Chapter 3

Neutrino Mixing Phenomenology: A_4 Discrete Flavor Symmetry with Type-I Seesaw Mechanism

3.1 Introduction

The discovery of neutrino oscillations has triggered a lot of theoretical and experimental effort to understand the physics of lepton masses and mixing. As flavor mixing happens due to the mismatch between the mass and flavor eigenstates, so neutrinos need to have small non-degenerate masses [1–3] Over the last twenty-five years, numerous experiments on neutrino oscillation have taken place, resulting in the precise determination of oscillation parameters [4–6]. The discovery of neutrino oscillations in 1998 by Japanese Super-Kamiokande (SK) collaborators and Canadian Sudbury Neutrino Observatory collaborators was the first evidence of physics beyond the Standard Model. A few latest reviews on neutrino physics are placed in references [7–30].

The main factors influencing neutrino oscillation probabilities are the mass-squared differences and the mixing angles. Therefore, these parameters are determined in neutrino oscillation experiments. The experimental data has shown two large mixing angles, one is atmospheric mixing angle θ_{23} and another is solar mixing angle θ_{12} and one small mixing angle, called reactor mixing angle θ_{13}. This pattern differs from quark mixing where all angles are small and the mixing matrix is close to the identity.

The tribimaximal (TBM) mixing pattern is one of the most extensively used lepton mixing patterns obtained utilizing discrete non-Abelian symmetries.

$$U_{TBM} = \begin{pmatrix} -\frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \tag{3.1}$$

But TBM has been ruled out due to a non-zero reactor mixing angle, [31, 32]. One of the admired ways to achieve realistic mixing is through either its extensions or through modifications. In the concept of tribimaximal mixing (TBM), the angle θ_{13}, which represents the degree of mixing in a reactor, is equal to zero and the CP phase δ_{CP}, which characterizes the violation of symmetry between matter and antimatter, cannot be determined or has no specific value. Albeit in 2012 the Daya Bay Reactor Neutrino Experiment ($\sin^2 2\theta_{13} = 0.089 \pm 0.010 \pm 0.005$) [33] and RENO Experiment $\sin^2 2\theta_{13} = 0.113 \pm 0.013 \pm 0.019$ [31] showed that $\theta_{13} \simeq 9°$. Moreover, several neutrino oscillation experiments like MINOS [34], Double Chooz [32], T2K [35], measured consistent non-zero values for θ_{13}. Other mixing angle values also show small deviations the TBM value.

The experiments investigating neutrino oscillation have discovered two mass-squared differences that vary significantly in their scales. The smaller mass squared differences, denoted $\Delta m^2_{21} = m^2_2 - m^2_1$, is positive and is of the order of $10^{-5} eV^2$ and the larger mass-squared difference, $\Delta m^2_{31} = m^2_3 - m^2_1$, is of order $10^{-3} eV^2$ but its sign is unknown. It leads to two possible mass hierarchies for neutrinos: normal hierarchy (NH) in which Δm^2_{31} is positive and $m_1 < m_2 < m_3$ and inverted hierarchy (IH) where $m_3 < m_1 < m_2$. Many experiments like INO [36–38], ICECube-PINGU [39–42] and and long baseline experiments [43, 44] has the primary objective of determining the sign of Δm^2_{31}. The values of mixing angles and mass-squared differences and δ_{CP} from the global analysis of data is summarized

Parameters	NH (3σ)	IH (3σ)
$\Delta m^2_{21}[10^{-5}eV^2]$	$6.82 \rightarrow 8.03$	$6.82 \rightarrow 8.03$
$\Delta m^2_{31}[10^{-3}eV^2]$	$2.428 \rightarrow 2.597$	$-2.581 \rightarrow -2.408$
$\sin^2\theta_{12}$	$0.270 \rightarrow 0.341$	$0.270 \rightarrow 0.341$
$\sin^2\theta_{13}$	$0.02029 \rightarrow 0.02391$	$0.02047 \rightarrow 0.02396$
$\sin^2\theta_{23}$	$0.405 \rightarrow 0.620$	$0.410 \rightarrow 0.623$
δ_{CP}	$105 \rightarrow 405$	$192 \rightarrow 361$

Table 3.1: The 3σ ranges of neutrino oscillation parameters from NuFIT 6.0 (2024) [45]

in given in Table 3.1.

To elucidate the small masses of neutrinos in comparison to charged leptons and quarks, a new mechanism involving Majorana nature of neutrinos, called the see-saw mechanism, was introduced in [46–50]. This method introduces right-handed neutrino companions with high-scale Majorana masses. In this mechanism, the right-handed partners of neutrinos are introduced with Majorana masses at a high scale. Furthermore, the neutrinos have Dirac masses of the order of charged lepton masses. Also the concept of modular symmetry [22, 51, 52] comes up in theoretical physics when discussing particle physics, particularly when attempting to explain specific regularities or patterns that are seen in the fundamental particles and their interactions. The idea behind modular symmetry in the neutrino mass matrix is to impose constraints on the matrix elements based on these modular transformations. These constraints might lead to specific patterns or relationships among the elements of the matrix, which in turn could potentially explain some of the observed features of neutrino masses and mixings. In the context of modular symmetry, various models and frameworks have been investigated to account for the neutrino mixing angles, mass hierarchies, and CP violation in the neutrino sector that have been observed. These attempts often involve introducing additional symmetries or structures beyond the Standard Model of particle physics to account for the specific patterns observed in neutrino oscillation experiments. There are a few modular symmetries in neutrino models. For example, A_4 modular symmetry predicts specific two-zero textures in the neutrino mass matrix, which can explain the observed hierarchy of neutrino masses and S_4 modular symmetry can be used to generate Dirac neutrino masses at the one-loop level, offering an alternative to the standard seesaw mechanism.

In addition, there are some other frameworks beyond the standard model (BSM) that can explain the origin of neutrino masses, for example, Supersymmetry [53], Minimal Supersymmetric Standard Model (MSSM) [54], Minimal seesaw model[55], Inverse seesaw model[56], Next-to-Minimal Supersymmetric Standard Model (NMSSM) [57], String theory [58], models based on extra dimensions [59], Radiative Seesaw Mechanism [60, 61] and also some other models. And also various models based on non-abelian discrete flavor symmetries [62] like A_4 [63–71], S_3 [72], S_4 [73–78], Δ_{27} [79–82], Δ_{54} [19, 83, 84] etc. have been proposed to obtain tribimaximal mixing (TBM) and deviation from TBM.

The mixing between the neutrino flavor eigenstates and their mass eigenstates is encoded by the commonly used PMNS matrix. This PMNS matrix is parameterized in a three-flavored paradigm using three mixing angles and three CP phases as given below:

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \cdot U_{Maj} \quad (3.2)$$

where, $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$. The diagonal matrix $U_{Maj} = diag(1, e^{i\alpha}, e^{i(\beta+\gamma)})$ contains the Majorana CP phases, α, β which become observable in case the neutrinos behave as Majorana particles. Identifying neutrinoless double beta decay will probably be necessary to prove that neutrinos are Majorana particles. Such decays have not yet been seen. Here, symmetry will play an important role in explaining these problems. In order to account for the fact that neutrino mass is zero within the standard model (SM) [85], it becomes necessary to develop a new framework that goes beyond the standard model. This entails incorporating a new symmetry and creating a mechanism that generates non-zero masses for neutrinos.

