(R) = \sqrt{\frac{G_N M(R)}{R}} \tag{1.4.41}$$

Here, M(R) represents the quantity of mass that exists within the galaxy at a given distance or radius, R. The gravitational constant is abbreviated GN. It is evident from the preceding calculation that rotational velocity and galaxy radius are inversely related. A consistent rotational velocity (v = 200 km/sec) was found in the outer luminosity area of the galaxies studied. The explanation for this can only be found if we suppose that $M(R) \propto R$ in the outer luminosity area, indicates the existence of dark matter.

• **Gravitational Lensing:** One method of indirect detection of DM is gravitational lensing [99]. According to the theory of relativity, light from a far-off source will bend when it comes into contact with a heavy object. This huge gravitational object functions similarly to a lens. DM was found to be present when various lensing patterns of various gravitational objects were analyzed. The existence of DM was confirmed by the matter distribution of the bullet cluster, which is nothing more than a subcluster of two merging galaxies, as determined by lensing. The presence of collisionless particles is confirmed by weak lensing of the mass profile of the merging process of two galaxies, which indirectly suggests the existence of dark matter.

• **Cosmic Microwave Background (CMB):**

The Cosmic Microwave Background (CMB) essentially contains details about the condition of the universe during the Big Bang [100]. This cosmic radiation offers crucial proof supporting the existence of Dark Matter (DM). Moreover, the CMB allows measurement of the overall matter density within the universe and its sensitivity extends to discerning the ratio between the density of ordinary matter and non-ordinary matter. Recent findings from experiments conducted by the Planck collaboration reveal that 26.8% of the universe's energy density is attributed to Dark Matter, while the relic density is expressed as:

$$\Omega_m h^2 = 0.1187 \pm 0.0017 \tag{1.4.42}$$

Even with these observations, the true characteristics and behavior of Dark Matter (DM) continue to puzzle the field of particle physics. Several literature works explore the criteria that any particle must meet to qualify as a potential DM candidate [101]. The Standard Model (SM) lacks a suitable DM candidate, as neutrinos are too small to meet the DM abundance. Consequently, numerous Beyond Standard Model (BSM) frameworks have been suggested to study DM further.

1.4.9 Discrete Flavor Symmetry In Particle Physics:

Symmetry plays a crucial role in particle physics by explaining how particles interact and the fundamental forces they exhibit. Noether's theory illustrates how symmetry leads to conservation laws in particle physics. Sometimes, broken symmetry is also significant, as seen in phenomena like spontaneous electroweak symmetry breaking and the Higgs mechanism discussed earlier. Continuous symmetries like Lorentz, Poincare, and gauge symmetry help us comprehend many occurrences in strong, weak, and electromagnetic interactions. Discrete symmetries such as Charge conjugation (C), Parity (P), and Time reversal (T) are crucial for accurately describing particle physics. Mathematically, symmetry is understood through group theory. A set of symmetry transformations maintaining the properties of a specific group is known as a symmetry group. These symmetry groups, also called Lie groups like SU(n), U(n) and O(n), are extensively utilized in theoretical particle physics.

In the preceding section, we explored the Standard Model (SM), which explains how elementary particles acquire mass through the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. These groups consist of both non-abelian $SU(3)_C$, $SU(2)_L$ and abelian $U(1)_Y$ continuous groups. However, the SM fails to account for neutrino mass and mixing. As experimental evidence confirms neutrino mass and mixing, the significance of studying flavor symmetry has grown. Observations indicate that the lepton sector displays less hierarchy and features substantial mixing compared to the quark sector. By incorporating flavor symmetry into an extended Standard Model through a corresponding non-abelian finite discrete group, we can potentially explain the neutrino mass and the observed mixing.

The structure of flavors in a specific model can be managed through discrete flavor symmetry [102, 103]. This type of symmetry finds extensive application in particle physics due to its capability. Symmetries like A_N, S_N, Δ_{27} and Z_N are among the discrete symmetries utilized in constructing models to explain various aspects of neutrino phenomenology, including their mass and mixing. Incorporating additional discrete symmetry aims to thoroughly study the flavor structure of a theory, ultimately improving the model's predictability. These symmetries are presumed to originate at high energy scales, breaking down into charged lepton and quark symmetries through various flavors as they move to lower energy levels. Flavors, introduced as scalar particles within a model, play a pivotal role in breaking these discrete symmetries. Several models have been proposed using flavor symmetries like A_4, S_4, Δ_{27}, Z_2, Z_3, Z_5, etc [104–110] to replicate the observed neutrino mixing consistent with experimental data.

Throughout this thesis, we have extensively applied the A_4 and Δ_{27} discrete flavor symmetry in relation to the Altarelli-Feruglio (A-F), A_4 discrete flavor symmetry model, $\Delta 27$ discrete flavor symmetry model, and Type-I seesaw mechanism. In the upcoming section, we will delve into discussing the characteristics and traits of the A_4 and Δ_{27} discrete symmetry group.