In this study, we put forward a model for neutrino masses to provide an explanation for the observed non-zero value of θ_{13}, as well as the existing data on neutrino masses and mixings. To get the deviation from exact TBM neutrino mixing pattern, we have extended the flavon sector of Altarelli-Feruglio (A-F) [86, 87] model by introducing extra flavons ξ', ξ'' and ρ which transform as $1''$, $1'$, and 1 respectively under A_4. Here, type-I see-saw framework [88, 89] is utilized to construct the model. Also, we incorporated a $Z_2 \times Z_3$ symmetry as well, which serves the purpose of avoiding undesired terms.

This model is constructed within the Type-I see-saw framework, in which the final form of the light neutrino mass matrix depends only on the non-trivial structure of the Dirac mass matrix. Moreover, the Dirac mass term derived from the anti-symmetric part arising from the product of two A_4 triplets [90] is considered to obtain non-zero θ_{13}. In this context, it's important to observe that several other models [91–93] exist where deviations from the Tri-bimaximal mixing (TBM) can occur by introducing extra flavon fields. However, in those studies, the anti-symmetric component either vanishes because of the Majorana nature of the mass terms or is not taken into account. In brief, thus this work distinguishes from others.

The content material of our work is organized as follows: In section 2, we give the overview of the framework of our model by specifying the fields involved and their transformation properties under the symmetries imposed. In section 3, we do a numerical analysis and study the results for the neutrino phenomenology. We finally conclude our work in section 4.

3.2 Framework of the Model

Here we provide a concise overview of the ways in which the non-Abelian discrete symmetry A_4 group can be represented [87, 94]. A_4 is a group of even permutations of four objects and it has 12 elements ($12= \frac{4!}{2}$). The group known as the tetrahedron group, or sometimes referred to as the group of orientation-preserving symmetries of a regular tetrahedron, is characterized by its ability to describe the symmetries of this particular geometric shape. This can be generated by two basic permutations S and T having properties $S^2 = T^3 = (ST)^3 = 1$. This group representations of A_4 include three one-dimensional unitary representations 1, $1'$, $1''$ with the generators S and T given, respectively as follows:

$$1 : S = 1, T = 1$$

$$1' : S = 1, T = \omega^2$$

$$1'' : S = 1, T = \omega$$

and a three-dimensional unitary representation with the generators[1]

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \tag{3.3}$$

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \tag{3.4}$$

. Here ω is the cubic root of unity, $\omega = exp(i2\pi)$, so that $1 + \omega + \omega^2 = 0$.
The multiplication rules corresponding to the specific basis of two generators S and T are as follows:

$$1 \times 1 = 1$$

$$1'' \times 1' = 1$$

$$1' \times \; 1'' = 1$$

$$3 \times 3 = 3 + 3_A + 1 + 1' + 1''$$

[1]Here the generator T has been chosen to be diagonal

Field	l	e^c	μ^c	τ^c	h_u	h_d	ν^c	Φ_S	Φ_T	ξ	ξ'	ξ''	ρ
SU(2)	2	1	1	1	2	2	1	1	1	1	1	1	1
A_4	3	1	$1''$	$1'$	1	1	3	3	3	1	$1''$	$1'$	1
Z_2	1	1	1	1	1	1	1	1	1	1	1	1	-1
Z_3	ω^2	ω	ω	ω	1	1	1	ω	1	ω	ω	ω	ω^2

Table 3.2: Full particle content of our model

For two triplets

$$a = (a_1, a_2, a_3)$$

$$b = (b_1, b_2, b_3)$$

we can write

$$1 \equiv (ab) = a_1b_1 + a_2b_3 + a_3b_2$$

$$1' \equiv (ab)' = a_3b_3 + a_1b_2 + a_2b_1$$

$$1'' \equiv (ab)'' = a_2b_2 + a_1b_3 + a_3b_1$$

Here 1 is symmetric under the exchange of the second and third elements of a and b, $1'$ is symmetric under the exchange of the first and second elements while $1''$ is symmetric under the exchange of the first and third elements.

$$3 \equiv (ab)_S = \frac{1}{3}(2a_1b_1 - a_2b_3 - a_3b_2, 2a_3b_3 - a_1b_2 - a_2b_1, 2a_2b_2 - a_1b_3 - a_3b_1)$$

$$3_A \equiv (ab)_A = \frac{1}{3}(a_2b_3 - a_3b_2, a_1b_2 - a_2b_1, a_1b_3 - a_3b_1)$$

Here 3 is symmetric and 3_A is anti-symmetric. For the symmetric case, we notice that the first element has 2-3 exchange symmetry, the second element has 1-2 exchange symmetry and the third element has 1-3 exchange symmetry.

The particle content and their charge assignment under the symmetry group is given in Table 3. The left-handed lepton doublets l and right-handed charged leptons (e^c, μ^c, τ^c) are assigned to triplet and singlet $(1, 1'', 1')$ representation under A_4 respectively and other particles transform as shown in Table-II. Here,

h_u and h_d are the standard Higgs doublets which remain invariant under A_4. The right-handed neutrino field ν^c is assigned to the triplet representation under A_4 flavor symmetry. There are six $SU(2) \otimes U_Y(1)$ Higgs singlets, four (ξ, ξ', ξ'' and ρ) of which singlets under A_4 and two (Φ_T and Φ_S) of which transform as triplets.

Consequently, the invariant Yukawa Lagrangian is as follows:

$$-\mathcal{L} = \frac{y_e}{\Lambda}(l\Phi_T)_1 h_d e^c + \frac{y_\mu}{\Lambda}(l\Phi_T)_{1'} h_d \mu^c + \frac{y_\tau}{\Lambda}(l\Phi_T)_{1''} h_d \tau^c + \frac{y_1}{\Lambda}(\xi)_1 (lh_u\nu^c)_1 + \frac{y_2}{\Lambda}(\xi')_{1''}(lh_u\nu^c)_{1'} + \frac{y_3}{\Lambda}(\xi'')_{1'}(lh_u\nu^c)_{1''} + y_4(\frac{\rho\rho}{\Lambda^2})_1(lh_u\nu^c)_1 + \frac{y_a}{\Lambda}\Phi_S(lh_u\nu^c)_A + \frac{y_b}{\Lambda}\Phi_S(lh_u\nu^c)_S + \frac{1}{2}M_N(\nu^c\nu^c) + h.c. \tag{3.5}$$

The terms y_e, y_μ, y_τ, y_1, y_2, y_3, y_4, y_a and y_b are coupling constant and Λ is the cut-off scale of the theorys. We assume Φ_T does not couple to the Majorana mass matrix and Φ_S does not couple to the charged leptons. After spontaneous symmetry breaking of flavour and electroweak symmetry, we obtain the mass matrices for the charged leptons and neutrinos. We assume the vacuum alignment of $\langle\Phi_T\rangle = (v_T, 0, 0)$ and $\langle\Phi_S\rangle = (v_s, v_s, v_s)$. Also, v_u, v_d are the VEVs of $\langle h_u\rangle$, $\langle h_d\rangle$ and u_1, u_2, u_3, u_4 are the VEVs of $\langle\xi_1\rangle$, $\langle\xi_2\rangle$, $\langle\xi_3\rangle$,$\langle\rho\rangle$ respectively.

The VEV pattern of the A_4 triplets, which is considered in our model, has been thoroughly examined in numerous A_4 models like [87, 95].

The charged lepton mass matrix is given as

$$M_l = \frac{v_d v_T}{\Lambda}\begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} \tag{3.6}$$

where, v_d and v_T are the VEVs of h_d and Φ_T respectively.

The structure of the Majorana neutrino mass matrix:

$$M_R = \begin{pmatrix} M_N & 0 & 0 \\ 0 & 0 & M_N \\ 0 & M_N & 0 \end{pmatrix} \tag{3.7}$$

The form of the Dirac mass matrix:

$$M_D = \begin{pmatrix} \frac{2b}{3}+c+f & -\frac{a}{3}-\frac{b}{3}+d & -\frac{a}{3}-\frac{b}{3}+e \\ \frac{a}{3}-\frac{b}{3}+d & \frac{2b}{3}+e & -\frac{a}{3}-\frac{b}{3}+c+f \\ \frac{a}{3}-\frac{b}{3}+e & \frac{a}{3}-\frac{b}{3}+c+f & \frac{2b}{3}+d \end{pmatrix} \tag{3.8}$$

Where, $a = \frac{y_a v_u v_s}{\Lambda}$, $b = \frac{y_b v_u v_s}{\Lambda}$, $c = \frac{y_1 v_u u_1}{\Lambda}$, $d = \frac{y_2 v_u u_2}{\Lambda}$, $e = \frac{y_3 v_u u_3}{\Lambda}$ and $f = \frac{y_4 v_u u_4}{\Lambda^2}$.

The Type-I seesaw method is used to determine the effective neutrino mass matrix $m_\nu = M_D^T M_R^{-1} M_D$

$$m_\nu = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{12} & m_{22} & m_{23} \\ m_{13} & m_{23} & m_{33} \end{pmatrix} \tag{3.9}$$

Where,

$m_{11} = \frac{1}{M_N}[2(\frac{a}{3}-\frac{b}{3}+d)(\frac{a}{3}-\frac{b}{3}+e)+(\frac{2b}{3}+c+f)^2]$

$m_{12} = m_{21} = \frac{1}{M_N}[(\frac{a}{3}-\frac{b}{3}+e)(\frac{2b}{3}+e)+(\frac{a}{3}-\frac{b}{3}+d)(\frac{a}{3}-\frac{b}{3}+c+f)+(-\frac{a}{3}-\frac{b}{3}+d)(\frac{2b}{3}+c+f)]$

$m_{13} = m_{31} = \frac{1}{M_N}[(\frac{a}{3}-\frac{b}{3}+d)(\frac{2b}{3}+d)+(\frac{a}{3}-\frac{b}{3}+e)(-\frac{a}{3}-\frac{b}{3}+c+f)+(-\frac{a}{3}-\frac{b}{3}+e)(\frac{2b}{3}+c+f)]$

$m_{22} = \frac{1}{M_N}[(-\frac{a}{3}-\frac{b}{3}+d)^2+2(\frac{2b}{3}+e)(\frac{a}{3}-\frac{b}{3}+c+f)]$

$m_{23} = m_{32} = \frac{1}{M_N}[(-\frac{a}{3}-\frac{b}{3}+d)(-\frac{a}{3}-\frac{b}{3}+e)+(\frac{2b}{3}+d)(\frac{2b}{3}+e)+(-\frac{a}{3}-\frac{b}{3}+c+f)(\frac{a}{3}-\frac{b}{3}+c+f)]$

$m_{33} = \frac{1}{M_N}[(-\frac{a}{3}-\frac{b}{3}+e)^2+2(\frac{2b}{3}+d)(-\frac{a}{3}-\frac{b}{3}+c+f)]$

To explain the smallness of active neutrino masses, we consider the heavy neutrino with masses $M_N \approx 10^{14}$ GeV. We can assume $c \simeq d \simeq e$. This is a reasonable assumption to make since the phenomenology does not change drastically unless the VEVs of the singlet Higgs vary by a huge amount [91, 95]. Thus, the neutrino mass matrix becomes with new matrix elements:

$m'_{11} = \frac{1}{9M_N}[2(a^2 - 2ab + 3b^2 + 6(a+b)c + 27c^2)]$

$m'_{12} = m'_{21} = \frac{1}{9M_N}[a^2 - 2a(b-3c) - 3(b-3c)(b+5c)]$

$m'_{13} = m'_{31} = \frac{1}{M_N}[-a^2 + 3(b-3c)(b+5c)]$

$m'_{22} = \frac{1}{9M_N}[a^2 + 6ab - 3b^2 + 12bc + 45c^2]$

$m'_{23} = m'_{32} = \frac{1}{9M_N}[2b(a+3b) - 6(a+b)c + 54c^2]$

$m'_{33} = \frac{1}{9M_N}[a^2 - 3b^2 + 12bc + 45c^2 - 2a(b+6c)]$

In section 3, we give a detailed phenomenological analysis of various neutrino oscillation parameters. Further, we present a numerical study of neutrinoless double-beta decay considering the allowed parameter space of the model.

3.3 Numerical Analysis and results

The neutrino mass matrix m_ν can be diagonalized by the PMNS matrix U as

$$U^\dagger m_\nu U^* = \mathrm{diag}(m_1, m_2, m_3) \tag{3.10}$$

.