1.4.10 Properties Of A_4 Group:

The non-Abelian discrete symmetry group A_4 is a group of even permutations of four objects and it has 12 elements ($12= \frac{4!}{2}$). It can describe the orientation-preserving symmetry of a regular tetrahedron, so this group is also known as a tetrahedron group. It can be generated by two basic permutations S and T having properties $S^2 = T^3 = (ST)^3 = 1$. This group representations include three one-dimensional unitary representations 1, $1'$, $1''$ with the generators S and T given, respectively as follows:

$$1 : S = 1, T = 1$$

$$1' : S = 1, T = \omega^2$$

$$1'' : S = 1, T = \omega$$

and a three-dimensional unitary representation with the generators[1]

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \tag{1.4.43}$$

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \tag{1.4.44}$$

. Here, ω is the cubic root of unity, $\omega = exp(i2\pi)$, so that $1 + \omega + \omega^2 = 0$. The multiplication rules corresponding to the specific basis of two generators S and T are as follows:

$$1 \times 1 = 1$$

$$1'' \times 1' = 1$$

$$1' \times \ 1'' = 1$$

$$3 \times 3 = 3 + 3_A + 1 + 1' + 1''$$

For two triplets

$$a = (a_1, a_2, a_3)$$

[1]Here the generator T has been chosen to be diagonal.

$$b = (b_1, b_2, b_3)$$

we can write

$$1 \equiv (ab) = a_1b_1 + a_2b_3 + a_3b_2$$

$$1' \equiv (ab)' = a_3b_3 + a_1b_2 + a_2b_1$$

$$1'' \equiv (ab)'' = a_2b_2 + a_1b_3 + a_3b_1$$

Here, 1 is symmetric under the exchange of second and third elements of a and b, $1'$ is symmetric under the exchange of the first and second elements while $1''$ is symmetric under the exchange of first and third elements.

$$3 \equiv (ab)_S = \frac{1}{3}(2a_1b_1 - a_2b_3 - a_3b_2, 2a_3b_3 - a_1b_2 - a_2b_1, 2a_2b_2 - a_1b_3 - a_3b_1)$$

$$3_A \equiv (ab)_A = \frac{1}{3}(a_2b_3 - a_3b_2, a_1b_2 - a_2b_1, a_3b_1 - a_1b_3)$$

Here 3 is symmetric and 3_A is anti-symmetric. For the symmetric case, we notice that the first element here has 2-3 exchange symmetry, the second element has 1-2 exchange symmetry and the third element has 1-3 exchange symmetry.

1.4.11 Properties of $\Delta(27)$ Group:

$\Delta(27)$ is the simplest non-trivial discrete symmetry group of $\Delta(3N^2)$. The $\Delta(27)$ group has nine singlets $1_{r,s}(r, s = 0, 1, 2)$ and two triplets, $3_{[0][1]}$ and $3_{[0][2]}$. Tensor products between triplets are obtained as-

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}_{3[0][1]} \otimes \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}_{3[0][1]} = \begin{pmatrix} x_1y_1 \\ x_2y_2 \\ x_3y_3 \end{pmatrix}_{3[0][2]} \oplus \begin{pmatrix} x_3y_1 \\ x_1y_2 \\ x_2y_3 \end{pmatrix}_{3[0][2]} \oplus \begin{pmatrix} x_1y_3 \\ x_2y_1 \\ x_3y_2 \end{pmatrix}_{3[0][2]} \quad (1.4.45)$$

$$\begin{pmatrix}\overline{x_1}\\ \overline{x_2}\\ \overline{x_3}\end{pmatrix}_{3[0][2]} \otimes \begin{pmatrix}\overline{y_1}\\ \overline{y_2}\\ \overline{y_3}\end{pmatrix}_{3[0][2]} = \begin{pmatrix}\overline{x_1}.\overline{y_1}\\ \overline{x_2}.\overline{y_2}\\ \overline{x_3}.\overline{y_3}\end{pmatrix}_{3[0][1]} \oplus \begin{pmatrix}\overline{x_3}.\overline{y_1}\\ \overline{x_1}.\overline{y_2}\\ \overline{x_2}.\overline{y_3}\end{pmatrix}_{3[0][1]} \oplus \begin{pmatrix}\overline{x_1}.\overline{y_3}\\ \overline{x_2}.\overline{y_1}\\ \overline{x_3}.\overline{y_2}\end{pmatrix}_{3[0][1]} \tag{1.4.46}$$

$$\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}_{3[0][1]} \otimes \begin{pmatrix}y_1\\ y_2\\ y_3\end{pmatrix}_{3[0][2]} = \sum_r \left(x_1.y_1 + \omega^{2r}x_2y_2 + \omega^r x_3y_3\right)_{1_{r,0}} \oplus \\ \sum_r \left(x_1.y_2 + \omega^{2r}x_2y_3 + \omega^r x_3y_1\right)_{1_{r,1}} \oplus \sum_r \left(x_1.y_3 + \omega^{2r}x_2y_1 + \omega^r x_3y_2\right)_{1_{r,2}} \oplus \tag{1.4.47}$$