We can numerically calculate U using the relation $U^\dagger h U = \mathrm{diag}(m_1^2, m_2^2, m_3^2)$, where, $h = m_\nu m_\nu^\dagger$. The neutrino oscillation parameters θ_{12}, θ_{13}, θ_{23} and δ_{CP} can be obtained from U as

$$s_{12}^2 = \frac{|U_{12}|^2}{1-|U_{13}|^2}, \qquad s_{13}^2 = |U_{13}|^2, \qquad s_{23}^2 = \frac{|U_{23}|^2}{1-|U_{13}|^2}, \tag{3.11}$$

and δ may be given by

$$\delta = \sin^{-1}\left(\frac{8\,\mathrm{Im}(h_{12}h_{23}h_{31})}{P}\right) \tag{3.12}$$

with

$$P = (m_2^2 - m_1^2)(m_3^2 - m_2^2)(m_3^2 - m_1^2)\sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\cos\theta_{13} \tag{3.13}$$

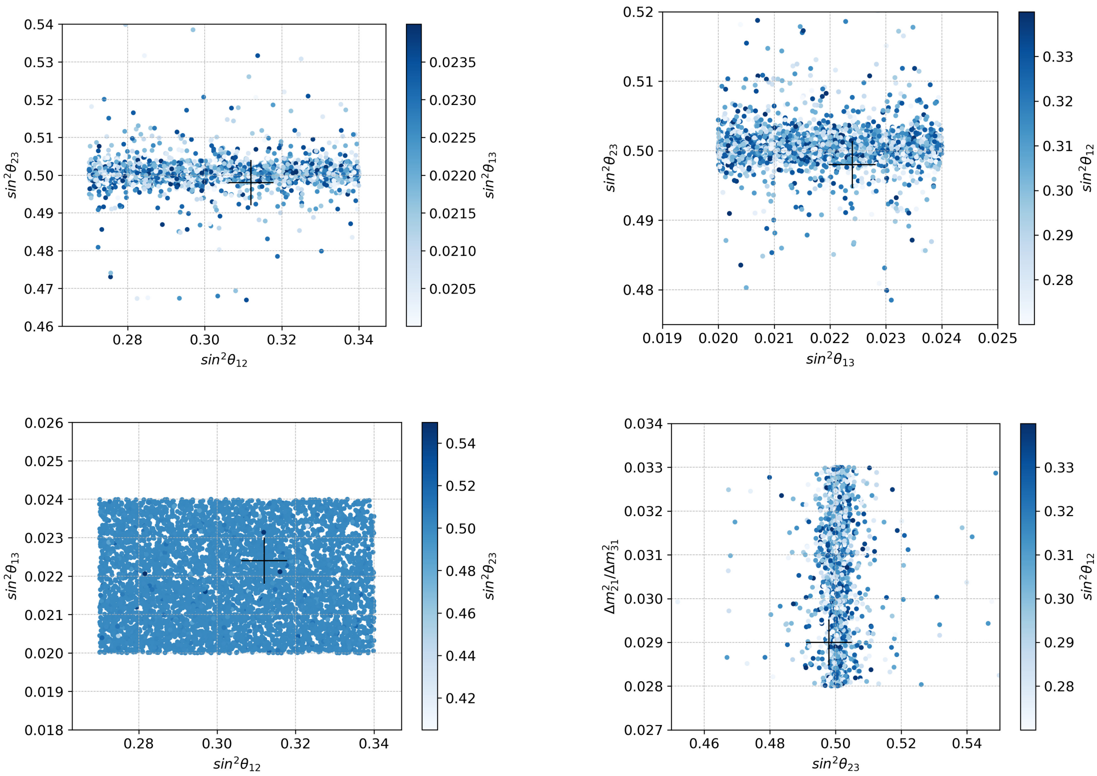

Figure 3.1: Correlation among the neutrino oscillation parameters $\sin^2\theta_{12}$, $\sin^2\theta_{23}$, $\sin^2\theta_{13}$ and ratio of mass-squared differences $\frac{\Delta m^2_{21}}{\Delta m^2_{31}}$

For the comparison of theoretical neutrino mixing parameters with the latest experimental data [45], the A_4 model is fitted to the experimental data by minimizing the following χ^2 function:

$$\chi^2 = \sum_i \left(\frac{\lambda_i^{model} - \lambda_i^{expt}}{\Delta\lambda_i} \right)^2 . \tag{3.14}$$

where λ_i^{model} is the i^{th} observable predicted by the model, λ_i^{expt} stands for the i^{th} experimental best-fit value and $\Delta\lambda_i$ is the 1σ range of the i^{th} observable.

In Fig. 3.1, correlation among the neutrino oscillation parameters $\sin^2\theta_{12}$, $\sin^2\theta_{23}$, $\frac{\Delta m^2_{21}}{\Delta m^2_{31}}$ and $\sin^2\theta_{13}$ for NH has shown, which is constrained using the 3σ bound on neutrino oscillation data. Fig. 3.2 is the correlation among the neutrino oscillation parameters $\sin^2\theta_{12}$, $\sin^2\theta_{23}$, $\sin^2\theta_{13}$, Dirac CP phase and Jarlskog parameter J for NH. We can see that there is a high correlation among different parameters of the model.

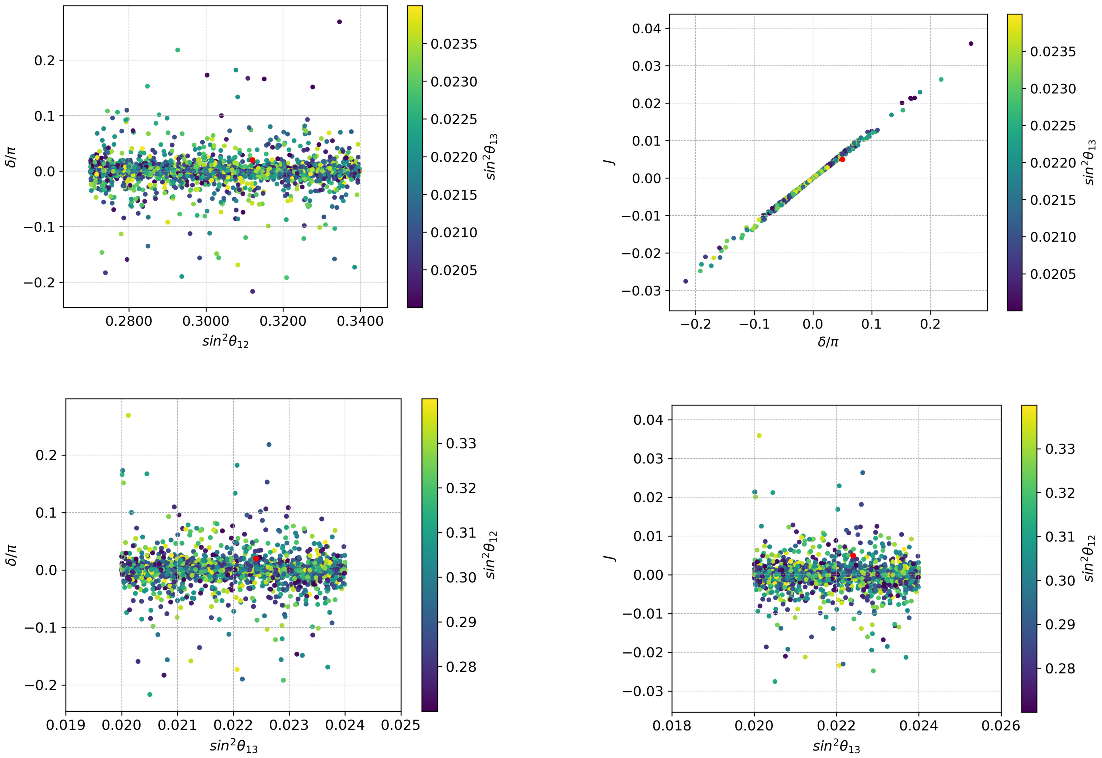

Figure 3.2: Correlation among the neutrino oscillation parameters $\sin^2\theta_{12}$, $\sin^2\theta_{23}$, $\sin^2\theta_{13}$, Dirac CP phase and Jarlskog parameter.

The calculated best-fit values of $\sin^2\theta_{12}$, $\sin^2\theta_{13}$ and $\sin^2\theta_{23}$ are (0.342, 0.0238, 0.556) which are within the 3 σ range of experimental values. Other parameters such as Δm^2_{21}, Δm^2_{31} and δ_{CP} have their best-fit values, corresponding to χ^2-minimum, at ($7.425 \times 10^{-5} eV^2$, $2.56 \times 10^{-3} eV^2$, -0.358π) respectively, which perfectly agreed with the latest observed neutrino oscillation experimental data. Thus, the model defined here clearly shows the deviation from exact tri-bimaximal mixing.

Neutrinoless double beta decay (NDBD):

Up until now, the question of whether neutrinos belong to the Dirac or Majorana category remains unanswered. If they are of the Majorana type, the investigation of Neutrinoless Double Beta Decay (NDBD) becomes highly significant. Several ongoing experiments are being conducted to ascertain the Majorana nature of neutrinos. The effective mass that controls this process is furnished by

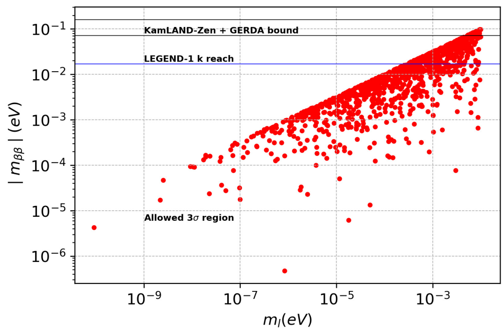

Figure 3.3: Variation of effective Majorana neutrino mass with lightest neutrino mass in NH with the KamLAND-Zen-Gerda bound on the effective mass.

$$m_{\beta\beta} = U_{Li}^2 m_i \tag{3.15}$$

where U_{Li} are the elements of the first row of the neutrino mixing matrix U_{PMNS} (Eq.3.2) which is dependent on known parameters θ_{12}, θ_{13} and the unknown Majorana phases α and β. U_{PMNS} is the diagonalizing matrix of the light neutrino mass matrix m_ν so that,

$$m_\nu = U_{PMNS} M_\nu^{(diag)} U_{PMNS}^T \tag{3.16}$$

where, $m_\nu^{(diag)}$ =diag(m_1, m_2, m_3). The effective Majorana mass can be parameterized using the diagonalizing matrix elements and the mass eigenvalues as follows:

$$m_{\beta\beta} = m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{2i\alpha} + m_3 s_{13}^2 e^{2i\beta} \tag{3.17}$$

Using the constrained parameter space we have evaluated the value of $m_{\beta\beta}$ for our model. The variation of $m_{\beta\beta}$ with the lightest neutrino mass is shown in figure 3.3. The sensitivity reach of NDBD experiments like KamLAND-Zen [96, 97], GERDA [98–100], LEGEND-1k [101] is also shown in figure 3.3. $m_{\beta\beta}$ is found to be well within the sensitivity reach of these NDBD experiments for our model.

3.4 Conclusions

We have developed a flavon-symmetric $A_4 \times Z_2 \times Z_3$ model using the seesaw type-I mechanism. This model aims to explain the recent experimental data on neutrino oscillation, which deviates from the Tribimaximal neutrino mixing pattern. The inclusion of the cyclic $Z_2 \times Z_3$ symmetric component has been done to remove undesired terms during the computations. The computed values unequivocally demonstrated that the neutrino mixing parameters deviate from the exact Tribimaximal neutrino mixing matrix. The resulting mass matrices give predictions for the neutrino oscillation parameters and their best-fit values are obtained using the chi^2 analysis, which are consistent with the latest global neutrino oscillation experimental data. In our model, we have also explored the concept of NDBD. The value of effective Majorana neutrino mass $|m_{\beta\beta}|$ is well fitted within the sensitivity reach of the recent NDBD experiments like KamLAND- Zen, GERDA, and LEGEND-1k. The identification of NDBD, cosmological mass, and the leptonic CP-violation phase δ_{CP}, which align with the most recent experimental information, will differentiate between various models of neutrino mass.

.1

Field	l	e^c	μ^c	τ^c	h_u	h_d	ν^c	Φ_S	Φ_T	ξ	ξ'	ξ''	ρ	ϕ_0^T	ϕ_0^s	ξ_0
SU(2)	2	1	1	1	2	2	1	1	1	1	1	1	1	-	-	-
A_4	3	1	$1''$	$1'$	1	1	3	3	3	$1'$	$1''$	1	1	3	3	1
Z_3	ω^2	ω	ω	ω	1	1	1	ω	1	ω	ω	ω	ω^2	1	ω	ω
Z_2	1	1	1	1	1	1	1	1	1	1	1	1	-1	1	1	1

Table 3: Full particle content of our model

The superpotential of the model with the "driving fields" ϕ_0^T, ϕ_0^S and ξ_0 that allows building the scalar potentials in the symmetry breaking sector, reads as

$$
\begin{aligned}
W =& M(\phi_0^T \phi_T) + g(\phi_0^T \phi_T \phi_T) + g_1(\phi_0^S \phi_S \phi_S) + g_2 \xi(\phi_0^S \phi_S) + g_3 \xi'(\xi_0^S \xi_S) \\
&+ g_4 \xi''(\xi_0^S \xi_S) + g_5 \xi_0(\phi_S \phi_S) + g_6 \xi_0 \xi^2 + g_7 \xi_0 \xi' \xi'' + g_8 \rho^2(\phi_0^S \phi_S)
\end{aligned} \tag{18}
$$

At this level there is no fundamental distinction among the singlets ξ, ξ' and ξ''. So, we can consider $\phi_0^S\phi_S$ is coupling with ξ only.