The tensor products between singlets and triplets are obtained as-

$$\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}_{3[0][1]} \otimes (Z_{r,s})_{1_{r,s}} = \begin{pmatrix}x_1Z_{r,s}\\ \omega^r x_2Z_{r,s}\\ \omega^{2r}x_3Z_{r,s}\end{pmatrix}_{3[s][s+1]} \tag{1.4.48}$$

$$\begin{pmatrix}y_1\\ y_2\\ y_3\end{pmatrix}_{3[0][2]} \otimes (Z_{r,s})_{1_{r,s}} = \begin{pmatrix}y_1Z_{r,s}\\ \omega^r y_2Z_{r,s}\\ \omega^{2r}y_3Z_{r,s}\end{pmatrix}_{3[s][s+2]} \tag{1.4.49}$$

The tensor products of singlets $1_{k,l}$ and $1_{k',l'}$ are obtained as-

$$1_{k,l} \otimes 1_{k',l'} = 1_{k+k',l+l'} \tag{1.4.50}$$

1.5 Thesis Outline

In chapter 1, we discuss the latest developments in experimental and theoretical neutrino physics-related research. We discuss about Standard Model of particle physics as well as the shortcomings of the model that demand its expansion.

Next, we discuss the oscillations of neutrino flavor in matter and vacuum. Lepton number violation and lepton flavor violation are two low-energy processes that we discuss here. This chapter has also covered two significant cosmological issues that require BSM frameworks: dark matter and baryon asymmetry of the universe. We discuss about the significance of discrete flavor symmetry in particle physics. We review different mechanisms of generating neutrino mass focusing on Altarelli-Feruglio A_4 discrete flavor symmetry model, Type-I seesaw mechanism, A_4 and $\Delta(27)$ discrete symmetry group, Neutrinoless Double Beta Decay as the thesis work is based on these frameworks.

In chapter 2, we study the modification of the Altarelli-Feruglio A_4 flavor symmetry model by adding three singlet flavons ξ', ξ'' and ρ and the model [104] is augmented with extra cyclic symmetry $Z_2 \times Z_3$ to prevent the unwanted terms in our study. The addition of these three flavors leads to two higher order corrections in the form of two perturbation parameters ϵ and ϵ'. These corrections yield the deviation from the exact tri-bimaximal (TBM) neutrino mixing pattern by producing a non-zero θ_{13} and other neutrino oscillation parameters which are consistent with the latest experimental data. In both corrections, the neutrino masses are generated via Weinberg operator. The analysis of the perturbation parameters ϵ and ϵ', shows that normal hierarchy (NH) and inverted hierarchy (IH) for ϵ does not change much. However, as the values of ϵ' increase, θ_{23} occupies the lower octant for NH case. We further investigate the neutrinoless double beta decay parameter $m_{\beta\beta}$ using the parameter space of the model for both normal and inverted hierarchies of neutrino masses.

In chapter 3, we study a neutrino mass model [105] with A_4 discrete flavor symmetry using a type-I seesaw mechanism. The inclusion of extra flavons in our model leads to deviations from the exact tribimaximal mixing pattern resulting in a nonzero θ_{13} consistent with the recent experimental results and a sum rule for light neutrino masses is also obtained. In this framework, a connection is established among the neutrino mixing angles- reactor mixing angle(θ_{13}), solar mixing angle(θ_{12}), and atmospheric mixing angle (θ_{23}). This model also allows us a predict of Dirac CP-phase and Jarlskog parameter J. The octant of the atmospheric mixing angle θ_{23} occupies the lower octant. Our model prefers normal hierarchy (NH) to inverted hierarchy (IH). We use the parameter space of our model of neutrino masses to study the neutrinoless double beta decay parameter m_{ee}.

In chapter 4, we present a neutrino mixing model based on the discrete flavor symmetry group $\Delta(27)$ and supplemented by other cyclic symmetries along with the seesaw mechanism to explain the observation of a non-zero reactor mixing angle θ_{13}. This kind of mass matrix easily produces mixing patterns that realistically deviate from tribimaximal mixing, including mixing patterns with non-zero θ_{13}. It explains the hierarchies of the charged leptons. Our model allows us to determine the Dirac CP violation phase as a function of the mixing angles θ_{12}, θ_{23}, θ_{13}. Both the normal ordering and the inverse ordering of the neutrino masses are quite close to the global fits of the experimental data.

Finally, **chapter 5** presents the summary and conclusion of the thesis work. We have also discussed about the future scope of the thesis in this chapter.

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