We use,

$$\phi_T = [\phi_T^1, \phi_T^2, \phi_T^3], \quad \phi_0^T = [\phi_{01}^T, \phi_{02}^T, \phi_{03}^T]$$

$$\phi_S = [\phi_S^1, \phi_S^2, \phi_S^3], \quad \phi_0^S = [\phi_{01}^S, \phi_{02}^S, \phi_{03}^S]$$

Now,

$$\frac{\delta W}{\delta \phi_{01}^T} = M\phi_T^1 + g.\frac{1}{3}.[2\phi_T^1\phi_T^1 - \phi_T^2\phi_T^3 - \phi_T^3\phi_T^2] = 0 \tag{19}$$
$$\frac{\delta W}{\delta \phi_{02}^T} = M\phi_T^3 + g.\frac{1}{3}.[2\phi_T^2\phi_T^2 - \phi_T^1\phi_T^3 - \phi_T^3\phi_T^1] = 0 \tag{20}$$
$$\frac{\delta W}{\delta \phi_{03}^T} = M\phi_T^2 + g.\frac{1}{3}.[2\phi_T^3\phi_T^3 - \phi_T^1\phi_T^2 - \phi_T^2\phi_T^1] = 0 \tag{21}$$

$$\frac{\delta W}{\delta \phi_{01}^S} = g_1.\frac{1}{3}[2\phi_S^1\phi_S^1 - \phi_S^2\phi_S^3 - \phi_S^3\phi_S^2] + g_2\xi\phi_S^1 = 0 \tag{22}$$
$$\frac{\delta W}{\delta \phi_{02}^S} = g_1.\frac{1}{3}[2\phi_S^2\phi_S^2 - \phi_S^1\phi_S^3 - \phi_S^3\phi_S^1] + g_2\xi\phi_S^3 = 0 \tag{23}$$
$$\frac{\delta W}{\delta \phi_{03}^S} = g_1.\frac{1}{3}[2\phi_S^3\phi_S^3 - \phi_S^1\phi_S^2 - \phi_S^2\phi_S^1] + g_2\xi\phi_S^2 = 0 \tag{24}$$

$$\frac{\delta W}{\delta \xi_0} = g_5.[\phi_S^1\phi_S^1 + \phi_S^2\phi_S^3 + \phi_S^3\phi_S^2] + g_6\xi^2 + g_7\xi'\xi'' = 0 \tag{25}$$

A solution of the first three equation is:

$$\phi_T = (v_T, 0, 0), v_T = -\frac{3M}{2g}$$

When we enforce $< \xi >= 0$, in a finite portion of the parameter space, we find the solution as:

$$< \xi >= 0, \quad < \xi' >= u', \quad < \xi'' >= u''$$

$$
\begin{aligned}
\phi_S &= (v_S, v_S, v_S) \\
v_S^2 &= -\frac{g_7 u' u''}{3 g_5}
\end{aligned}
$$

Bibliography

[1] Maki, Z. *et al.* Remarks on the unified model of elementary particles. *Prog. Theor. Phys.* **28**, 870–880, 1962.

[2] Fusaoka, H. & Katayama, Y. A unified model of elementary particles based on leptons. *Prog. Theor. Phys.* **48**, 1753–1754, 1972.

[3] Oberauer, L. & von Feilitzsch, F. Neutrino oscillations. *Rept. Prog. Phys.* **55**, 1093–1163, 1992.

[4] Forero, D. V. *et al.* Global status of neutrino oscillation parameters after Neutrino-2012. *Phys. Rev. D* **86**, 073012, 2012. 1205.4018.

[5] Aartsen, M. G. *et al.* Neutrino oscillation studies with IceCube-DeepCore. *Nucl. Phys. B* **908**, 161–177, 2016.

[6] Gonzalez-Garcia, M. C. *et al.* Global fit to three neutrino mixing: critical look at present precision. *JHEP* **12**, 123, 2012. 1209.3023.

[7] Kang, S. K. *et al.* Revisiting A_4 model for leptons in light of NuFIT 3.2. *PTEP* **2018** (8), 083B01, 2018. 1804.10468.

[8] Razzaghi, N. Neutrino mixing matrix and masses from a particular two-zero texture based on A $_4$ symmetry. *Commun. Theor. Phys.* **75** (5), 055204, 2023.

[9] Frank, M. *et al.* Dark matter in the Alternative Left Right model. *JHEP* **12**, 032, 2022. 2211.04286.

[10] Ahn, Y. H. *et al.* Confronting the prediction of leptonic Dirac CP-violating phase with experiments. *Phys. Rev. D* **106** (9), 095002, 2022. 2205.02796.

[11] Nath, A. & Francis, N. K. Detection techniques and investigation of different neutrino experiments. *Int. J. Mod. Phys. A* **36** (13), 2130008, 2021. 1804.08467.

[12] Puyam, V. *et al.* Deviation from Tribimaximal mixing using A4 flavour model with five extra scalars. *Nucl. Phys. B* **983**, 115932, 2022. 2204.10122.

[13] Kang, S. K. & Dasgupta, A. Phenomenology of minimal seesaw model with S_4 symmetry. *PoS* **ICHEP2018**, 410, 2019.

[14] Zhao, Z.-h. Breakings of the neutrino $\mu - \tau$ reflection symmetry. *JHEP* **09**, 023, 2017. 1703.04984.

[15] Buravov, L. I. Confining potential and mass of elementary particles. *J. Mod. Phys.* **7** (1), 129–133, 2017. 1502.00958.

[16] Nishi, C. C. & Silva-Marcos, J. I. Leptonic CP violation from the seesaw , 2023. 2305.08980.

[17] Hagedorn, C. *et al.* Flavour and CP symmetries in the inverse seesaw. *Eur. Phys. J. C* **82** (3), 194, 2022. 2107.07537.

[18] Thapa, B. *et al.* A minimal inverse seesaw model with S_4 flavour symmetry , 2023. 2305.09306.

[19] Bora, H. *et al.* Neutrino Mass Model in the Context of $\Delta(54) \otimes Z_2 \otimes Z_3 \otimes Z_4$ Flavor Symmetries with Inverse Seesaw Mechanism , 2023. 2305.08963.

[20] Loualidi, M. A. Trimaximal Mixing with One Texture Zero from Type II Seesaw and $\Delta(54)$ Family Symmetry. In *Beyond Standard Model: From Theory to Experiment*, 2021. 2104.13734.

[21] Khatun, A. *et al.* Three Flavor Quasi-Dirac Neutrino Mixing, Oscillations and Neutrinoless Double Beta Decay. *Symmetry* **12** (8), 1310, 2020.

[22] Okada, H. & Orikasa, Y. Neutrino mass model with a modular S_4 symmetry , 2019. 1908.08409.

[23] Bhattacharyya, S. & Datta, A. Dark matter perspective of left-right symmetric gauge model. *Nucl. Phys. B* **991**, 116197, 2023. 2206.13105.

[24] Boruah, B. B. & Das, M. K. Neutrinoless double-beta decay and lepton flavor violation in discrete flavor symmetric left–right symmetric model. *Int. J. Mod. Phys. A* **37** (06), 2250026, 2022. 2111.10341.

[25] Girardi, I. *Discrete Symmetry Approach to Lepton Flavour, Neutrino Mixing and Leptonic CP Violation, and Neutrino Related Physics Beyond the Standard Theory*. Ph.D. thesis, SISSA, Trieste, 2015.

[26] Mishra, P. *et al.* Neutrino phenomenology, W-mass anomaly, and muon (g-2) in a minimal type-III seesaw model using a T' modular symmetry. *Phys. Rev. D* **107** (11), 115004, 2023. 2302.00494.

[27] Datta, A. *et al.* Flavored leptogenesis and neutrino mass with A_4 symmetry. *JHEP* **12**, 051, 2021. 2106.06773.

[28] Kang, S. K. & Tanimoto, M. Prediction of Leptonic CP Phase in A_4 symmetric model. *Phys. Rev. D* **91** (7), 073010, 2015. 1501.07428.

[29] Ahn, Y. *et al.* Spontaneous c p violation in a 4 flavor symmetry and leptogenesis. *Physical Review D* **87** (11), 113012, 2013.

[30] Ding, G.-J. *et al.* Trimaximal neutrino mixing from scotogenic A_4 family symmetry. *Phys. Lett. B* **815**, 136122, 2021. 2009.04750.

[31] Ahn, J. K. *et al.* Observation of Reactor Electron Antineutrino Disappearance in the RENO Experiment. *Phys. Rev. Lett.* **108**, 191802, 2012. 1204.0626.

[32] Abe, Y. *et al.* Indication of Reactor $\bar{\nu}_e$ Disappearance in the Double Chooz Experiment. *Phys. Rev. Lett.* **108**, 131801, 2012. 1112.6353.

[33] An, F. P. *et al.* Observation of electron-antineutrino disappearance at Daya Bay. *Phys. Rev. Lett.* **108**, 171803, 2012. 1203.1669.

[34] Adamson, P. *et al.* Improved search for muon-neutrino to electron-neutrino oscillations in MINOS. *Phys. Rev. Lett.* **107**, 181802, 2011. 1108.0015.

[35] Abe, K. *et al.* Indication of Electron Neutrino Appearance from an Accelerator-produced Off-axis Muon Neutrino Beam. *Phys. Rev. Lett.* **107**, 041801, 2011. 1106.2822.

[36] Blennow, M. & Schwetz, T. Identifying the neutrino mass ordering with ino and nova. *Journal of High Energy Physics* **2012** (8), 1–17, 2012.

[37] Ghosh, A. *et al.* Determining the neutrino mass hierarchy with ino, t2k, nova and reactor experiments. *Journal of High Energy Physics* **2013** (4), 1–33, 2013.

[38] Ahmed, S. *et al.* Physics Potential of the ICAL detector at the India-based Neutrino Observatory (INO). *Pramana* **88** (5), 79, 2017. 1505.07380.

[39] Ribordy, M. & Smirnov, A. Y. Improving the neutrino mass hierarchy identification with inelasticity measurement in PINGU and ORCA. *Phys. Rev. D* **87** (11), 113007, 2013. 1303.0758.

[40] Quinn, L. *Determining the Neutrino Mass Hierarchy with KM3NeT/ORCA*. Ph.D. thesis, Aix-Marseille U., 2018.

[41] Jia, J. *et al.* On the Possibility to Determine Neutrino Mass Hierarchy via Supernova Neutrinos with Short-Time Characteristics. *Chin. Phys. C* **43** (9), 095102, 2019. 1709.09453.

[42] Winter, W. Neutrino mass hierarchy determination with IceCube-PINGU. *Phys. Rev. D* **88** (1), 013013, 2013. 1305.5539.

[43] Agarwalla, S. *et al.* The lbno long-baseline oscillation sensitivities with two conventional neutrino beams at different baselines. *arXiv preprint arXiv:1412.0804* , 2014.

[44] Agarwalla, S. K. Neutrino mass hierarchy in future long-baseline experiments. *Nuclear Physics B-Proceedings Supplements* **237**, 196–198, 2013.

[45] Esteban, I. *et al.* The fate of hints: updated global analysis of three-flavor neutrino oscillations. *Journal of High Energy Physics* **2020** (9), 1–22, 2020.

[46] Minkowski, P. $\mu \rightarrow$ eγ at a rate of one out of 109 muon decays? *Physics Letters B* **67** (4), 421–428, 1977.

[47] Yanagida, T. Proc. workshop on unified theory and the baryon number in the universe. *KEK Report No. 79-18* **95**, 1979.

[48] Mohapatra, R. N. & Senjanović, G. Neutrino mass and spontaneous parity nonconservation. *Physical Review Letters* **44** (14), 912, 1980.

[49] Gell-Mann, M. *et al.* Supergravity ed p van nieuwenhuizen and dz freedman. *Amsterdam: North-Holland) p* **315**, 79–18, 1979.

[50] Achiman, Y. & Stech, B. Quark-lepton symmetry and mass scales in an e6 unified gauge model. *Physics Letters B* **77** (4-5), 389–393, 1978.

[51] Ding, G.-J. & King, S. F. Neutrino Mass and Mixing with Modular Symmetry , 2023. `2311.09282`.

[52] Nomura, T. & Okada, H. A two loop induced neutrino mass model with modular A_4 symmetry. *Nucl. Phys. B* **966**, 115372, 2021. `1906.03927`.

[53] Ma, E. Supersymmetry and neutrino masses. *arXiv preprint hep-ph/9902450* , 1999.

[54] Csaki, C. The Minimal supersymmetric standard model (MSSM). *Mod. Phys. Lett. A* **11**, 599, 1996. `hep-ph/9606414`.

[55] Kang, S. K. Low-energy cp violation and leptogenesis in a minimal seesaw model. *Journal of the Korean Physical Society* **78**, 743–749, 2021.

[56] Hue, L. T. *et al.* An explanation of experimental data of $(g-2)_{e,\mu}$ in 3-3-1 models with inverse seesaw neutrinos. *Eur. Phys. J. C* **82** (8), 722, 2022. 2109.06089.

[57] Ellwanger, U. *et al.* The Next-to-Minimal Supersymmetric Standard Model. *Phys. Rept.* **496**, 1–77, 2010. 0910.1785.

[58] Ibanez, L. E. & Uranga, A. M. *String theory and particle physics: An introduction to string phenomenology*, Cambridge University Press, 2012.

[59] Arkani-Hamed, N. *et al.* Neutrino masses from large extra dimensions. *Phys. Rev. D* **65**, 024032, 2001. hep-ph/9811448.

[60] Thapa, B. & Francis, N. K. Connecting low-energy CP violation, resonant leptogenesis and neutrinoless double beta decay in a radiative seesaw model. *Nucl. Phys. B* **986**, 116054, 2023. 2209.06263.

[61] Ma, E. Verifiable radiative seesaw mechanism of neutrino mass and dark matter. *Phys. Rev. D* **73**, 077301, 2006. hep-ph/0601225.

[62] King, S. F. & Luhn, C. Neutrino Mass and Mixing with Discrete Symmetry. *Rept. Prog. Phys.* **76**, 056201, 2013. 1301.1340.

[63] Ishimori, H. *et al.* Non-Abelian Discrete Symmetries in Particle Physics. *Prog. Theor. Phys. Suppl.* **183**, 1–163, 2010. 1003.3552.

[64] Shimizu, Y. *et al.* Breaking tri-bimaximal mixing and large $\theta 13$. *Progress of Theoretical Physics* **126** (1), 81–90, 2011.

[65] King, S. F. & Luhn, C. Trimaximal neutrino mixing from vacuum alignment in a 4 and s 4 models. *Journal of High Energy Physics* **2011** (9), 1–25, 2011.

[66] Nguyen, T. P. *et al.* Low-energy phenomena of the lepton sector in an A4 symmetry model with heavy inverse seesaw neutrinos. *PTEP* **2022** (2), 023B01, 2022. 2011.12181.

[67] Ganguly, J. *et al.* Common origin of θ_{13} and dark matter within the flavor symmetric scoto-seesaw framework. *JHEP* **11**, 074, 2022. 2209.08610.

[68] Behera, M. K. *et al.* Implications of A4 modular symmetry on neutrino mass, mixing and leptogenesis with linear seesaw. *Phys. Dark Univ.* **36**, 101027, 2022. 2007.00545.

[69] Vien, V. V. & Long, H. N. A $_4$-based model with linear seesaw scheme for lepton mass and mixing. *Phys. Scripta* **98** (1), 015301, 2023. 2302.03195.

[70] Ma, E. Tribimaximal neutrino mixing from a supersymmetric model with A4 family symmetry. *Phys. Rev. D* **73**, 057304, 2006. hep-ph/0511133.

[71] Ma, E. Neutrino mixing: A_4 variations. *Phys. Lett. B* **752**, 198–200, 2016. 1510.02501.

[72] Ma, E. Non-abelian discrete symmetries and neutrino masses: two examples. *arXiv preprint hep-ph/0405152* , 2004.

[73] Bazzocchi, F. & Merlo, L. Neutrino Mixings and the S4 Discrete Flavour Symmetry. *Fortsch. Phys.* **61**, 571–596, 2013. 1205.5135.

[74] Vien, V. V. & Long, H. N. Multiscalar $B - L$ extension based on S_4 flavor symmetry for neutrino masses and mixing. *Chin. Phys. C* **45** (4), 043112, 2021. 2012.01715.

[75] Thapa, B. & Francis, N. K. Resonant leptogenesis and TM_1 mixing in minimal type-I seesaw model with S_4 symmetry. *Eur. Phys. J. C* **81** (12), 1061, 2021. 2107.02074.

[76] Ma, E. Tetrahedral family symmetry and the neutrino mixing matrix. *Mod. Phys. Lett. A* **20**, 2601–2606, 2005. hep-ph/0508099.

[77] Dev, A. *et al.* Non-zero θ 13 and δ cp in a neutrino mass model with a4 symmetry. *Journal of High Energy Physics* **2015** (11), 1–15, 2015.

[78] Grossman, Y. & Ng, W. H. Nonzero θ_{13} in $SO(3) \rightarrow A_4$ lepton models. *Phys. Rev. D* **91** (7), 073005, 2015. 1404.1413.

[79] Ma, E. Near tribimaximal neutrino mixing with Delta(27) symmetry. *Phys. Lett. B* **660**, 505–507, 2008. 0709.0507.

[80] Hernández, A. C. *et al.* A 3-3-1 model with right-handed neutrinos based on the\vardelta\left (27\right) δ 27 family symmetry. *The European Physical Journal C* **76**, 1–17, 2016.

[81] de Medeiros Varzielas, I. *et al.* Neutrino tri-bi-maximal mixing from a non-abelian discrete family symmetry. *Physics Letters B* **648** (2-3), 201–206, 2007.

[82] Cárcamo Hernández, A. E. & de Medeiros Varzielas, I. $\Delta(27)$ framework for cobimaximal neutrino mixing models. *Phys. Lett. B* **806**, 135491, 2020. 2003.01134.

[83] Ishimori, H. *et al.* Lepton flavor model from δ (54) symmetry. *Journal of High Energy Physics* **2009** (04), 011, 2009.

[84] Carballo-Pérez, B. *et al.* δ (54) flavor phenomenology and strings. *Journal of High Energy Physics* **2016** (12), 1–24, 2016.

[85] Liu, J.-H. *et al.* Majorana Neutrino Masses from Neutrinoless Double-Beta Decays and Lepton-Number-Violating Meson Decays. *Phys. Lett. B* **760**, 571–576, 2016. 1606.04886.

[86] Altarelli, G. & Feruglio, F. Tri-bimaximal neutrino mixing, A(4) and the modular symmetry. *Nucl. Phys. B* **741**, 215–235, 2006. hep-ph/0512103.

[87] Altarelli, G. & Feruglio, F. Discrete Flavor Symmetries and Models of Neutrino Mixing. *Rev. Mod. Phys.* **82**, 2701–2729, 2010. 1002.0211.

[88] Borah, D. & Karmakar, B. A_4 flavour model for Dirac neutrinos: Type I and inverse seesaw. *Phys. Lett. B* **780**, 461–470, 2018. 1712.06407.

[89] Crivellin, A. *et al.* Comprehensive analysis of charged lepton flavour violation in the symmetry protected type-I seesaw. *JHEP* **12**, 031, 2022. 2208.00020.

[90] Memenga, N. *et al.* A_4 flavor symmetry model for Dirac neutrinos and sizable U_{e3}. *Phys. Rev. D* **87** (5), 053021, 2013. 1301.2963.

[91] Brahmachari, B. *et al.* The A(4) flavor symmetry and neutrino phenomenology. *Phys. Rev. D* **77**, 073008, 2008. [Erratum: Phys.Rev.D 77, 119901 (2008)], 0801.3554.

[92] Shimizu, Y. *et al.* Breaking Tri-bimaximal Mixing and Large θ_{13}. *Prog. Theor. Phys.* **126**, 81–90, 2011. 1105.2929.

[93] He, X.-G. & Zee, A. Minimal modification to the tri-bimaximal neutrino mixing. *Phys. Lett. B* **645**, 427–431, 2007. hep-ph/0607163.

[94] Kobayashi, T. *et al.* *An Introduction to Non-Abelian Discrete Symmetries for Particle Physicists*, Springer, 2022.

[95] Barman, A. *et al.* Nonzero $\theta 13$, CP-violation and neutrinoless double beta decay for neutrino mixing in the A4×Z2×Z3 flavor symmetry model. *Int. J. Mod. Phys. A* **38** (02), 2350012, 2023. 2203.05536.

[96] Gando, Y. First results of KamLAND-Zen 800. *J. Phys. Conf. Ser.* **1468** (1), 012142, 2020.

[97] Biller, S. D. Combined constraints on Majorana masses from neutrinoless double beta decay experiments. *Phys. Rev. D* **104** (1), 012002, 2021. 2103.06036.

[98] Di Marco, N. Searching for neutrinoless double-beta decay with GERDA. *Nucl. Instrum. Meth. A* **958**, 162112, 2020.

[99] Agostini, M. *et al.* Searching for neutrinoless double beta decay with GERDA. *J. Phys. Conf. Ser.* **1342** (1), 012005, 2020. 1710.07776.

[100] D'Andrea, V. Neutrinoless double beta decay search with the GERDA experiment. *Nuovo Cim. C* **43** (2-3), 24, 2020.

[101] Abgrall, N. *et al.* Legend-1000 preconceptual design report. *arXiv preprint arXiv:2107.11462* , 2021.